2018 秋季本科时间序列

第三次作业答案

2018 年 10 月 26 日

1.

$$\begin{aligned}
\mathbb{E}X_t &= \int_{-\pi}^{\pi} \frac{1}{2\pi}\cos(\frac{t}{2\pi}+u)\mathrm{d}u \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi}[\cos(\frac{t}{2\pi})\cos(u)-\sin(\frac{t}{2\pi})\sin(u)]\mathrm{d}u \\
&= \frac{1}{2\pi}[\cos(\frac{t}{2\pi})\int_{-\pi}^{\pi}\cos(u)\mathrm{d}u-\sin(\frac{t}{2\pi})\int_{-\pi}^{\pi}\sin(u)\mathrm{d}u)] \\
&= 0
\end{aligned}$$

且

$$\mathbb{E}X_{t+k} = \mathbb{E}X_t = 0$$

故有

$$\begin{aligned}
\mathrm{cov}(X_{t+k}, X_t) &= \mathbb{E}[X_{t+k}, X_t] \\
&= \mathbb{E}[(\cos(\frac{t}{2\pi}+U+\frac{k}{2\pi})(\cos(\frac{t}{2\pi}+U))] \\
&= \frac{1}{2}\mathbb{E}[\cos(\frac{1}{2\pi}(2t+k)+2U)+\cos(\frac{1}{2\pi}k)] \\
&= \frac{1}{2}\frac{1}{2\pi}\underbrace{\int_{-\pi}^{\pi}\cos(\frac{1}{2\pi}(2t+k)+2u)\mathrm{d}u}_{=0}+\frac{1}{2}\cos(\frac{1}{2\pi}k) \\
&= \frac{1}{2}\cos(\frac{k}{2\pi})
\end{aligned}$$

2. 协方差具有双线性, 且 $\mathrm{cov}(\varepsilon_i, \varepsilon_j) = 0$ 当 $i \neq j$：

$$\begin{aligned}
\mathrm{cov}(a_1\varepsilon_1 + \cdots + a_T\varepsilon_T, b_1\varepsilon_1 + \cdots + b_T\varepsilon_T) &= \sum_{i=1}^{T} \mathrm{cov}(a_i\varepsilon_i, b_1\varepsilon_1 + \cdots + b_T\varepsilon_T) \\
&= \sum_{i=1}^{T} a_i \sum_{j=1}^{T} \mathrm{cov}(\varepsilon_i, b_j\varepsilon_j) \\
&= \sum_{i=1}^{T} a_i b_i \mathrm{cov}(\varepsilon_i, \varepsilon_i) \\
&= \sum_{i=1}^{T} a_i b_i \mathrm{var}(\varepsilon_i)
\end{aligned}$$

当 $a_i = 1, b_I = 1$, 对任意 $i, j = 1 \cdots T$

$$\begin{aligned}
\mathrm{var}(\varepsilon_1 + \cdots \varepsilon_T) &= \mathrm{cov}(\varepsilon_1 + \cdots \varepsilon_T) \\
&= \sum_{i=1}^{T} a_i b_i \mathrm{var}(\varepsilon_i) \\
&= \sum_{i=1}^{T} \mathrm{var}(\varepsilon_i)
\end{aligned}$$