Optimal Automatic Stabilizers[*]

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Abstract

Should the generosity of unemployment benefits and the progressivity of income taxes depend on the presence of business cycles? This paper proposes a tractable model where there is a role for social insurance against uninsurable shocks to income and unemployment, as well as business cycles that are inefficient due to the presence of matching frictions and nominal rigidities. We derive an augmented Baily-Chetty formula showing that the optimal generosity of the social insurance system depends on a macroeconomic stabilization term. This term pushes for an increase in generosity when the level of economic activity is more responsive to social programs in recessions than in booms. A calibration to the U.S. economy shows that taking concerns for macroeconomic stabilization into account substantially raises the optimal unemployment insurance replacement rate, but has a negligible impact on the optimal progressivity of the income tax. More generally, the role of social insurance programs as automatic stabilizers affects their optimal design.

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1 Introduction

The usual motivation behind large social welfare programs, like unemployment insurance or progressive income taxation, is to provide social insurance and engage in redistribution. An extensive literature therefore studies the optimal progressivity of income taxes, typically by weighing the disincentive effect on individual labor supply and savings against concerns for redistribution and for insurance against idiosyncratic income shocks.[1] In turn, the optimal generosity of unemployment benefits is often stated in terms of a Baily-Chetty formula, which weighs the moral hazard effect of unemployment insurance on job search and creation against the social insurance benefits that it provides.[2]

For the most part, this literature abstracts from aggregate shocks, so that the optimal generosity and progressivity do not take into account business cycles. Yet, from their inception, an auxiliary justification for these social programs was that they were also supposed to automatically stabilize the business cycle.[3] The classic work that focused on the automatic stabilizers relied on a Keynesian tradition that ignored the social insurance that these programs provide and their disincentive effects on employment. Some recent work brings these two literatures together, but so far it has focused on the positive effects of the automatic stabilizers, falling short of computing optimal policies.[4]

The goal of this paper is to answer two classic questions—How generous should unemployment benefits be? How progressive should income taxes be?—but taking into account their automatic stabilizer benefits as well as their social insurance benefits. We present a model in which there is both a role for social insurance as well as aggregate shocks and inefficient business cycles. We introduce unemployment insurance and progressive income taxes as automatic stabilizers, that is, programs that do not directly depend on the aggregate state of the economy, even if the aggregate size of the programs changes with the composition of income in the economy. We then solve for the ex ante socially optimal replacement rate of unemployment benefits and progressivity of personal income taxes in the presence of uninsured income risks, precautionary savings motives, labor market frictions, and nominal rigidities.

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[1]Mirrlees (1971) and Varian (1980) are classic references, and more recently see Benabou (2002), Conesa and Krueger (2006), Heathcote et al. (2017), Krueger and Ludwig (2013), and Golosov et al. (2016).

[2]See the classic work by Baily (1978) and Chetty (2006).

[3]Musgrave and Miller (1948) and Auerbach and Feenberg (2000) are classic references, while Blanchard et al. (2010) is a recent call for more modern work in this topic.

[4]See McKay and Reis (2016) for a recent model, DiMaggio and Kermani (2016) for recent empirical work, and IMF (2015) for the shortcomings of the older literature.

Our first contribution is to provide a new, theoretical definition of an automatic stabilizer. We present necessary conditions for optimal policy that capture the trade-offs between insurance, incentives, and macroeconomic stabilization. In the case of unemployment insurance, this condition is a variant of the Baily-Chetty formula for unemployment insurance but augmented by a new macroeconomic stabilization term. This term equals the expectation of the product of the welfare gain from raising the level of activity and the elasticity of activity with respect to the unemployment benefits. Our optimality condition for progressivity has the same structure and an analogous macroeconomic stabilization term. Even if the economy is efficient on average, economic fluctuations may lead to more generous unemployment insurance or more progressive income taxes, relative to standard analyses that ignore the automatic stabilizer properties of these programs. This term captures the automatic stabilizer nature of social insurance programs.

The second contribution is to characterize this macroeconomic stabilization term to understand the different economic mechanisms behind it. Fluctuations in aggregate economic activity can lead to welfare losses through four separate channels. First, they may create a wedge between the marginal disutility of hours worked and the social benefit of work. This inefficiency appears in standard models of inefficient business cycles, and is sometimes described as a result of time-varying markups (Chari et al., 2007; Galí et al., 2007). Second, a tighter labor market raises the level of employment but also raises recruiting costs. The equilibrium level of unemployment need not be efficient as hiring and search decisions do not necessarily internalize these trade-offs. This is the source of inefficiency common to search models (e.g. Hosios, 1990). Third, the state of the business cycle alters the extent of uninsurable risk that households face. This is the source of welfare costs of business cycles that has been studied by Storesletten et al. (2001), Krebs (2003, 2007), and De Santis (2007). Finally, with nominal rigidities, inflation dynamics lead to dispersion in relative prices, as emphasized by the new Keynesian business cycle literature (Woodford, 2010; Gali, 2011). Our measure isolates these four effects into separate additive terms in the condition determining the optimal extent of the social insurance programs.

As for the elasticity of activity with respect to social programs, unemployment benefits and progressive taxes can stabilize the economy even if these policies are themselves not responsive to the business cycle. For one, these policies mitigate precautionary savings motives by providing social insurance. Because the risk in pre-tax incomes rises in a recession, the effect of this social

insurance on aggregate demand rises as well, so these policies stabilize aggregate demand. We further show that if prices are flexible so aggregate demand matters little, or if monetary policy aggressively stabilizes the business cycle, then little role is left for the social programs to work as stabilizers.

Our third contribution is to investigate the magnitude of the macroeconomic stabilization term and the key mechanisms behind it. We calculate the optimal unemployment replacement rate and tax progressivity and we compare these values to what one would find in the absence of aggregate risk. We find that macroeconomic stabilization considerations imply substantially more generous benefits when there is aggregate risk. On the other hand, we find that optimal tax progressivity is approximately independent of the presence of aggregate risks.

The analytical results provide an interpretation of these numerical results by quantifying the trade-offs between incentives, social insurance, and macroeconomic stabilization, as well as the constituent mechanisms of the macroeconomic stabilization term. This highlights the usefulness of the propositions for isolating the key forces at hand. Quantitatively, the automatic stabilizer term is large in the case of unemployment benefits because of the interaction between two forces. First, unemployment benefits stabilize the business cycle by dampening the destabilizing feedback loop between unemployment fears, precautionary savings, and aggregate demand. Second, stabilizing the business cycle is important in welfare terms because recessions lead to concentrated and long-lasting losses for individuals through the cyclicality of uninsurable idiosyncratic risk. More progressive taxes, however, have a small stabilization benefit in comparison to the incentive costs, which is why the results suggest little role for tax progressivity as an automatic stabilizer.

There are large literatures on the three topics that we touch on: business cycle models with incomplete markets and nominal rigidities, social insurance and public programs, and automatic stabilizers. Our model of aggregate demand has some of the key features of new Keynesian models with labor markets frictions (Gali, 2011) but that literature focuses on optimal monetary policy, whereas we study the optimal design of the social insurance system. Our model of incomplete markets builds on McKay and Reis (2016), Ravn and Sterk (2017), and Heathcote et al. (2017) to generate a tractable model of incomplete markets and automatic stabilizers, where a rich distribution of consumption and income across households co-exists with a degenerate wealth distribution. The model's simplicity allows us to analytically express optimality conditions for generosity and

progressivity, and to, even in a more general case, easily solve the model numerically and so be able to search for the optimal policies. Finally, our paper is part of a surge of work on the interplay of nominal rigidities and precautionary savings, but this literature has mostly been positive whereas this paper's focus is on optimal policy.[5] We emphasize the effect of automatic stabilizers on precautionary savings, whereas other discussions have instead highlighted the macroeconomic transmission of fiscal transfers through high and heterogeneous marginal propensities to consume.

On the generosity of unemployment insurance, our work is closest to Landais et al. (2018) and Kekre (2019). They also couch their analysis in terms of the standard Baily-Chetty formula by considering the general equilibrium effects of unemployment insurance. The main difference is that they study benefits as policy instruments that vary over the business cycle, while we study how the presence of business cycles affects the ex ante fixed level of benefits.[6] Our focus is on automatic stabilizers, an ex ante passive policy, while they consider active stabilization policy.[7] This focus leads us to investigate channels through which even constant policies shape the dynamics of the business cycle. Moreover, our model includes aggregate uncertainty, and we also study income tax progressivity.

On income taxes, our work is closest to Benabou (2002) and Bhandari et al. (2018). Our dynamic heterogeneous-agent model with progressive income taxes is similar to the one in Benabou (2002), but our focus is on business cycles, so we complement it with aggregate shocks and nominal rigidities. Bhandari et al. (2018) is one of the very few studies of optimal income taxes with aggregate shocks. Like us, those authors emphasize the interaction between business cycles and the desire for redistribution.[8] However, they solve for the Ramsey optimal fiscal policy, which adjusts the tax instruments every period in response to shocks, while we choose the ex ante optimal rules for generosity and progressivity. This is consistent with our focus on automatic stabilizers, which are ex ante fiscal systems, rather than counter-cyclical policies.

Finally, this paper is related to the modern study of automatic stabilizers and especially our earlier work in McKay and Reis (2016). There, we considered the positive question of how the

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[5]See Oh and Reis (2012); Guerrieri and Lorenzoni (2017); Auclert (2019); McKay et al. (2016); Kaplan et al. (2018); Werning (2015).

[6]See also Mitman and Rabinovich (2015), Jung and Kuester (2015), and Den Haan et al. (2018).

[7]Unlike the United States, where the duration of unemployment benefits is often increased during recessions, in most OECD countries the terms of unemployment insurance programs do not change over the business cycle, as described in http://www.oecd.org/els/soc/.

[8]Werning (2007) also studies optimal income taxes with aggregate shocks and social insurance.

actual automatic stabilizers implemented in the US alter the dynamics of the business cycle. Here we are concerned with the optimal fiscal system as opposed to the observed one.

The paper is structured as follows. Section 2 presents the model, and Section 3 discusses its equilibrium properties. Section 4 derives the macroeconomic stabilization term in the optimality conditions for the two social programs. Section 5 discusses its qualitative properties, the economic mechanisms that it depends on, and its likely sign. Section 6 calibrates the model and quantifies the macro stabilization term and its effect on the optimal automatic stabilizers. Section 7 concludes.

2 The Model

The main ingredients in the model are: uninsurable income and employment risks, social insurance programs, and nominal rigidities so that aggregate demand matters for equilibrium allocations. Time is discrete and indexed by t.

2.1 Agents and commodities

There are two groups of private agents in the economy: households and firms. Households are indexed by i in the unit interval, and their type is given by their productivity $\alpha_{i,t} \in \mathbb{R}_0^+$ and employment status $n_{i,t} \in \{0,1\}$. Every period, an independently drawn share δ dies, and is replaced by newborn households with no assets and productivity normalized to $\alpha_{i,t} = 1$. Households derive utility from consumption, $c_{i,t}$, and publicly provided goods, G_t, and derive disutility from working for pay, $h_{i,t}$, searching for work, $q_{i,t}$, and being unemployed according to the utility function:

$$\mathbb{E}_0 \sum_t \beta^t \left[\log(c_{i,t}) - \frac{h_{i,t}^{1+\gamma}}{1+\gamma} - \frac{q_{i,t}^{1+\kappa}}{1+\kappa} + \chi \log(G_t) - \xi\left(1 - n_{i,t}\right) \right]. \tag{1}$$

The parameter β captures the joint discounting effect from time preference and mortality risk, while ξ is a non-pecuniary cost of being unemployed.[9]

The final consumption good is provided by a competitive final goods sector in the amount Y_t that sells for price p_t. It is produced by combining varieties of goods in a Dixit-Stiglitz aggregator with elasticity of substitution $\mu/(\mu - 1)$. Each variety $j \in [0,1]$ is monopolistically provided by a

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[9]If $\hat{\beta}$ is pure time discounting, then $\beta \equiv \hat{\beta}(1 - \delta)$. While the unemployed may benefit from an increase in leisure time, they also show higher rates of mortality (Sullivan and Von Wachter, 2009) and report low levels of subjective well being (Krueger and Mueller, 2011).

firm with output $y_{j,t} = \eta_t^A l_{j,t}$, where $l_{j,t}$ is the effective units of labor hired by the firm and η_t^A is an exogenous productivity shock.

2.2 Asset markets and social programs

Households can insure against mortality risk by buying an annuity, but they cannot insure against risks to their individual skill or employment status. The simplest way to capture this market incompleteness is by assuming that households can only hold a single risk-free annuity, $a_{i,t}$, that has a gross real return R_t.[10]

The net supply of inside assets is zero, while there is a stock of government bonds B. Following Krusell et al. (2011), Ravn and Sterk (2017), and Werning (2015), we use a strong assumption that will make the distribution of wealth tractable: households cannot borrow, $a_{i,t} \geq 0$, and $B = 0$ so bonds are in zero supply. This assumption facilitates our analysis and we relax it in an extension of our quantitative analysis.

The government provides two social insurance programs. The first is a progressive income tax such that if $z_{i,t}$ is pre-tax income, the after-tax income is $\lambda_t z_{i,t}^{1-\tau}$. The object of our study is $\tau \in [0,1]$, which determines the progressivity of the tax system. If $\tau = 0$, there is a flat tax at rate $1 - \lambda_t$, while if $\tau = 1$ everyone ends up with the same after-tax income. In between, a higher τ implies a more convex tax function, or a more progressive income tax system. The scale of the tax system is determined by $1 - \lambda_t \in [0,1]$, which is linked to the size of government purchases, G_t, through the government budget constraint.

The second social program is unemployment insurance. A household qualifies as long as it is unemployed ($n_{i,t} = 0$) and collects benefits that are paid in proportion to what the unemployed worker would earn if she were employed. Suppose the worker's productivity is such that she would earn pre-tax income $z_{i,t}$ if she were employed, then her after-tax unemployment benefit is $b\lambda_t z_{i,t}^{1-\tau}$.[11]

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[10]A standard formulation for asset markets that gives rise to these annuities is the following: a financial intermediary sells claims that pay one unit if the household survives and zero units if the household dies, and supports these claims by trading a risk-less bond with return $\tilde{R}$. If a_i are the annuity holdings of household i, the law of large numbers implies the intermediary pays out in total $(1-\delta)\int a_i di$, which is known in advance, and the cost of the bond position to support it is $(1-\delta)\int a_i di/\tilde{R}$. Because the risk-less bond is in zero net supply, then the net supply of annuities is zero $\int a_i di = 0$, and for the intermediary to make zero profits, $R_t = \tilde{R}_t/(1-\delta)$.

[11]It would be more realistic, but less tractable, to assume that benefits are a proportion of the income the agent earned when she lost her job. But, given the persistence in earnings, both in the data and in our model, our formulation will not be quantitatively too different from this case. Also, in our notation, it may appear that unemployment benefits are not subject to the income tax, but this is just the result of a normalization: if they were taxed and the replacement rate was $\tilde{b}$, then the model would be unchanged and $b \equiv \tilde{b}^{1-\tau}$.

Our focus is on the replacement rate $b \in [0, 1]$, with a more generous program understood as having a higher b.[12]

The goal is to characterize the optimal fixed levels of b and τ set ex ante as automatic stabilizers. These are programs that can automatically stabilize the business cycle without policy intervention, so b and τ do not depend on time or on the state of the business cycle. In this design problem, we are following the tradition in the literature on automatic stabilizers that makes a sharp distinction between built-in properties of programs as opposed to feedback rules or discretionary choices that adjust these programs in response to current or past information.[13] While our focus is on the role of the social insurance system in stabilizing the business cycle, these are not the only policy parameters that are relevant to the cyclical dynamics of the economy.[14]

2.3 Key frictions

There are three key frictions in the economy that create the policy trade-offs that we analyze.

2.3.1 Individual productivity risk

Labor income for an employed household is $\alpha_{it} w_t h_{it}$, where $\alpha_{i,t}$ is idiosyncratic productivity or skill and w_t is the wage per effective unit of labor. The productivity of households evolves as

$$\alpha_{i,t+1} = \alpha_{i,t} \epsilon_{i,t+1} \quad \text{with} \quad \epsilon_{i,t+1} \sim F(\epsilon, u_t), \tag{2}$$

and where $\int \epsilon dF(\epsilon, u_t) = 1$ for all t, which implies that the average idiosyncratic productivity in the population is constant and equal to one.[15] The distribution of shocks varies over time so that the model generates cyclical changes in the distribution of earnings risks. We capture this cyclical dependence using the unemployment rate. In the quantitative analysis, cyclical idiosyncratic productivity risk is important because it leads business cycles to have substantial welfare costs.

Our function $F(\epsilon, u_t)$ stands in for the concentrated and long-lasting costs of recessions (see Storesletten et al., 2004; Guvenen et al., 2014). For instance, Davis and von Wachter (2011) find

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[12]In our model, focusing on the duration of unemployment benefits instead of the replacement rate would lead to similar trade-offs, so we refer to b more generally as the generosity of the program.

[13]Perotti (2005) among many others.

[14]For an analysis of proportional income taxes see Galí (1994) and McKay and Reis (2016). For an analysis of the level of government spending see Andrés et al. (2008).

[15]Since newborn households have productivity 1, the assumption is that they have average productivity.

that workers laid off as part of a mass layoff have twice as large long-term earnings losses when the layoff occurs in a recession rather than an expansion. This evidence supports the perspective that aggregate business cycle conditions have long-term consequences for individual workers. The loss of skills during non-employment spells is one potential economic mechanism that could explain these earnings dynamics. A premise of the model is that these costs can be reduced by stabilizing the business cycle. To date, the literature has not settled on a theoretical understanding of these cyclical dynamics of income changes, which leads us to adopt a reduced-form approach instead.

2.3.2 Employment risk

The second source of risk is unemployment. At the start of the period, a fraction $v \in [0,1]$ of employed households loses employment and must search to regain employment. Search effort $q_{i,t}$ leads to employment with probability $M_t q_{i,t}$, where M_t is the job-finding rate per unit of search effort. Subject to some conditions, which we will discuss below, all households choose a common search effort q_t. In this case, the unemployment rate evolves as:

$$u_t = [u_{t-1} + v (1 - u_{t-1})] (1 - q_t M_t). \tag{3}$$

Firms incur costs to hire workers. Following Blanchard and Galí (2010), we abstract from vacancies that are posted at a cost and filled with some probability and instead specify a deterministic process in which the firm pays a cost to hire a worker. The cost per hire is increasing in aggregate labor market tightness, which is equal to the ratio of hires to searchers, or the job-finding rate M_t. This formulation captures the feature of search models that each vacancy is less likely to be filled as the labor market becomes tighter so the expected cost of hiring a worker rises. The hiring cost per hire is $\psi_1 M_t^{\psi_2}$, denominated in units of final goods, where ψ_1 and ψ_2 are parameters that govern the level and elasticity of the hiring costs. Since aggregate hires are the difference between the beginning of period employment rate $(1-v)(1-u_{t-1})$ and the realized employment rate $1-u_t$, aggregate hiring costs are:

$$J_t \equiv \psi_1 M_t^{\psi_2} [(1 - u_t) - (1 - v)(1 - u_{t-1})]. \tag{4}$$

We assume a law of large numbers within the firm so the average productivity of hires is 1.

In this model of the labor market, there is a surplus in the employment relationship since, on one side, firms would have to pay hiring costs to replace the worker and, on the other side, a worker who rejects a job must continue searching for a job thereby foregoing wages. This surplus creates a bargaining set for wages. There are many alternative models of how wages are chosen within this set, from Nash bargaining to wage stickiness, as emphasized by Hall (2005). We assume the equilibrium wage can be represented by a general function of the following form

$$w_t = w(\eta_t^A, u_t, b, \tau). \tag{5}$$

That is, wages can depend on the exogenous productivity, η_t^A, labor market slack measured by the unemployment rate, u_t, and the policy parameters b and τ.

2.3.3 Intermediate goods production and nominal rigidities

Firms may not be able to set the actual price for their goods equal to their desired price because of nominal rigidities. For our theoretical analysis, we assume a simple, canonical model of nominal rigidities that captures most of the qualitative insights from New Keynesian economics (Mankiw and Reis, 2010). Every period an i.i.d. fraction θ of firms can set their prices $p_{j,t} = p_t^*$ to the desired markup over marginal cost, while the remaining set their price to equal what they expected their optimal price would be: $p_{j,t} = \mathbb{E}_{t-1} p_t^*$. The analytical benefit of this assumption is that the degree of price dispersion in the economy reflects only current conditions and is not itself a state variable. For our quantitative analysis, we will use Calvo-style nominal rigidities.

Firms must adjust production to meet demand at their posted price. In principle, there are two margins of labor supply to adjust, however, we assume that workers choose their own hours on the intensive margin taking the wage as given. The firms then choose how many workers to hire in order to meet demand. Appendix A.4 states the firm's decision problem under both formulations of nominal rigidities. The equations that relate marginal cost to the optimal reset price and inflation dynamics are standard, with one novelty: marginal cost incorporates the cost of hiring workers net of expected savings on hiring costs next period.

The aggregate profits of these firms are distributed among employed workers in proportion to their skill, which can be thought of as representing bonus payments in a sharing economy. This assumption allows for equal hours across workers and implies total income is proportional to skill,

which keeps the model tractable.[16]

2.4 Other government policy

Aside from the two social programs that are the focus of our study, the government also chooses policies for nominal interest rates, government purchases, and the public debt. Starting with the first, we assume an interest rate rule for nominal interest rates I_t:

$$I_t = \bar{I}\pi_t^{\omega_\pi}\left(\frac{1-u_t}{1-\bar{u}}\right)^{\omega_u}\eta_t^I, \tag{6}$$

where $\bar{u}$ is the steady state unemployment rate, $\omega_\pi > 1$ and $\omega_u \geq 0$. The exogenous η_t^I represent shocks to monetary policy.[17]

Turning to the second, government purchases follow:

$$G_t = \chi C_t \eta_t^G, \tag{7}$$

where η_t^G are random shocks. Absent these shocks, this rule states that the marginal utility benefit of public goods offsets the marginal utility loss from diverting goods from private consumption.

Finally, third, the government runs a balanced budget by adjusting λ_t to satisfy:

$$G_t = \int n_{i,t}\left(z_{i,t} - \lambda_t z_{i,t}^{1-\tau}\right) - (1-n_{i,t})\,b\lambda_t z_{i,t}^{1-\tau}di, \tag{8}$$

where $z_{i,t}$ denotes the income of household i should they be employed. It is well known, at least since Aiyagari and McGrattan (1998), that in an incomplete markets economy like ours, changes in the supply of safe assets will affect the ability to accumulate precautionary savings. Deficits or surpluses may stabilize the business cycle by changing the cost of self-insurance. In the same way that we abstracted above from the stabilizing properties of changes in government purchases, this lets us likewise abstract from the stabilizing property of public debt, in order to focus on our two

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[16]Broer et al. (2020) show that the allocation of profits is crucial to the transmission mechanism of a New Keynesian model with balanced growth preferences and workers on their labor supply curves. Their argument applies to the determination of the intensive margin of labor supply in our model, but not to the extensive margin. With respect to the intensive margin, our approach is neutral in the sense that, for a given wage and aggregate dividend, hours per worker are the same as in a representative agent economy.

[17]As usual, the real and nominal interest rates are linked by the Fisher equation $R_t = I_t/\mathbb{E}_t[\pi_{t+1}]$.

social programs.[18]

3 Equilibrium and the role of policy

Our model combines idiosyncratic risk, incomplete markets, and nominal rigidities, and yet it is structured so as to be tractable enough to analytically investigate optimal policy. An aggregate equilibrium is a solution for 17 endogenous variables using a system of equations summarized in Appendix A.4, together with the exogenous processes η_t^A, η_t^G, and η_t^I. To further simplify the analytical investigation we make use of three parameter restrictions. First, agents never die, $\delta = 0$, which allows us to easily compute the welfare effects of skill risk. Second, the job separation rate, υ, is one, which means that the unemployment rate is not a state variable. Third, the volatility of fiscal shocks is zero so $\eta_t^G = 1$ for all t, which means public spending is at the efficient level. In addition to these parameter assumptions, we make use of the sticky-information form of nominal rigidities. These assumptions are in effect until we arrive at our quantitative analysis in Section 6 where we relax them, but not all of them are necessary for the results derived below, which we present in as general form as we can.

3.1 Inequality and heterogeneity

The following result plays a crucial role in simplifying the analysis:

Lemma 1. *All households choose the same asset holdings, hours worked, and search effort, so $a_{i,t} = 0$, $h_{i,t} = h_t$, and $q_{i,t} = q_t$ for all i.*

To prove this result, note that the decision problem of a household searching for a job at the start of the period is:

$$V^s(a, \alpha, \mathcal{S}) = \max_q \left\{ MqV(a, \alpha, n = 1, \mathcal{S}) + (1 - Mq)V(a, \alpha, n = 0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}, \qquad (9)$$

where we used $\mathcal{S}$ to denote the collection of aggregate states. The decision problem of an employed

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[18]In previous work (McKay and Reis, 2016), we found that allowing for deficits and public debt had little effect on the effectiveness of stabilizers. This is because, in order to match the concentration of wealth in the data, almost all of the public debt is held by richer households who are already close to fully self insured.

household is:

$$V(a, \alpha, n = 1, \mathcal{S}) = \max_{c,h,a' \geq 0} \left\{ \log c - \frac{h^{1+\gamma}}{1+\gamma} + \chi \log(G) + \beta \mathbb{E} \left[(1-v) V(a', \alpha', 1, \mathcal{S}') + v V^s(a', \alpha', \mathcal{S}') \right] \right\},$$

(10)

subject to the budget constraint:

$$a' + c = Ra + \lambda \left(n + (1-n)b \right) \left[\alpha(wh + d) \right]^{1-\tau}.$$

(11)

The decision problem of an unemployed household is:

$$V(a, \alpha, n = 0, \mathcal{S}) = \max_{c,a' \geq 0} \left\{ \log c + \chi \log(G) - \xi + \beta \mathbb{E} \left[V^s(a', \alpha', \mathcal{S}') \right] \right\}.$$

(12)

The budget constraint of an unemployed household is the same as that for the employed household except that the unemployment benefit payment is exogenous and the h that appears in the budget constraint should be understood as a parameter equal to the equilibrium choice of their employed counterpart, $h(a, \alpha, 1, \mathcal{S})$.

Starting with asset holdings, since no agent can borrow and bonds are in zero net supply, then it must be that $a_{i,t} = 0$ for all i in equilibrium because there is no gross supply of bonds for savers to own (see Krusell et al., 2011). Turning to hours worked, the intra-temporal labor supply condition for an employed household is:

$$c_{i,t} h_{i,t}^{\gamma} = (1 - \tau) \lambda_t z_{i,t}^{-\tau} w_t \alpha_{i,t},$$

(13)

where the left-hand side is the marginal rate of substitution between consumption and leisure, and the right-hand side is the after-tax return to working an extra hour to raise income $z_{i,t}$. More productive agents want to work more. However, they are also richer and consume more. The combination of our preferences and the budget constraint imply that these two effects exactly cancel out so that in equilibrium all employed households work the same hours:

$$h_t^{\gamma} = \frac{(1-\tau)w_t}{w_t h_t + d_t},$$

(14)

where d_t is aggregate dividends per employed worker.[19]

Finally, the optimality condition for search effort is:

$$q_{i,t}^\kappa = M_t \left[V(a_{i,t}, \alpha_{i,t}, 1, \mathcal{S}) - V(a_{i,t}, \alpha_{i,t}, 0, \mathcal{S}) \right]. \tag{15}$$

Intuitively, the household equates the marginal disutility of searching on the left-hand side to the expected benefit of finding a job on the right-hand side, which is the product of the job-finding probability M_t and the increase in value of becoming employed. Appendix A.1 shows that this increase in value is independent of $\alpha_{i,t}$. The key assumption that ensures this is that unemployment benefits are indexed to income $z_{i,t}$ so the after-tax income with and without a job scales with idiosyncratic productivity in the same way. Equation (15) then implies that $q_{i,t}$ is the same for all households.

The Lemma clearly limits the scope of our study. We cannot speak to the effect of policy on asset holdings, and differences in labor supply are reduced to having a job or not, which ignores diversity in part-time jobs and overtime. At the same time, it has the substantial payoff that we do not need to keep track of the cross-sectional distribution of wealth to characterize an equilibrium. Thus, our model can be studied analytically and global, non-linear, numerical solutions are easy to compute. Moreover, the social programs that we study are arguably more concerned with income inequality, rather than wealth inequality, and the vast majority of studies of the automatic stabilizers also ignores direct effects of wealth inequality (as opposed to income inequality) on the business cycle.

Even though there is no wealth inequality, there is a rich distribution of income and consumption driven by heterogeneity in employment status $n_{i,t}$ and skill $\alpha_{i,t}$ in our model. In Section 6, we are able to fit the more prominent features of income inequality in the United States by parameterizing the distribution $F(\epsilon, u)$. Moreover, there is a rich distribution of individual prices and output across firms in the model, $(p_{j,t}, y_{j,t})$, driven by nominal rigidities. And finally, the exogenous aggregate shocks to productivity, monetary policy, and government purchases, $(\eta_t^A, \eta_t^I, \eta_t^G)$, affect all of these distributions, which therefore vary over time and over the business cycle. In spite of the simplifications and their limitations, the model still has a rich amount of inequality and heterogeneity.

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[19]To derive this, substitute $z_{i,t} = \alpha_{i,t}(w_t h_{i,t} + d_t)$ and $c_{i,t} = \lambda_t z_{i,t}^{1-\tau}$ into equation (13).

3.2 Quasi-aggregation and consumption

Define $\tilde{c}_t$ as the consumption of the average-skilled ($\alpha_{i,t} = 1$), employed agent. The consumption of individual i is given by:

$$c_{i,t} = \alpha_{i,t}^{1-\tau}(n_{i,t} + (1 - n_{i,t})b)\tilde{c}_t. \tag{16}$$

Integrating across i gives aggregate consumption:

$$C_t = \underbrace{\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]}_{\equiv\Omega_t}(1 - u_t + u_t b)\tilde{c}_t. \tag{17}$$

The next property that simplifies our model is proven in Appendix A.3.

Lemma 2. *Consumption dynamics follow a modified Euler equation:*

$$\frac{1}{\tilde{c}_t} = \beta R_t \, \mathbb{E}_t\left\{\frac{1}{\tilde{c}_{t+1}}Q_{t+1}\right\}, \tag{18}$$

$$\text{with:} \quad Q_{t+1} \equiv \left[1 + \upsilon\left(1 - q_{t+1}M_{t+1}\right)\left(b^{-1} - 1\right)\right]\mathbb{E}\left[\epsilon_{i,t+1}^{-(1-\tau)}\right]. \tag{19}$$

and equation (17) *gives* $c_{i,t}$.

The variable Q_{t+1} captures how uninsurable risk affects aggregate consumption dynamics through precautionary savings motives. The more uncertain is income, the larger is Q_{t+1} and so the stronger are savings motives leading to steeper consumption growth. This Euler equation is the key equation through which precautionary savings motives affect fluctuations in output. In particular, the term $\upsilon\left(1 - q_{t+1}M_{t+1}\right)\left(b^{-1} - 1\right)$ is central to our analysis. With probability $\upsilon\left(1 - q_{t+1}M_{t+1}\right)$ the employed worker becomes unemployed next period, in which case their marginal utility is increased by a factor $b^{-1} - 1$ relative to remaining employed.

A more generous unemployment insurance system dampens the precautionary savings motive stemming from employment risk by reducing the effect of unemployment on expected marginal utility of consumption. This is a key mechanism by which unemployment insurance stabilizes the economy. Similarly, a more progressive income tax lowers the dispersion of after-tax income growth due to skill shocks.

3.3 Policy distortions and redistribution over the business cycle

Social policies not only affect aggregate consumption, but also all individual choices in the economy, introducing both distortions and redistribution. As can be seen in equation (14), a more progressive income tax lowers hours worked by increasing the ratio of the marginal tax rate to the average tax rate.

Moving to search effort, Appendix A.2 shows that with $\upsilon = 1$:

$$q_t^\kappa = M_t \left[\log(1/b) + \xi - \frac{h_t^{1+\gamma}}{1+\gamma} \right].$$ (20)

This states that the marginal disutility of searching for a job is equal to the probability of finding a job times the increase in utility of having a job. This utility gain is equal to the utility gain from consuming a factor $1/b$ more, plus the gain from avoiding the non-pecuniary cost of unemployment ξ, less the value of leisure. More generous benefits therefore lower search effort, because the consumption gain from employment is reduced.

Equation (16) shows that more productive and employed households consume more, as expected. Social policies redistribute income and equalize consumption. A higher b requires larger contributions from all households, lowering $\tilde{c}_t$, but raises the consumption of the unemployed relative to the employed. In turn, a higher τ lowers the cross-sectional dispersion of consumption because it reduces the income of the rich more than that of the poor.

Finally, nominal rigidities lead otherwise identical firms to charge different prices, and this relative-price dispersion lowers efficiency. The social insurance system will alter the dynamics of aggregate demand leading to different dynamics for nominal marginal costs, inflation, and price dispersion. The degree of price dispersion is given by:

$$S_t \equiv \int (p_t(j)/p_t)^{\mu/(1-\mu)} \, dj = \left(\frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)} \left[\theta + (1-\theta) \left(\frac{\mathbb{E}_{t-1} p_t^*}{p_t^*} \right)^{\mu/(1-\mu)} \right].$$ (21)

Integrating over the intermediate good production functions and using the demand for each variety it follows that $Y_t = A_t h_t (1 - u_t)$ where $A_t \equiv \eta_t^A / S_t$.

3.4 The structure of the labor market

Our model has several margins along which aggregate hours worked can adjust. First, there is the intensive margin h_t. Second, there is the extensive margin on the number of workers employed, which can change either because the unemployed increase their search effort or because of changes in the job-finding rate, M_t. Appendix B proves the following result:

Lemma 3. *There is a function $\mathcal{H}_h$ such that h_t satisfies $h_t = \mathcal{H}_h(b, \tau, M_t, \eta_t^A)$. Similarly, there are functions such that $q_t = \mathcal{H}_q(b, \tau, M_t, \eta_t^A)$, $u_t = \mathcal{H}_u(b, \tau, M_t, \eta_t^A)$, and $Y_t = \mathcal{H}_Y(b, \tau, M_t, \eta_t^A)$.*

The Lemma says that, for fixed values of b and τ, hours worked, the unemployment rate, search effort, and output can all be determined from the job-finding rate M_t and the productivity shock η_t^A. In this sense, the job-finding rate serves as a useful measure of the level of activity in the economy. How is M_t itself determined? Suppose there is a change in aggregate demand, say, a shock to government purchases. Firms that are subject to nominal rigidities will have to expand production to meet demand and the margin for doing so is to hire more workers. The $\mathcal{H}_Y(.)$ function is increasing in M_t, so M_t must rise to increase the supply of goods to meet demand and satisfy the aggregate resource constraint. (Of course, in general equilibrium, this equation is not the sole determinant of M_t.)

4 Optimal policy and insurance versus incentives

All agents in the economy are identical ex ante, making it natural to take as the target of policy the utilitarian social welfare function. Using equations (16) and (17) and integrating the utility function in equation (1) gives the objective function for policy $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t W_t$, where period-welfare is:

$$W_t = \mathbb{E}_i \log\left(\alpha_{i,t}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\right) + u_t \log b - \log\left(1 - u_t + u_t b\right)$$
$$+ \log(C_t) - (1 - u_t)\frac{h_t^{1+\gamma}}{1+\gamma} - \frac{q_t^{1+\kappa}}{1+\kappa} + \chi \log(G_t) - \xi u_t. \tag{22}$$

The first line shows how inequality affects social welfare. Productivity differences and unemployment introduce costly idiosyncratic risk, which is attenuated by the social insurance policies. The second line captures the usual effect of aggregates on welfare. While these would be the terms that

would survive if there were complete insurance markets, recall that the incompleteness of markets also affects the evolution of aggregates, as we explained in the previous Section.

It is useful to express the welfare loss from skill dispersion in terms of the initial dispersion and the capitalized welfare cost of the shocks that occur at each date, which we denote $\mathcal{R}_t$.

Lemma 4. *Under no mortality, $\delta = 0$:*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, \mathbb{E}_i \left[\log \left(\frac{\alpha_{i,t}^{1-\tau}}{\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right]} \right) \right] = \frac{1}{1-\beta} \, \mathbb{E}_i \left[\log \left(\frac{\alpha_{i,0}^{1-\tau}}{\mathbb{E}_i \left[\alpha_{i,0}^{1-\tau} \right]} \right) \right] + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{R}_t$$

where

$$\mathcal{R}_t \equiv \frac{\beta}{1-\beta} \int \log \left(\frac{\epsilon_{i,t+1}^{1-\tau}}{\int \left[\epsilon_{i,t+1}^{1-\tau} \right] dF(\epsilon, u_t)} \right) dF(\epsilon, u_t). \tag{23}$$

See Appendix B for the proof.

The policy problem is then to pick b and τ once and for all to maximize equation (22) subject to the equilibrium conditions.

4.1 Optimal unemployment insurance

Appendix B derives the following optimality condition for b:

Proposition 1. *Under rigid prices, $\theta = 0$, the optimal choice of the generosity of unemployment insurance b satisfies:*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \underbrace{u_t \left(\frac{1}{b} - 1 \right) \left. \frac{\partial \log (b\tilde{c}_t)}{\partial \log b} \right|_{u,h}}_{\text{Insurance}} \\[2em] + \underbrace{\left. \frac{\partial \log \tilde{c}_t}{\partial u_t} \right|_{\Omega,G,M} \left. \frac{\partial u_t}{\partial b} \right|_M + \frac{d\mathcal{R}_t}{du_t} \left. \frac{\partial u_t}{\partial b} \right|_M + (1 - u_t) \left(\frac{A_t}{C_t} - h_t^{\gamma} \right) \left. \frac{\partial h_t}{\partial b} \right|_M}_{\text{Incentives}} \\[2em] + \underbrace{\frac{dW_t}{dM_t} \frac{dM_t}{db}}_{\text{Macro stabilization}} \end{array} \right\} = 0. \tag{24}$$

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Equation (24) is closely related to the Baily-Chetty formula for optimal unemployment insurance. The first row captures the social insurance value of changing the replacement rate. It is equal to the percentage difference between the marginal utility of unemployed and employed agents times the elasticity of the consumption of the unemployed with respect to the benefit. If unemployment came with no differences in consumption, this term would be zero, and likewise if giving higher benefits to the unemployed had no effect on their consumption. But as long as employed agents consume more, and raising benefits closes some of the consumption gap, then this term will be positive and call for higher unemployment benefits.

More generous unemployment insurance benefits will lead unemployed workers to search less for a job, resulting in an increase in the equilibrium unemployment rate. This moral hazard channel is the primary incentive effect of unemployment insurance. It is captured by the first term on the second row, and it is equal to the product of the elasticity of the consumption of the employed with respect to the unemployment rate, which is negative, and the elasticity of the unemployment rate with respect to the benefit, which arises out of reduced search effort. As higher replacement rates induce agents to search less, the unemployment rate rises and leads to higher taxes to finance benefits.[20] Another consequence of changing the unemployment rate in our model is that the distribution of idiosyncratic skill risk changes and the welfare consequences of this change in risk are captured by the term $d\mathcal{R}/du$. In addition, unemployment insurance potentially influences the intensive margin of labor supply because it potentially affects the wages that workers face when they choose their hours. This channel is the third term on the second row, and includes the product of the utility benefit of raising hours on the intensive margin, $(1 - u_t)(A_t/C_t - h_t^\gamma)$ and the change in h_t.

The result assigns a special role to the job-finding rate, M_t, as a measure of labor market tightness, since the partial derivatives in the incentives terms hold M_t fixed. Standard treatments of optimal unemployment insurance (e.g. Chetty, 2006) assume there is a fixed mapping from search effort to employment status. In those models, this mapping is assumed to not change with general equilibrium effects of unemployment benefits. In our model, that corresponds to a fixed job-finding rate. With general equilibrium effects, there is an extra term that comes from changing M_t, which

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[20] The derivative of $\tilde{c}$ with respect to unemployment captures the gain in resources from putting more people to work and reducing taxes to fund unemployment insurance benefits, but omits the effect of the unemployment rate on the distribution of income and therefore the level of tax revenue raised by the progressive tax system, $\Omega_t \equiv \mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right]$, and holds the level of government purchases fixed.

is shown on the third row. This term reflects the logic that an increase in unemployment benefits can make jobs easier or harder to find. As M_t is closely linked to the level of activity in the economy (see Lemma 3), we can interpret this channel as the effect of benefits on the level of activity. We call this the macroeconomic stabilization term and the larger it is, the more generous optimal unemployment benefits should be. We explain this shortly, but first, we turn to the income tax.

4.2 Optimal progressivity of the income tax

Appendix B shows the following:

Proposition 2. *Under rigid prices, $\theta = 0$, the optimal progressivity of the tax system τ satisfies:*

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{
\begin{array}{c}
\underbrace{\frac{\mathrm{Cov}\left(\alpha_{i,0}^{1-\tau}, \log \alpha_{i,0}\right)}{\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]} + \left.\frac{\partial \mathcal{R}_t}{\partial \tau}\right|_u}_{\text{Insurance}} \\[2em]
+ \underbrace{\left.\frac{\partial \log \tilde{c}_t}{\partial u_t}\right|_{M,\Omega,G} \left.\frac{\partial u_t}{\partial \tau}\right|_M + \frac{d\mathcal{R}_t}{du_t} \left.\frac{\partial u_t}{\partial \tau}\right|_M + \left(\frac{A_t}{C_t} - h_t^\gamma\right)(1-u_t)\left.\frac{\partial h_t}{\partial \tau}\right|_M}_{\text{Incentives}} \\[2em]
+ \underbrace{\frac{dW_t}{dM_t}\frac{dM_t}{d\tau}}_{\text{Macro stabilization}}
\end{array}
\right\} = 0. \quad (25)
$$

The three rows again capture the trade-offs between insurance, incentives, and macroeconomic stabilization, respectively. Starting with the first row, the first term gives the welfare benefit of redistributing already existing differences in income, as captured by the initial dispersion of skills. The second term gives the welfare benefits of reducing the dispersion in after-tax incomes due to skill shocks that the household is exposed to in the future. This term holds fixed the unemployment rate and therefore the distribution of pre-government risk the household face. What it captures is then the fact that more progressive taxes reduce the dispersion of post-government income. Using equation (23), note that:

$$
\left.\frac{\partial \mathcal{R}_t}{\partial \tau}\right|_u = \frac{\beta}{1-\beta} \frac{\mathrm{Cov}\left(\epsilon_{i,0}^{1-\tau}, \log \epsilon_{i,0}\right)}{\mathbb{E}_i\left[\epsilon_{i,0}^{1-\tau}\right]}.
$$

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Both terms in the first row therefore have a similar structure and are both positive.[21]

The second row gives the incentive costs of raising progressivity. These incentive costs have the same structure as in Proposition 1. The tax system affects the relative rewards to being employed and therefore alters household search effort and the unemployment rate, which has consequences for skill risk. These effects on search effort are captured by the first two terms on the row. The classic incentive effect of a more progressive tax system in raising marginal tax rates and reducing the incentive to supply labor on the intensive margin is captured by the third term as $\frac{\partial h}{\partial \tau}\big|_M$ captures the effect of progressivity on the intensive margin of labor supply.

Finally, the third row captures the concern for macroeconomic stabilization in a very similar way to the term for unemployment benefits. A larger stabilization term in equation (25) justifies a more progressive tax.

Both Propositions assume rigid prices to eliminate a term related to price dispersion, S_t. If we relax this assumption, there is an additional term that captures the efficiency loss from the effect of the policies on price dispersion. Specifically, $-\frac{Y_t}{S_t C_t} \frac{\partial S_t}{\partial b}\big|_M$ is appended to Proposition 1 and an analogous term to Proposition 2. This term reflects the fact that policy changes the unemployment rate even holding fixed M_t by changing the incentives for search, and therefore may change the level of wages and marginal costs for the intermediate goods producers.

4.3 The macroeconomic stabilization term

The two previous Propositions clearly isolate the automatic-stabilizing role of the social insurance programs in a single term. It equals the product of the welfare benefit of changing the level of economic activity, captured by the job-finding rate, and the response of activity to policy. If business cycles are efficient, the macroeconomic stabilization term is zero. That is, if the economy is always at an efficient level of activity, so that $dW_t/dM_t = 0$, then there is no reason to take macroeconomic stabilization into account when designing the stabilizers. Intuitively, the business cycle is of no concern for policymakers in this case.

Even if business cycles are efficient on average or the stabilizers have no effect on the average

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[21]Each of the terms involves the covariance of two increasing functions of a single random variable, which is positive if the underlying random variable has positive variance. The denominators are positive because α_i and ϵ_i take positive values.

level of activity, the stabilizers can still have stabilization benefits. This is because:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{dW_t}{dM_t} \frac{dM_t}{db} \right\} = \sum_{t=0}^{\infty} \beta^t \left\{ \mathbb{E}_0 \left[\frac{dW_t}{dM_t} \right] \mathbb{E}_0 \left[\frac{dM_t}{db} \right] + \mathrm{Cov} \left[\frac{dW_t}{dM_t}, \frac{dM_t}{db} \right] \right\}, \tag{26}$$

so that, even if $\mathbb{E}_0 \left[\frac{dW_t}{dM_t} \right] \mathbb{E}_0 \left[\frac{dM_t}{db} \right] = 0$, a positive covariance term would still imply a positive aggregate stabilization term and an increase in benefits (or more progressive taxes). Our model therefore provides a definition of a social policy that serves as an automatic stabilizer: it stimulates the economy more in recessions, when activity is inefficiently low. The stronger this effect, the larger the program should be. In the next Section, we discuss the sign of this covariance and what affects it.

5 Inspecting the macroeconomic stabilization term

Understanding the automatic stabilizer nature of social programs requires understanding separately the effect of activity on welfare, dW_t/dM_t, and the effect of the social policies on activity, dM_t/db and $dM_t/d\tau$. Trying to measure the covariance between these two unobservables in the data is a daunting task. Instead, we proceed to characterize their structural determinants in terms of familiar economic channels that have been measured elsewhere.

5.1 Activity and welfare

There are five separate channels through which the business cycle may be inefficient in our model, characterized in the following result:

Proposition 3. *The effect on welfare of real activity, captured by the job finding rate, can be decomposed into:*

$$\frac{dW_t}{dM_t} = \underbrace{(1 - u_t) \left[\frac{A_t}{C_t} - h_t^{\gamma} \right] \frac{dh_t}{dM_t}}_{labor\text{-}wedge} - \underbrace{\frac{Y_t}{C_t S_t} \frac{dS_t}{dM_t}}_{price\text{-}dispersion} + \underbrace{\frac{1}{C_t} \frac{\partial C_t}{\partial u_t} \bigg|_{M,G} \frac{du_t}{dM_t} - \frac{1}{C_t} \frac{\partial J_t}{\partial M_t} \bigg|_{u}}_{extensive\text{-}margin} \tag{27}$$

$$\underbrace{- \left(\xi - \log b - \frac{h_t^{1+\gamma}}{1+\gamma} \right) \frac{\partial u_t}{\partial M_t} \bigg|_q + \frac{1-b}{1 - u_t + u_t b} \frac{du_t}{dM_t}}_{unemployment\text{-}risk} + \underbrace{\frac{d\mathcal{R}}{du_t} \frac{du_t}{dM_t}}_{income\text{-}risk}$$

The first term captures the effect of the labor wedge or markups. In the economy, A_t/C_t is the marginal product of an extra hour worked in utility units, while h_t^γ is the marginal disutility of working. If the first exceeds the second, the economy is under-producing, and increasing hours worked would raise welfare. The gap between the marginal rate of substitution between consumption and leisure and the marginal product of labor may arise due to markups and fluctuations in this gap are a key cost of business cycles in the new Keynesian literature (see Galí et al., 2007).

The second term captures the effect of activity on price dispersion. Because of nominal rigidities, aggregate shocks will lead to price dispersion. In that case, changes in aggregate activity will affect inflation, via the Phillips curve, and so price dispersion. This is the conventional welfare cost of inflation in new Keynesian models.

The third and fourth terms capture the standard extensive margin trade-off in models with costly matching. On the one hand, tightening the labor market lowers unemployment and raises consumption. On the other hand, it increases hiring costs. If $\frac{\partial C_t}{\partial u_t} \frac{du_t}{dM_t} > \frac{\partial J_t}{\partial M_t}$, welfare rises as the labor market gets tighter.[22] These considerations are the focus of the analysis of unemployment insurance by Landais et al. (2018). They discuss the macroeconomic effects of unemployment benefits from the perspective of their effect on labor market tightness by changing the worker's bargaining position and wages on the one hand and, on the other hand, their impact on dissuading search effort.

The terms in the second line of equation (27) focus on inequality and its effect on welfare. If the extent of income risk is counter-cyclical, which the literature has demonstrated starting with Storesletten et al. (2004), then raising economic activity reduces income risk and so raises welfare. In our model, there is both unemployment and income risk, so this works through two channels.

For a given aggregate consumption, more unemployment has two effects on welfare. First, there are more unemployed who consume a lower amount. The term $\xi - \log b - h_t^{1+\gamma}/(1+\gamma)$ is the utility loss from becoming unemployed. Second, those who are employed consume a larger share (dividing the pie among fewer employed people).

The final term captures the effect of activity on the distribution of skill shocks. Activity affects welfare by changing the distribution $F(\epsilon, u_t)$ and therefore $\mathcal{R}_t$. Since the skewness of this

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[22]The partial derivative of C_t with respect to u_t given M_t is defined mathematically in Appendix B. It is the gain in consumption from putting more people to work but without changing wages, hours on the intensive margin, price dispersion, or the other consequences of changing M_t.

distribution tends to be procyclical, by the concavity of the log function, a more negatively skewed $F(.)$ in recessions results in more welfare losses. The costs of business cycles due to cyclical risk in persistent shocks to income has been emphasized by Storesletten et al. (2001), Krebs (2003, 2007), and De Santis (2007).

5.2 The stabilizing effect of social programs

We now turn attention to the second component of the macroeconomic stabilization term, either dM_t/db in the case of unemployment benefits, or $dM_t/d\tau$ in the case of tax progressivity. Here we describe why dM_t/db and $dM_t/d\tau$ can be counter-cyclical even if b and τ are constant. When we turn to the quantitative analysis, we will see that this cyclicality is important.

To build intuition, focus on the Euler equation (18), which determines the dynamics of $\tilde{c}_t$ and depends on the precautionary savings motive Q_{t+1} given by equation (19). The risk to employment and skills is dampened by the social insurance system so Q_{t+1} is decreasing in b and τ. Moreover, the level of social insurance is more important when the pre-tax-and-transfer risks are more severe. As these risks are more pronounced in recessions, the precautionary motive, Q_{t+1}, is more sensitive to policy in a recession. For this reason, aggregate demand becomes more sensitive to the level of b and τ in recessions.

Equation (17) shows an additional effect of unemployment benefits. Taking $\tilde{c}_t$ as given, aggregate consumption increases in b and more so the higher is u_t. This channel is a classic argument for unemployment insurance payments as an automatic stabilizer by not letting the consumption of the unemployed fall as much as their income drop. In the model, the consumption of the unemployed is a fraction b of the consumption of the employed and increases one-for-one with the transfers.

General equilibrium forces may either dampen or reinforce these mechanisms. On the one hand, real interest rates will typically adjust to stabilize the economy and therefore attenuate the effect of other factors on aggregate demand including the automatic stabilizers. This effect pushes dM_t/db and $dM_t/d\tau$ toward zero. On the other hand, the dynamics of the job-finding rate can amplify fluctuations in aggregate demand because low aggregate demand leads to a slack labor market, which in turn increases the risk of becoming unemployed further reducing aggregate demand.[23] This effect amplifies dM_t/db and $dM_t/d\tau$.

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[23]Similar reinforcing dynamics arise out of unemployment risk in Ravn and Sterk (2017), Den Haan et al. (2018), and Heathcote and Perri (2018).

If business cycles are inefficient in the sense that activity is inefficiently low in a recession, then we expect a positive covariance between dW_t/dM_t and the elasticities of M_t with respect to policy. This positive covariance implies a positive aggregate stabilization term, so more generous unemployment benefits and more progressive tax system, even if the business cycle is efficient on average.

6   Quantitative analysis

We have shown that the presence of business cycles leads to a macroeconomic stabilization term in the determination of the optimal generosity of unemployment insurance and the progressivity of income taxes, and that this term likely makes these programs more generous and progressive, respectively. We now turn to numerical solutions to evaluate whether the macroeconomic stabilization term is quantitatively significant, while using the analytical formulas to make sense of the mechanisms driving the numerical results.

6.1   Calibration and solution of the model

We solve the model using global methods, as described in Appendix D.2, so that we can accurately compute social welfare, assuming that the economy starts at date 0 at the deterministic steady state. We then numerically search for the values of b and τ that maximize the social welfare function, and compare these with the maximal values in a counterfactual economy without aggregate shocks, but otherwise identical.

In Section 3, we introduced several assumptions for tractability that we now relax. Specifically, we allow for mortality, persistent unemployment, government spending shocks, and Calvo-style pricing. For the wage rule, we adopt the specification:

$$w(\eta_t^A, u_t, b, \tau) = \bar{w}\eta_t^A \left(\frac{1 - u_t}{1 - \bar{u}}\right)^\zeta, \tag{28}$$

where ζ controls the elasticity of the wage with respect to labor market slack measured by the unemployment rate. In Section 6.4.1 we consider alternative specifications of the wage rule.

Table 1 shows the calibration of the model, dividing the parameters into different groups. The first group has parameters set ex ante to standard choices in the literature. Only the last

one deserves some explanation. ψ_2 is the elasticity of hiring costs with respect to labor market tightness, and we set it at 1 as in Blanchard and Galí (2010), in order to be consistent with an elasticity of the matching function with respect to unemployment of 0.5 as suggested by Petrongolo and Pissarides (2001).[24]

Panel B contains parameters individually calibrated to match time-series moments. For the preference for public goods, we target the observed average ratio of government purchases to GDP in the US in 1984-2007. For the monetary policy rule, we use OLS estimates of equation (6). We calibrate the job separation rate, υ, so that we match the employment-to-unemployment transition rate from Table 2 of Krusell et al. (2017).[25] Finally, we estimate a version of the income innovation process specified in equation (2) using a mixture of normals as a flexible parameterization of the distribution $F(\epsilon, u_t)$. Two of the mixture components shift with the unemployment rate to match the observed pro-cyclical skewness of earnings growth rates documented by Guvenen et al. (2014). This parametric income process is similar to the one in McKay (2017) and Appendix D.1 provides additional details.[26] As a check on our calibration, the model implies a cross-sectional variance of log consumption of 0.40, while in 2005 CEX data the variance of log consumption of non-durables was around 0.35 (Heathcote et al., 2010).

Panel C has parameters chosen jointly to target a set of moments. We target the average unemployment rate between 1960 to 2014 and recruiting costs of 3 percent of quarterly pay, consistent with Barron et al. (1997). The parameter κ controls the marginal disutility of effort searching for a job, and we set it to target a micro-elasticity of unemployment with respect to benefits of 0.5 as reported by Landais et al. (2018). Last in the panel is ξ, the non-pecuniary costs of unemployment. In the model, the utility loss from unemployment is $\log(1/b) - h^{1+\gamma}/(1+\gamma) + \xi$, reflecting the loss in consumption, the gain in leisure, and other non-pecuniary costs of unemployment. We set $\xi = h^{1+\gamma}/(1+\gamma)$ in the steady state of our baseline calibration so that the benefit of increased leisure in unemployment is dissipated by the non-pecuniary costs.

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[24]Suppose there is a matching function $m_t = v_t^{1/2} u_t^{1/2}$, where m is the number of matches and v is the number of vacancies. If each vacancy has a cost ψ_1 the expected cost to hire a worker is ψ_1 divided by the job filling rate $m_t/v_t = (u_t/v_t)^{1/2}$. Note $\psi_1(u_t/v_t)^{-1/2}$ can be expressed as $\psi_1 M_t$, where $M_t = m_t/u_t = (v_t/u_t)^{1/2}$ is the job-finding rate.

[25]We use the Abowd-Zellner panel of the table converted to a quarterly frequency. As our model abstracts from non-participation we measure the EU transition rate as $f_{EU}/(1 - f_{EN})$ where f_{EU} and f_{EN} represents the entries in the transformed table.

[26]We include unemployment fluctuations in the income process we simulate to match the empirical moments so the contribution of unemployment to observed changes in income distributions is accounted for.

Table 1: Calibrated parameter values and targets

Symbol	Value	Description	Target
Panel A. Parameters chosen ex ante			
δ	1/200	Mortality rate	50-year expected lifetime
$\frac{\mu}{\mu-1}$	6	Elasticity of substitution	Basu and Fernald (1997)
θ	1/3.5	Probability of price reset	Klenow and Malin (2010)
$1/\gamma$	1/2	Frisch elasticity	Chetty (2012)
ψ_2	1	Elasticity of hiring cost	Blanchard and Galí (2010)
Panel B. Parameters individually calibrated			
χ	0.262	Preference for public goods	$G/Y = 0.207$
ω_π	1.66	Mon. pol. response to π	Estimated interest rate rule
ω_u	0.133	Mon. pol. response to u	Estimated interest rate rule
υ	0.094	Job separation rate	3.8% EU transition probability
$F(\epsilon, .)$	mix-normals	Productivity-risk process	See Appendix D.1
Panel C. Parameters jointly calibrated to steady-state moments			
β	0.982	Discount factor	3% annual real interest rate
$\bar{w}$	0.831	Average wage	Unemployment rate = 6.1%
ψ_1	0.0309	Scale of hiring cost	Recruiting costs of 3% of pay
$1/\kappa$	0.0476	Search effort elasticity	$d\log u/d\log b|_M = 0.5$
ξ	0.236	Pain from unemployment	Leisure benefit of unemployment
Panel D. Parameters jointly calibrated to volatilities			
ρ	0.9	Autocorrelation of shocks	See text
$\text{StDev}(\eta^A)$	0.46%	TFP innovation	Smets and Wouters (2007)
$\text{StDev}(\eta^I)$	0.44%	Monetary policy innovation	$\text{StDev}(u_t) = 1.59\%$
$\text{StDev}(\eta^G)$	4.63%	Gov't purchases innovation	$\text{StDev}(G_t/Y_t) = 1.75\%$
ζ	1.68	Elas. of wage w.r.t. $1-u$	$\text{StDev}(h_t)/\text{StDev}(1-u_t) = 0.568$
Panel E: Automatic stabilizers			
b	0.810	UI replacement rate	See text
τ	0.151	Progressivity of tax system	Heathcote et al. (2017)

Panel D calibrates the three aggregate shocks in our model that perturb productivity, monetary policy, and public expenditures. In each case, the exogenous process is an AR(1) in logs. We set the variance of the TFP shock to the posterior mean of the estimates in Smets and Wouters (2007). The other two variances are set to match the standard deviation of the unemployment rate and the standard deviation of G_t/Y_t. In the model, monetary shocks are particularly important in that they account for the majority of the variance of the unemployment rate. It is especially important for the model to match the persistence of unemployment risk, since the aggregate demand consequences of the precautionary savings motive accumulate in the forward-looking Euler equation. We therefore set the autocorrelation coefficient of these shocks to 0.9 in order to generate a quarterly persistence of the unemployment rate that matches the data, which is is between 0.92 and 0.98 depending on how one accounts for low-frequency components. Our results are largely insensitive to the persistence of the fiscal and technology shocks, so we set them to 0.9 as well. Finally, we set $\zeta = 1.68$ to match the standard deviation of hours per worker relative to the standard deviation of the employment-population ratio.

Finally, panel E has the baseline values for the automatic stabilizers. For τ we adopt the estimate of 0.151 from Heathcote et al. (2017). In calibrating b, we target the observed degree of insurance that households have against unemployment shocks, as measured by the change in consumption upon unemployment. We set $b = 0.81$, consistent with a 19% decline in consumption when unemployed since the literature has found consumption changes between 16% and 21%.[27] These calibrated values for b and τ do not directly enter our analysis of optimal policy but are used to jointly calibrate the other structural parameters of the model.

As a check on the model's performance, the standard deviation of hours, output, and inflation in the model are 0.80%, 2.65%, and 0.86%. The corresponding moments in the US data 1960-2014 are 0.84%, 1.32%, and 0.55%.

6.2 Stabilization and optimal unemployment benefits

To assess how business cycle stabilization alters the design of the social insurance system, we compare the optimal values of b and τ to what would be optimal in the absence of aggregate shocks. While we jointly optimized over b and τ, we will first discuss the results for b, before

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[27]See Stephens Jr (2004), Aguiar and Hurst (2005), Saporta-Eksten (2014), and Chodorow-Reich and Karabarbounis (2016).

turning to the results for τ in Section 6.3.

6.2.1 The impact of business cycles on optimal unemployment benefits

Our first main quantitative result is that introducing aggregate shocks increases the optimal b from 0.746 to 0.824. Our interpretation of the magnitude of b is the level of insurance against unemployment shocks at the level of a household. Specifically, in an unemployment spell, consumption is a fraction b of what it would have been had the household been employed. b is large relative to typical unemployment insurance replacement rates because many households have other sources of insurance with multiple earners being especially important. So, to express b in terms of a replacement rate, we conduct a simple calculation in which the hypothetical unemployment spell only affects one of two workers in a household and the UI system partially replaces that worker's income while the other worker continues working as normal. We also adjust for the fact that b is an after-tax measure while UI replacement rates are typically pre-tax. We then have the relationship:

$$\left(\text{replacement rate} \times \frac{1}{2} + \frac{1}{2}\right)^{1-\tau} = b. \tag{29}$$

Using this conversion of b and τ we find an optimal replacement rate of 56% with aggregate shocks as compared to 35% percent without. Comparing the social welfare function at $b = 0.824$ and $b = 0.746$ shows a 0.3% gain in consumption-equivalent welfare.

Figure 1 shows the positive effects of changing the unemployment benefit. Raising the generosity of unemployment benefits hurts the incentives for working, so unemployment rises somewhat (left panel). However, it significantly stabilizes the job finding rate and through it lowers the variation in most macroeconomic aggregates (right panel).

6.2.2 What drives the large automatic stabilization role for unemployment benefits?

The analytical results in Propositions 1 and 3 provide guidance on the key economic channels at play. Those Propositions were derived under some restrictive assumptions, which Appendix D.3 partly relaxes to extend their insights to the richer setting considered here.

Figure 2 splits the welfare gain from raising b into the terms we defined in the propositions. Starting with Proposition 1 in the top panel, as b rises, the incentive costs worsen, because the value of working gets closer to the value of unemployment. In addition, higher b comes with lower

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Figure 1: Effects of changing b for fixed τ
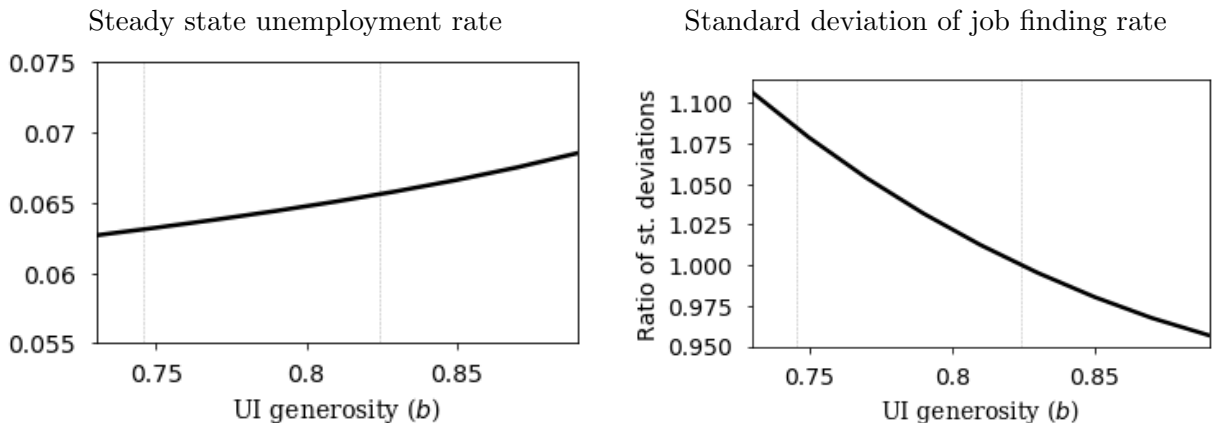
Notes: Vertical lines show optimal b with and without aggregate shocks. Right panel shows the standard deviation of M_t, relative to its value when $b = 0.81$.

insurance gains, as the consumption (and therefore marginal utilities) of employed and unemployed become closer. The macro-stabilization term is split into two pieces: $\mathbb{E}\left[\frac{dW}{dM}\right]\mathbb{E}\left[\frac{dM}{db}\right]$ and the co-variance term. The former is negative, since more generous unemployment benefits distort labor market outcomes on average. The latter is large and positive, so that more unemployment benefits are particularly valuable in stabilizing the economy at times when labor demand is inefficiently low. This role of the program, which has been neglected so far, is as important as the incentives and redistribution roles that the literature has emphasized instead. A concern for automatic stabilizers makes the unemployment insurance system significantly more generous.

The lower panel of Figure 2 uses Proposition 3 to unpack the macro stabilization term into the different sources of inefficient fluctuations. The dominant component is clearly the reduction in idiosyncratic risk that results from more generous unemployment benefits. By stabilizing the economy, higher b reduces the risk that households face in their pre tax-and-transfer incomes. This channel is distinct from the insurance benefit, which is the smoothing of post tax-and-transfer income for a given risk to pre tax-and-transfer income. There are two sources of idiosyncratic risk, skills and unemployment, but the welfare effects are driven almost entirely by the skills component. The inefficient utilization of labor on the extensive margin is negative, as raising b raises the unemployment rate on average. The other components that are plotted in the figure are small in contrast.

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Figure 2: Marginal welfare gain from changing b for fixed τ
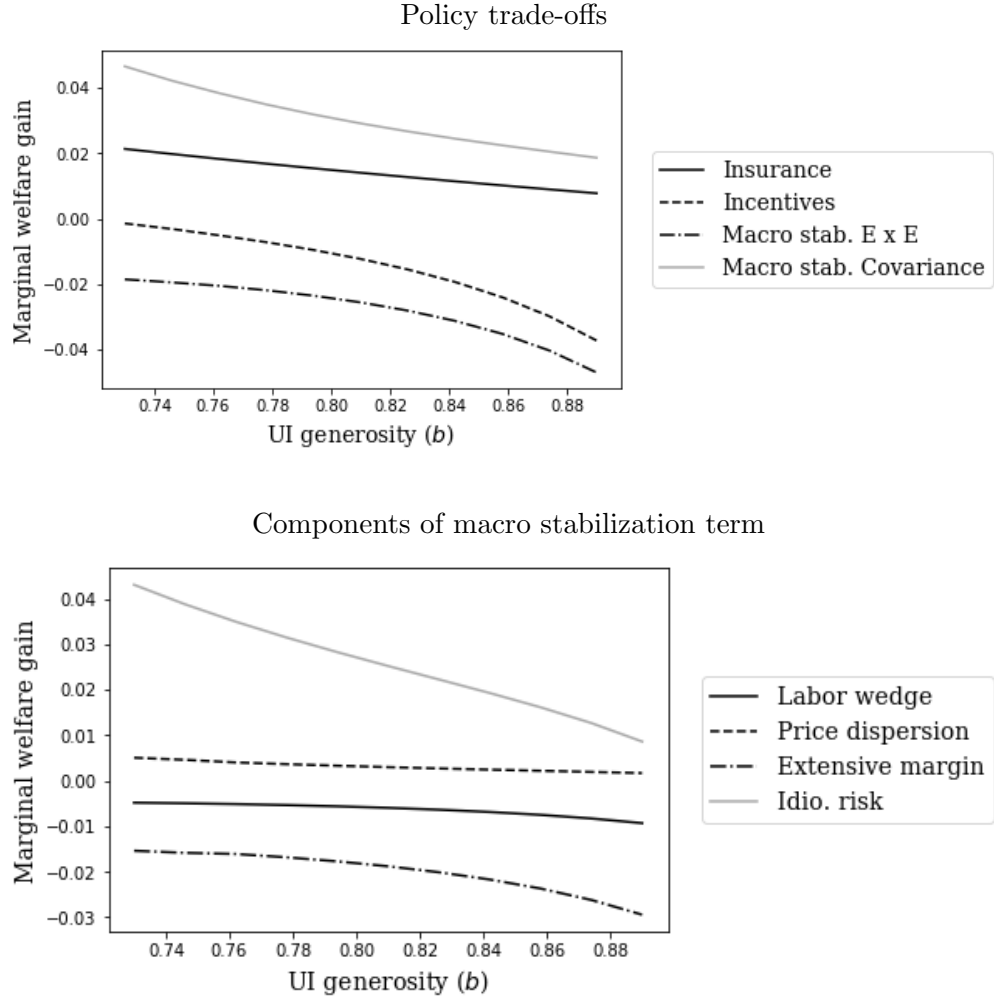
Policy trade-offs

Components of macro stabilization term

Notes: The curves in the top panel correspond to the terms in Proposition 1. The macroeconomic stabilization term is split into the piece reflecting $\mathbb{E}\left[\frac{dW_t}{dM_t}\right]\mathbb{E}\left[\frac{dM_t}{db}\right]$ and the covariance term. The curves in the bottom panel correspond to $\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{dW_t}{dM_t}\frac{dM_t}{db}$ where $\frac{dW_t}{dM_t}$ is broken into the components in Proposition 3. Both figures are scaled to units of consumption equivalent welfare per unit change in b.

6.2.3 Which features of the economy create the stabilization role for unemployment benefits?

The macro stabilization covariance term, $\text{Cov}\left(\frac{dW}{dM}, \frac{dM}{db}\right)$, can be split into a correlation and two standard deviations. The correlation is high in the model because the labor market variables move closely together in response to changes in the level of activity (Lemma 3) and welfare is primarily determined by labor market variables (Proposition 3). Most of the action from policy changes is instead in the standard deviations.

To interpret them, suppose welfare is $W_t = -(M_t - M^*)^2$ where M^* is the efficient level of activity. If $\mathbb{E}(M_t) = M^*$, then the welfare cost of business cycles would be $-\mathbb{E}(W_t) = \text{Var}(M_t) = (1/4)\text{Var}\left(\frac{dW}{dM}\right)$. Instead, if $M_t = M^* + f(b)\eta_t$, where η_t is an exogenous shock with standard deviation σ and $f(\cdot)$ is a positive and decreasing function, then $|f'(b)|$ would measure how effective the benefits are at stabilizing the economy by reducing the exposure to η_t. In this case, $\text{StDev}\left(\frac{dM}{db}\right) = |f'(b)|\sigma$ would be proportional to this welfare benefit. The two standard deviations that we focus on would then characterize the marginal welfare effect of changing the level of benefits.

Table 2 reports the covariance term, as well as the two standard deviations, for different specifications of the parameters, so we can learn what is driving the baseline results. A large value for $\text{StDev}\left(\frac{dW}{dM}\right)$ indicates that there is a substantial welfare cost from inefficient fluctuations. A large value of $\text{StDev}\left(\frac{dM}{db}\right)$ indicates that unemployment benefits are quite effective in stabilizing the economy. Rows (i) and (ii) of Table 2 summarize the baseline economy with and without aggregate shocks. Aggregate shocks raise the optimal generosity of unemployment benefits, as the macro-stabilization term is quantitatively significant.

Row (iii) of the table shows that there is almost no role for automatic stabilizers when prices are flexible. All the terms are approximately zero, just as in the case without aggregate shocks. This confirms the important role of aggregate demand and inefficient business cycles.

Row (iv) increases the coefficient on inflation in the monetary policy rule to 2.50 from 1.66. With this more aggressive monetary policy rule, there is less need for fiscal policy to manage aggregate demand. Therefore, stabilization plays a smaller role in the design of the optimal social insurance system than in the baseline calibration. Both flexible prices and aggressive monetary policy make the real interest rate respond strongly to changes in the output gap, which stabilizes the economy.

Table 2: Optimal policies under alternative specifications

		b	τ	StDev $\left(\frac{dW}{dM}\right)$	StDev $\left(\frac{dM}{db}\right)$	Cov $\left(\frac{dW}{dM}, \frac{dM}{db}\right)$
(i)	No aggregate shocks	0.746	0.248	–	–	–
(ii)	Baseline	0.824	0.216	0.279	0.098	0.027
(iii)	Flexible prices	0.740	0.246	–	See text	–
(iv)	Aggressive mon. pol.	0.753	0.244	0.161	0.034	0.005
(v)	Smaller mon. shock	0.773	0.252	0.147	0.052	0.008
(vi)	No Q^u	0.775	0.231	0.252	0.021	0.004
(vii)	Acyclical skill risk	0.753	0.245	0.074	0.099	0.006
(viii)	Acyclical G	0.807	0.224	0.267	0.089	0.023
(ix)	Acyclical G, $G/Y = 0.26$	0.811	0.250	0.272	0.088	0.024

Notes: With flexible prices, M_t is pinned down by the firm's first order condition and we find $|dM/db| < 3 \times 10^5$ across the state space. Moments of dW/dM and dM/db are computed at the optimal policy for the baseline model.

The term dM/db is therefore small as M is already stabilized. This is reflected in a small value for StDev(dM/db).

Row (v) shows that shocks to monetary policy are a key source of inefficient fluctuations in the model, and the main driver of the unemployment rate. In this experiment we cut the standard deviation of the monetary shock by half and we see that the standard deviations of both dW/db and dM/db fall by about half. This finding is consistent with the view that discount rate fluctuations are an important source of unemployment volatility (e.g., Hall, 2017).

Rows (vi) and (vii) illustrate two central mechanisms in our quantitative analysis: unemployment benefits stabilize the economy by dampening the counter-cyclical fluctuations in the precautionary savings motive generated by unemployment risk, and stabilizing the economy is beneficial because of the counter-cyclical dynamics of idiosyncratic skill risk. Starting with the first, let $Q_t^u = \mathbb{E}_t \left[1 + \upsilon \left(1 - q_{t+1} M_{t+1}\right) \left(b^{-1} - 1\right)\right]$ be the component of the Euler equation that reflects unemployment risk. Row (vi) considers a monetary policy rule:

$$I_t = \bar{I} \pi_t^{\omega_\pi} \left(\frac{1 - u_t}{1 - \bar{u}}\right)^{\omega_u} \eta_t^I \frac{1}{Q_t^u}.$$

This neutralizes this precautionary savings motive. Under this policy, aggregate demand is quite insensitive to the benefits as shown by StDev(dM/db). In this case, a main mechanism through which unemployment insurance can help stabilize the economy has been removed.

Row (vii) fixes the distribution of skill risk at its steady state distribution. Here we see that StDev(dW/dM) is much lower than in the baseline because counter-cyclical skill risk contributes a considerable amount to the welfare cost of business cycles. The combination of these two mechanisms create the rationale for increasing unemployment benefits for stabilization purposes.

Some discussions of automatic stabilizers argue for more public consumption and higher average tax rates on the grounds that this shifts aggregate demand towards a more stable source of expenditure (e.g., Andrés et al., 2008). We next ask how this logic interacts with the generosity of unemployment benefits. Rows (viii) and (ix) explore whether this consideration reduces the need for benefits to act as an automatic stabilizer. Specifically, Row (viii) modifies the model so that government purchases are no longer proportional to private consumption as dictated by equation (7), but instead given by $G_t = \bar{G}\eta_t^G$, where η_t^G is an exogenous disturbance as before. Row (ix) is similar, but raises $\bar{G}$ so the steady state G/Y ratio increases from 21% to 26%. The role of unemployment benefits as stabilizers is slightly weakened when G is acyclical, but relatively unaffected by the level of $\bar{G}$.

6.2.4 Summary

The optimal unemployment benefits are substantially more generous in the presence of aggregate risk. This is no longer true with flexible prices, since the result reflects the effect of unemployment insurance on the dynamics of aggregate demand. There are two key mechanisms at work: first, unemployment insurance dampens the precautionary savings motive that fluctuates in response to changes in unemployment risk, and second, changes in aggregate demand have a large effect on welfare through counter-cyclical income risk.

6.3 Stabilization and optimal tax progressivity

The optimal τ actually falls slightly to 0.216, as compared to 0.248 without aggregate shocks. This is shown in the top two rows of table 2. The finding of a lower τ in the presence of business cycle risk results from the joint optimization over b and τ. If we hold b fixed and optimize only over τ, then τ is hardly affected at all by business cycles. The interaction between the two policies comes from reducing τ to partly undo the labor supply distortion from raising b.

To map τ into empirical measures of progressivity, note that the ratio between the 80th and

20th percentiles of the pre-tax wage income distribution, among those 25 to 60 years old in the 2001 Survey of Cosumer Finances, is 4.76. A τ of 0.248 implies a ratio of 3.24. The optimal progressivity taking into account macroeconomic stabilization, $\tau = 0.216$, leads to a P80-20 ratio that is only slightly higher, at 3.40.
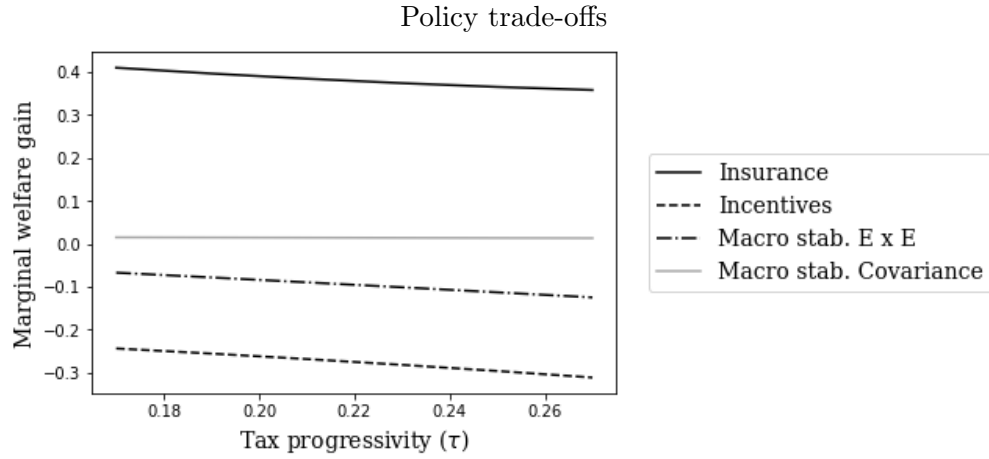
6.3.1 Why does macroeconomic stabilization not factor significantly into the choice of tax progressivity?

Figure 3 shows the policy trade-offs for the choice of τ. The insurance and incentives terms are much higher in the top panel than the stabilization covariance term. Dividing Cov $\left(\frac{dW}{dM}, \frac{dM}{d\tau}\right)$ by the incentives term in the Proposition gives a measure of the macro welfare benefits relative to the welfare loss from distorting the economy. The ratio is a mere 0.05. By comparison, dividing Cov $\left(\frac{dW}{dM}, \frac{dM}{db}\right)$ by the marginal effect of b on incentives gives 1.8. Thus, the stabilization benefit per unit of incentive distortion is much higher for b than for τ.

A back of the envelope calculation provides some insight into why the incentives-stabilization trade-off is much more attractive for unemployment benefits than for progressive taxation. A 10% increase in benefits reduces steady-state output by approximately 0.5%. It is so since the calibrated micro-elasticity of unemployment to benefits is 1/2 and the steady state unemployment rate is 6 percent. For tax progressivity, since the Frisch elasticity is 1/2, then an increase in τ of only about 0.01 would lead to the same 0.5% percent reduction in output (see equation 14). But, the 10% increase in b offers more social insurance than does raising τ by 0.01. To see this, we compute how these changes in policy affect the precautionary savings motive Q_t in the Euler equation. The increase in b reduces the standard deviation of the precautionary savings motive by 27% while the increase in τ has essentially no effect on it. Therefore, these two policy changes have the same effect on incentives by the metric of a change in steady state output, but have significantly different effects on precautionary savings motives.

The disincentive effects of unemployment insurance are much smaller than those for progressive taxes. This is partly because the former applies to the relatively small group of people who are searching for a job, while the latter applies to the much larger group of people who are employed.

Figure 3: Marginal welfare gain from changing τ for fixed b
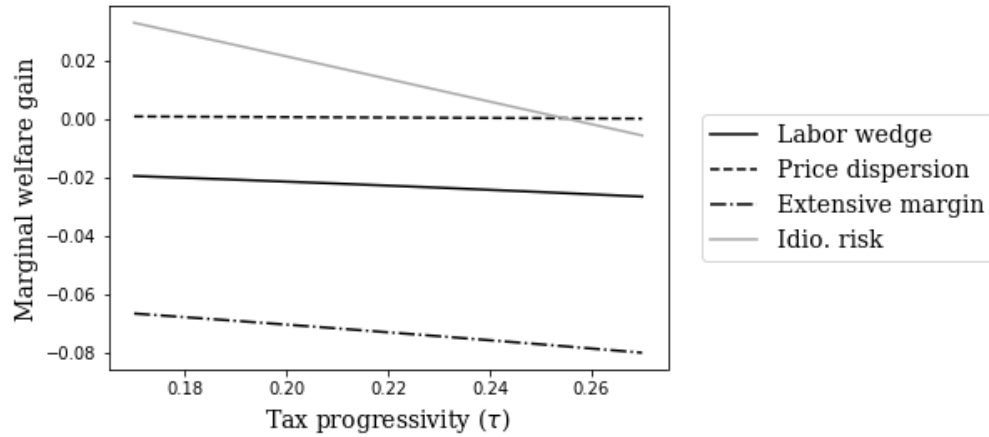
Policy trade-offs

Components of macro stabilization term

Notes: The curves in the top panel correspond to the terms in Proposition 1. The macroeconomic stabilization term is split into the piece reflecting $\mathbb{E}\left[\frac{dW_t}{dM_t}\right]\mathbb{E}\left[\frac{dM_t}{db}\right]$ and the covariance term. The curves in the bottom panel correspond to $\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{dW_t}{dM_t}\frac{dM_t}{db}$ where $\frac{dW_t}{dM_t}$ is broken into the components in Proposition 3. Both figures are scaled to units of consumption equivalent welfare per unit change in τ.

6.3.2 How do the results depend on features of the economy?

The second column of Table 2 shows the optimal τ for the alternative specifications considered there. For all of them, the optimal τ lies in a narrow range. Business cycle stabilization has only a limited effect on optimal tax progressivity.

In addition to the specifications in Table 2, we also consider two cases that may specifically affect tax progressivity. First, with more elastic labor supply, the optimal tax progressivity is lower. This familiar result reflects the larger incentive costs of progressive taxation with more elastic labor supply. Specifically, with a labor supply elasticity of 1 rather than our baseline of 1/2 we find an optimal steady state τ of 0.169 rather than our baseline of 0.248. However, the effect of business cycles is relatively similar, with an optimal τ of 0.144 in the presence of aggregate shocks, roughly the same as was the case without shocks.

Second, we consider an alternative specification in which skill risk is more cyclical. Specifically, we raise the sensitivity of the distribution of idiosyncratic risk to the unemployment rate by 25%. More cyclical skill risk raises the benefit of progressivity for two reasons. First, it makes the precautionary savings motive against skill risk more volatile, so there is more stabilization benefit from progressive taxation. Second, the welfare consequences of the cyclical fluctuations in skill risk are now more severe, and the benefit of social insurance is therefore larger. While these forces push towards a more progressive tax system, the effect is again small and we find that the optimal τ is 0.244 as compared to our baseline 0.216.

6.3.3 Summary

The optimal tax progressivity is largely independent of the business cycle because the trade-off between stabilizing the economy and distorting the economy is much less favorable for τ than it was for b.

6.4 Extensions

We now evaluate the role of these assumptions in the analysis, in particular the robustness of our finding that the stabilization role of unemployment insurance leads to a larger value of the optimal unemployment benefits. They are: the specification of the wage rule, the assumption of no savings, and the imposition of unemployment benefits that are constant over time.

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6.4.1 The wage rule

There are two features of wage determination that are important for the effectiveness of the stabilizers. The first is the cyclicality of wages, since this affects the amplitude of the business cycle. In our wage rule, the parameter ζ captures the elasticity of wages with respect to resource utilization. Making the wage more cyclical makes the intensive margin of labor supply more volatile while the unemployment rate (the extensive margin) becomes less volatile. With less variability in the unemployment rate, there is less amplification through the precautionary savings motive, and so less value in using the automatic stabilizers to stabilize the business cycle. When we double ζ, the optimal b is now 0.764 rather than 0.824 under our baseline specification. While this change reduces the role of the automatic stabilizers, it implies a counterfactually low level of unemployment volatility, at odds with a key calibration target.

The second feature of wage determination is whether there is a direct response of wages to changes in policy. This might arise from changes in benefits or take-home pay affecting the bargaining position of workers for wages. In the baseline specification, the steady state wage is independent of b, consistent with empirical work that fails to find an effect of unemployment benefits on wages.[28] However, some studies have found large effects of unemployment benefits on the equilibrium unemployment rate possibly reflecting general equilibrium effects operating through wages (Hagedorn et al., 2016). We explored how our analysis is affected when the steady state wage is increasing in the unemployment benefit. We considered an alternative in which we assume that the steady state wage in equation (28) is increasing in b such that the elasticity of the unemployment rate to benefits is 1, twice what it is in the baseline specification.

When steady state wages increase with benefits, the steady state unemployment rate is more sensitive to benefits because the incentives for hiring now fall with benefits in addition to the effect of benefits on search effort. The greater sensitivity of the steady state unemployment rate to the unemployment benefit leads to a lower optimal benefit in the absence of aggregate shocks: 0.478 as opposed to 0.746 in our baseline. However, when there are aggregate shocks, low benefits lead to strong de-stabilizing dynamics because of the precautionary savings motive. Because there is little social insurance, the precautionary savings motive is stronger and more cyclical. This causes stronger internal amplification of shocks, so fluctuations become larger and therefore costlier. The

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[28]Card et al. (2007), Van Ours and Vodopivec (2008), Lalive (2007), and Johnston and Mas (2018) find no evidence that UI generosity affects earnings upon re-employment.

automatic-stabilizer nature of unemployment benefits is substantially stronger as a result. In this case, we find that aggregate shocks lead to a large change in the optimal benefit relative to what is optimal in steady state: the optimal b rises from 0.478 to 0.755 with aggregate shocks. Therefore, a positive wage elasticity with respect to benefits leads to lower unemployment benefits overall, but a much larger effect of the business cycle on the optimal policy.

6.4.2 Aggregate borrowing and saving

Next, we evaluate the assumption that there are no assets in gross supply. This was important so far to keep both the analytical and the numerical models tractable, since we did not have to keep track of a changing wealth distribution. We now assume instead that there is a constant and policy-invariant stock of government debt that private households can hold to self insure against unemployment shocks. There is now a distribution of wealth that changes over time and responds to policy. The level of taxes adjusts to keep the level of government debt constant.

At the same time, to simplify the numerical challenged, since our model with savings focuses on unemployment risk and the role of unemployment benefits, we omit time-varying idiosyncratic skill risk. Instead, we incorporate a simpler form of heterogeneity in skills, patience and job-loss rates. Specifically, we assume that households take one of three types, which we associate with phases of the life cycle so we call them "young", "middle-age," and "old." Each age group, has its own time-preference rate β, labor productivity α, and job-loss rate υ. An individual starts young and stochastically ages to middle-age and so on. We calibrate the model to be consistent with the moments of the distribution of liquid asset holdings, earnings, and unemployment across age groups. Appendix D.4 provides a detailed description of the equilibrium conditions of the extended model and its calibration.

Evaluating the welfare benefits of varying b in this model is computationally infeasible, because it requires an accurate global solution in the presence of a time-varying distribution of skill risk and wealth. Instead we give a positive description of how unemployment benefits affect the economy to argue that there is a still a substantial effect of unemployment insurance on macroeconomic volatility albeit smaller than in our baseline model. We solve the extended model using the Reiter (2009) method, which gives a non-linear solution with respect to idiosyncratic state variables, but a linearized solution with respect to aggregate states.

Figure 4: Standard deviation of unemployment rate as function of steady state unemployment rate.
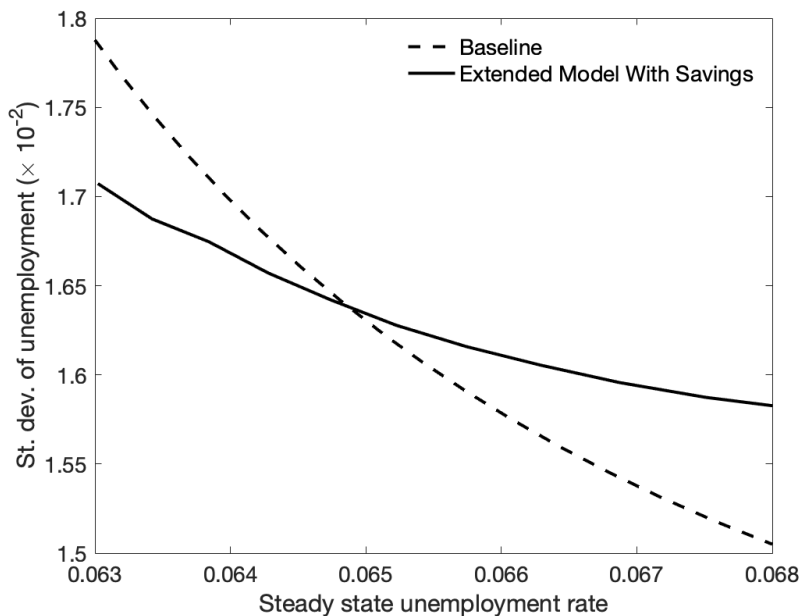

Notes: The variation along the horizontal axis is induced by changing unemployment benefit generosity.

The key implication of aggregate savings is that unemployment benefits are not the only source of insurance against unemployment spells. As a result, the precautionary savings motive is not as sensitive to changes in the level of benefits and the trade-off between the stabilization benefit of unemployment benefits and the worsening of incentives is less favorable than in our baseline analysis. Figure 4 shows how the volatility of the unemployment rate varies with the steady state unemployment rate. The variation along the horizontal axis is induced by changes in b. The slopes of the lines in the figure reveal how much stabilization is achieved for a given increase in distortion to the steady state, which is an important consideration in the policy trade-off. The line with savings is flatter than that without savings—the slope is half as large—meaning that the stabilization benefit of unemployment insurance may be weaker than our baseline model implies. Nevertheless, there is still a non-trivial stabilization benefit to raising the generosity of unemployment benefits.

6.4.3 Cylical unemployment insurance

The baseline model assumes that the unemployment benefit is a constant fraction b of the earnings of an employed household with the same level of skills. Our focus has been on this constant policy in

Figure 5: An empirical measure of unemployment benefit generosity
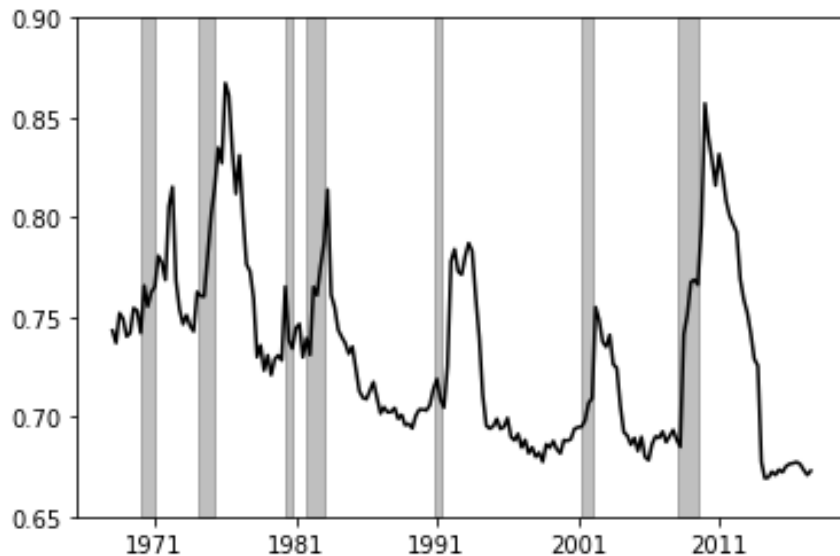
line with the view of unemployment insurance as an automatic stabilizer. However, the generosity of unemployment benefits, notably their maximum duration, can vary over the cycle and this may also serve to stabilize the economy. Does this cyclical generosity obviate the need for more generous benefits on average?

Figure 5 shows an empirical measure of unemployment insurance generosity, measured by:

$$b_t = \left(\frac{1}{2} + \frac{1}{2} \frac{\text{Total UI Payments}}{\text{Continuing claims for UI}} \times \frac{1}{\text{DPI per capita trend}} \right)^{1-\tau}. \tag{30}$$

At the heart of this estimate is the average benefit payment per recipient, which is then normalized by the trend in disposable personal income per capita, and converted from a replacement rate to a level of insurance. This measure does not control for the skill composition of the pool of unemployed. Therefore, it may overstate the cyclicality of generosity since more high-skill people become unemployed during recessions (see Mueller, 2017). The figure shows clear counter-cyclical spikes in benefit generosity. Regressing the time series for benefit generosity shown in Figure 5 on the unemployment rate gives a coefficient of 1.88.

We consider an extension to the model in which $b_t = \bar{b} + 1.88(u_t - \bar{u})$, where $\bar{b}$ and $\bar{u}$ are steady state values. Our analysis continues to focus on the optimal value of the constant $\bar{b}$. We find that the optimal $\bar{b}$ rises from 0.746 without cycles to 0.802 with them, while tax progressivity falls from

0.248 to 0.228. The implied replacement rate rises from 35% to 50%. The observed cyclicality of unemployment benefits reduces the need for more generous benefits on average, but macroeconomic stabilization still pushes for a higher average level of benefits.

7   Conclusion

Policy debates take as given that there are stabilizing benefits of unemployment insurance and income tax progressivity, but there are few systematic studies of what factors drive these benefits and how they interact with the insurance and incentive effects of these policies. In contrast, the study of these social programs in the academic literature rarely takes into account their macroeconomic stabilization role, instead treating it as a fortuitous side benefit.

This paper tries to remedy this situation. It provides a theoretical characterization of an automatic stabilizer as a fixed policy for which there is a positive covariance between the effect of macroeconomic activity on welfare, and the effect of the policy on activity. If a policy tool has this property of stimulating the economy more in recessions when resources are under-utilized, then its role in stabilizing the economy calls for expanding the use of the policy beyond what would be appropriate in a stationary environment. Overall, we found that the role of social insurance programs as automatic stabilizers affects their optimal design and, in the case of unemployment insurance, it can lead to substantial differences in the generosity of the system.

Our focus on the automatic stabilizing nature of existing social programs led us to take a Ramsey approach to the ex ante design of fiscal policy. Future work might explore how these forces affect the design of the social insurance system from a Mirrleesian perspective. Another question is how these fiscal policy programs can adjust to the state of the business cycle, taking into account measurement difficulties, time inconsistency, political economy, and other challenges of implementing state-dependent stabilization policy. There is already some research on these two and, hopefully, our analysis will provide some insights to guide their further development.

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Online Appendix

This appendix contains four sections, which in turn: (i) provide auxiliary steps to some results stated in section 3; (ii) prove the propositions in section 4; (iii) prove the propositions in section 5; and (iv) describe the methods used to solve the model in section 6.

A   Additional steps for deriving the results in section 3

A.1   The value of employment

In equilibrium, $a_{i,t} = 0$, and search effort is determined by comparing the value of working and not working according to equation (15). This section of the appendix derives the two key steps that make this difference independent of the household's skill, so that all households choose the same search effort.

Lemma 5. *The household's value function has the form*

$$V(\alpha, n, \mathcal{S}) = V^{\alpha}(\alpha, \mathcal{S}) + n V^{n}(\mathcal{S}) \tag{31}$$

for some functions V^{α} and V^{n} where $\mathcal{S}$ is the aggregate state. The choice of search effort is then the same for all searching households regardless of α.

Proof: Suppose that the value function is of the form given in equation (31). We will establish that the Bellman equation maps functions in this class into itself, which implies that the fixed point of the Bellman equation is in this class by the contraction mapping theorem. The household's search problem is

$$V^{s}(\alpha, \mathcal{S}) = \max_{q} \left\{ MqV(\alpha, 1, \mathcal{S}) + (1 - Mq)V(\alpha, 0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}.$$

Substitute for the value functions to arrive at

$$V^{s}(\alpha, \mathcal{S}) = \max_{q} \left\{ Mq \left[V^{\alpha}(\alpha, \mathcal{S}) + V^{n}(\mathcal{S}) \right] + (1 - Mq) \left[V^{\alpha}(\alpha, \mathcal{S}) \right] - \frac{q^{1+\kappa}}{1 + \kappa} \right\} \tag{32}$$

$$V^{s}(\alpha, \mathcal{S}) = \max_{q} \left\{ MqV^{n}(\mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\} + V^{\alpha}(\alpha, \mathcal{S}) \tag{33}$$

where we have brought $V^{\alpha}(\alpha, \mathcal{S})$ outside the max operator as it appears in an additively separable

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manner. As there is no α inside the max operator, the optimal q is independent of α. Note that we can write V^s as $V^s(\alpha, \mathcal{S}) = g(\mathcal{S}) + V^\alpha(\alpha, \mathcal{S})$ where g is the solution to the maximization problem above.

The Bellman equation for employed and unemployed are

$$V^\alpha(\alpha, \mathcal{S}) + V^n(\mathcal{S}) = \log\left[\lambda\left(\alpha(wh + d)\right)^{1-\tau}\right] - \frac{h^{1+\gamma}}{1+\gamma} + \beta\mathbb{E}\left[(1-\upsilon)\left(V^\alpha(\alpha', \mathcal{S}') + V^n(\mathcal{S}')\right) + \upsilon V^s(\alpha', \mathcal{S}')\right]$$

$$V^\alpha(\alpha, \mathcal{S}) = \log\left[\lambda b\left(\alpha(wh + d)\right)^{1-\tau}\right] - \xi + \beta\mathbb{E}\left[V^s(\alpha', \mathcal{S}')\right], \tag{34}$$

where we have used the budget constraint to substitute for consumption and the result that h is independent of α. Taking the difference yields

$$V^n(\mathcal{S}) = -\log(b) - \frac{h^{1+\gamma}}{1+\gamma} + \xi + \beta(1-\upsilon)\mathbb{E}\left[V^n(\mathcal{S}') - g(\mathcal{S}')\right] \tag{35}$$

and plugging $V^s(\alpha, \mathcal{S}) = g(\mathcal{S}) + V^\alpha(\alpha, \mathcal{S})$ into the continuation value of the unemployed in (34) gives

$$V^\alpha(\alpha, \mathcal{S}) = \log\left[\lambda b\left(\alpha(wh + d)\right)^{1-\tau}\right] - \xi + \beta\mathbb{E}\left[g(\mathcal{S}') + V^\alpha(\alpha', \mathcal{S}')\right].$$

$\square$

A.2 Optimal search effort

To derive optimal search effort in equation (20) we use the results of Lemma 5, specifically equations (33) and (35), using $\upsilon = 1$. This leads to

$$\max_q\left\{Mq\left[\log\frac{1}{b} - \frac{h^{1+\gamma}}{1+\gamma} + \xi\right] - \frac{q^{1+\kappa}}{1+\kappa}\right\}.$$

The first order condition yields equation (20).

A.3 Proof of lemma 2

First, the Euler equation for a household is

$$\frac{1}{c_{i,t}} \geq \beta R_t \mathbb{E}\left[\frac{1}{c_{i,t+1}}\right]$$

as usual. Using (16) we have

$$\left[\alpha_{i,t}^{1-\tau}(n_{i,t} + (1 - n_{i,t})b)\tilde{c}_t\right]^{-1} \geq \beta R_t \mathbb{E}\left[\left[\alpha_{i,t+1}^{1-\tau}(n_{i,t+1} + (1 - n_{i,t+1})b)\tilde{c}_{t+1}\right]^{-1}\right].$$

Notice that $\mathbb{E}\left[\frac{\alpha_{i,t}^{1-\tau}}{\alpha_{i,t+1}^{1-\tau}}\right] = \mathbb{E}\left[\epsilon_{i,t+1}^{\tau-1}\right]$ is common across households and is known at date t. Now consider the two cases for $n_{i,t}$ and use the EU and UU transition probabilities to arrive at

$$\tilde{c}_t^{-1} \geq \beta R_t \mathbb{E}\left[\left[1 + \upsilon\left(1 - q_{t+1}M_{t+1}\right)\left(b^{-1} - 1\right)\right]\tilde{c}_{t+1}^{-1}\right]\mathbb{E}\left[\epsilon_{i,t+1}^{\tau-1}\right] \tag{36}$$

$$\tilde{c}_t^{-1} \geq \beta R_t \mathbb{E}\left[b\left[1 + \left(1 - q_{t+1}M_{t+1}\right)\left(b^{-1} - 1\right)\right]\tilde{c}_{t+1}^{-1}\right]\mathbb{E}\left[\epsilon_{i,t+1}^{\tau-1}\right]. \tag{37}$$

The right-hand side of these inequalities is larger for the employed (we establish this formally below), so there are two possibilities: the Euler equation of the employed holds with equality or both inequalities are strict.

Here we follow Krusell et al. (2011), Ravn and Sterk (2017), and Werning (2015) in assuming that the Euler equation of the employed/high-income household holds with equality. This household is up against its constraint $a' = 0$ so there could be other equilibria in which the Euler equation does not hold with equality. The equilibrium we focus on is the limit of the unique equilibrium as the borrowing limit approaches zero from below. See Krusell et al. (2011) for further discussion of this point. Equation (36) holding with equality yields the desired result.

The right-hand side of equation (36) weakly exceeds that of (37) if

$$q_{t+1}M_{t+1} + \frac{\upsilon}{b}\left(1 - q_{t+1}M_{t+1}\right) \geq 0.$$

Notice that qM is a job finding rate $\in [0, 1]$ and $\upsilon \in [0, 1]$ and $b \in [0, 1]$.

A.4 Equilibrium definition

We first state the intermediate firm's problem and some additional equilibrium conditions and then state a definition of an equilibrium.

Firm's problem and inflation with Calvo pricing. The intermediate firm's problem is

$$
\max_{p_t^*, \{y_{j,s}, n_{j,s}, v_{j,s}\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left[\frac{p_t^*}{P_s} y_{j,s} - n_{j,s} h_{j,s} w_s - \psi_1 M_s^{\psi_2} v_{j,s} \right]
$$

subject to

$$
y_{j,s} = (p_t^*/P_s)^{\mu/(1-\mu)} Y_s
$$

$$
y_{j,s} = \eta_s^A h_s n_{j,s}
$$

$$
n_{j,s} = (1-\upsilon) n_{j,s-1} + v_{j,s}.
$$

The solution to this problem satisfies

$$
\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t^*}{p_s} \right)^{\mu/(1-\mu)} Y_s \mu \left[\frac{w_s h_s + \psi_1 M_s^{\psi_2} - R_{s+1}^{-1}(1-\upsilon)\psi_1 M_{s+1}^{\psi_2}}{\eta_s^A h_s} \right]}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t^*}{p_s} \right)^{1/(1-\mu)} Y_s}. \tag{38}
$$

The term in square brackets is real marginal cost at date s. As is standard, inflation and price dispersion evolve according to:

$$
\pi_t = \left[(1-\theta) / \left[1 - \theta \left(\frac{p_t^*}{p_t} \right)^{1/(1-\mu)} \right] \right]^{1-\mu} \tag{39}
$$

$$
S_t = (1-\theta) S_{t-1} \pi_t^{-\mu/(1-\mu)} + \theta \left(\frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)}. \tag{40}
$$

where p_t^*/p_t is the relative price chosen by firms that adjust their price in period t.

Firm's problem and inflation with sticky information. Under the assumption of a unit separation rate, the real marginal cost of the firm is:

$$\frac{w_t + \psi_1 M_t^{\psi_2}/h_t}{\eta_t^A}.$$

Marginal costs are the sum of the wage paid per effective unit of labor and the hiring costs that had to be paid, divided by productivity. The price-setting first order condition is

$$\frac{p_t^*}{P_t} = \mu\left(\frac{w_t h_t + \psi_1 M_t^{\psi_2}}{\eta_t^A h_t}\right) \tag{41}$$

for firms with full information and $\mathbb{E}_{t-1}(p_t^*)$ for others. The price level satisfies

$$P_t^{\frac{1}{1-\mu}} = \theta p_t^{*\left(\frac{1}{1-\mu}\right)} + (1-\theta)\,\mathbb{E}_{t-1}(p_t^*)^{\frac{1}{1-\mu}} \tag{42}$$

and price dispersion is given by

$$S_t = \left(\frac{p_t^*}{P_t}\right)^{\mu/(1-\mu)} \left[\theta + (1-\theta)\left[\frac{1}{1-\theta}\left(\frac{p_t^*}{P_t}\right)^{\frac{1}{\mu-1}} - \frac{\theta}{1-\theta}\right]^{\mu}\right]. \tag{43}$$

Equilibrium The aggregate resource constraint is:

$$Y_t - J_t = C_t + G_t. \tag{44}$$

The Fisher equation is:

$$R_t = I_t/\mathbb{E}_t\left[\pi_{t+1}\right]. \tag{45}$$

The link between $\tilde{c}_t$ and C_t depends on $\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]$. This evolves according to:

$$\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right] = (1-\delta)\mathbb{E}_i\left[\alpha_{i,t-1}^{1-\tau}\right]\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right] + \delta.. \tag{46}$$

The net revenues of the firm are paid out to the employed workers in the form of wages and

dividends so we have

$$Y_t - J_t = (w_t h_t + d_t)(1 - u_t) \tag{47}$$

using the aggregate production function

$$Y_t = A_t h_t (1 - u_t) \tag{48}$$

and substituting into equation (14) we arrive at

$$h_t^\gamma = \frac{(1-\tau)w_t}{A_t h_t \frac{Y_t - J_t}{Y_t}}. \tag{49}$$

An equilibrium of the economy can be calculated from a system equations in 17 variables and three exogenous processes. The variables are

$$C_t, \tilde{c}_t, u_t, \mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right], Q_t, R_t, I_t, \pi_t, Y_t, G_t, h_t, w_t, S_t, \frac{p_t^*}{p_t}, J_t, q_t, M_t.$$

And the equations are: (3), (4), (5), (6), (7), (17) (18), (19), (20), (41), (42), (43), (44), (45), (46), (48), and (49). The exogenous processes are η_t^A, η_t^G, and η_t^I.

In the quantitative model with Calvo pricing, we replace (41) with (38), (42) with (39), and (43) with (40). Moreover, with persistent employment we replace (20) with (15) and we must keep track of the value of employment in excess of unemployment, which is forward-looking, independent of α and can be calculated from (35).

B   Proofs for section 4

Proof of Lemma 3. Given b, τ, M_t, η_t^A, the variables h_t, q_t, u_t, Y_t, A_t, w_t, J_t, S_t, $\frac{p_t^*}{P_t}$ satisfy a system of 9 equations in these 9 variables. The equation that determines h_t is (49). In what follows we establish by guess and verify that if h_t satisfies the form $\mathcal{H}_h(b, \tau, M_t, \eta_t^A)$, then the solution to equation (49) satisfies the same form. In order to do so, we first establish that other variables in the system have equilibrium mappings of the same form.

Start with q_t, which is given by equation (20). Substitute $h_t = \mathcal{H}_h(b, \tau, M_t, \eta_t^A)$ into (20) to

obtain a mapping $q_t = q(b, \tau, M_t, \eta_t^A)$. Next, note that by $u_t = 1 - q_t M_t$ (i.e. (3) with $v = 1$) . Substitute in $q_t = q(b, \tau, M_t, \eta_t^A)$ to obtain a mapping $u_t = u(b, \tau, M_t, \eta_t^A)$. In turn, w_t is given by the wage rule in equation (5) and we substitute for u_t. Using equation (41) we substitute for w_t and h_t to establish a mapping for p_t^*/P_t of the same form and equation (43) then gives the mapping for $S_t = S(b, \tau, M_t, \eta_t^A)$. Using the aggregate production function $Y_t = \frac{\eta_t^A}{S_t} h_t (1 - u_t)$ we substitute to obtain a similar mapping and so too with the resources spent on recruiting $J_t = \psi_1 M_t^{\psi_2}(1 - u_t)$. Finally, we get to equation (49) rearranged as

$$
h_t = \left[\frac{(1 - \tau)w_t}{\frac{\eta_t^A}{S_t}\left(1 - \frac{J_t}{Y_t}\right)} \right]^{1/(1+\gamma)}. \tag{50}
$$

As all the variables on the right-hand side are functions of b, τ, M_t, and η_t^A, the solution has the form $h_t = \mathcal{H}_h(b, \tau, M_t, \eta_t^A)$ for some function $\mathcal{H}_h(.)$. $\qquad \square$

Proof of Lemma 4: When there is no mortality, $\delta = 0$, we can compute the cumulative welfare effect of a change in $F(\epsilon_{i,t}, u_t)$ including the effects on current and future skill dispersion. In particular

$$
\begin{aligned}
\mathbb{E}_i \log\left(\alpha_{i,t}^{1-\tau}\right) &= \mathbb{E}_i \log\left(\alpha_{i,t-1}^{1-\tau}\epsilon_{i,t}^{1-\tau}\right) \\
&= \mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau}\epsilon_{i,1}^{1-\tau}\cdots\epsilon_{i,t}^{1-\tau}\right) \\
&= \mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau}\right) + \mathbb{E}_i \log\left(\epsilon_{i,1}^{1-\tau}\right) + \cdots + \mathbb{E}_i \log\left(\epsilon_{i,t}^{1-\tau}\right).
\end{aligned}
$$

Similarly

$$
\begin{aligned}
\log\left(\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\right) &= \log\left(\mathbb{E}_i\left[\alpha_{i,t-1}^{1-\tau}\right]\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]\right) \\
&= \log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\mathbb{E}_i\left[\alpha_{i,1}^{1-\tau}\right]\cdots\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]\right) \\
&= \log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\right) + \log\left(\mathbb{E}_i\left[\alpha_{i,1}^{1-\tau}\right]\right) + \cdots + \log\left(\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]\right)
\end{aligned}
$$

Notice that in this no-mortality case, the date-t loss from skill dispersion can be written as:

$$
\mathbb{E}_i \log\left(\alpha_{i,t}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\right) = \mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\right) + \sum_{s=1}^{t}\left[\mathbb{E}_i \log\left(\epsilon_{i,s}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\epsilon_{i,s}^{1-\tau}\right]\right)\right].
$$

Finally, take the expected discounted sum of this expression and rearrange to prove the result. □

Lemma 6. *For a random variable X,*

$$\frac{d}{d\tau}\left\{\mathbb{E}\left[\log\left(X^{1-\tau}\right)\right] - \log\left(\mathbb{E}\left[X^{1-\tau}\right]\right)\right\} = \frac{\operatorname{Cov}\left(X^{1-\tau}, \log X\right)}{\mathbb{E}\left[X^{1-\tau}\right]}$$

Proof:

$$
\begin{aligned}
\frac{d}{d\tau}\left\{\mathbb{E}\left[\log\left(X^{1-\tau}\right)\right] - \log\left(\mathbb{E}\left[X^{1-\tau}\right]\right)\right\} &= -\mathbb{E}\left[\log\left(X\right)\right] + \frac{\mathbb{E}\left[X^{1-\tau}\log X\right]}{\mathbb{E}\left[X^{1-\tau}\right]} \\
&= -\mathbb{E}\left[\log\left(X\right)\right] + \frac{\mathbb{E}\left[X^{1-\tau}\right]\mathbb{E}\left[\log X\right] + \operatorname{Cov}\left(X^{1-\tau}, \log X\right)}{\mathbb{E}\left[X^{1-\tau}\right]} \\
&= \frac{\operatorname{Cov}\left(X^{1-\tau}, \log X\right)}{\mathbb{E}\left[X^{1-\tau}\right]}
\end{aligned}
$$

□

Proof of Proposition 1. For this proof, in addition to the social welfare function, (22), the relevant equations of the model are (3), (48), (4), (44), (20), and (49). Conceptually we can write the period t contribution to the objective function as $W_t = W(b, \tau, q, h, M, \eta_t^A)$ where h and q are functions of (b, τ, M_t, η_t^A) by Lemma 3. Specifically

$$
\begin{aligned}
W_t = {} & \left[\mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\right)\right] + \mathcal{R}_t + u_t \log b - \log(1 - u_t + u_t b) \\
& + (1 + \chi)\log\left(\frac{\eta_t^A}{S(b, \tau, M_t, \eta_t^A)} h_t(1 - u_t) - \psi_1 M_t^{\psi_2}(1 - u_t)\right) - (1 - u_t)\frac{h_t^{1+\gamma}}{1+\gamma} - \frac{q_t^{1+\kappa}}{1+\kappa} - \xi u_t
\end{aligned}
\tag{51}
$$

where one should interpret u_t as $1 - q_t M_t$.

The first order condition of the objective function with respect to b is then

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{dW}{db} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left.\frac{\partial W_t}{\partial b}\right|_{M,q,h} + \left.\frac{\partial W_t}{\partial q_t}\right|_M \left.\frac{\partial q_t}{\partial b}\right|_M + \left.\frac{\partial W_t}{\partial h_t}\right|_M \left.\frac{\partial h_t}{\partial b}\right|_M + \frac{dW_t}{dM_t}\frac{dM_t}{db} \right\} = 0.
$$

The first term on the right-hand side corresponds to the insurance term, the next two terms give the effect of incentives, and finally we have the macro-stabilization term. The rest of the proof expresses these derivatives as the terms shown in the proposition.

Begin with the insurance term

$$
\begin{aligned}
\left.\frac{\partial W_t}{\partial b}\right|_{M,q,h} &= \frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} \\
&= u_t \left(\frac{1}{b} - 1 + 1 - \frac{1}{1 - u_t + u_t b} \right) \\
&= u_t \left(\frac{1}{b} - 1 \right) \left(1 - \frac{u_t b}{1 - u_t + u_t b} \right).
\end{aligned}
$$

Here we have made use of the rigid price assumption to treat $S_t = 1$. Note that

$$
\begin{aligned}
\left.\frac{\partial \log (b\tilde{c}_t)}{\partial \log b}\right|_{u,h} &= \frac{\partial}{\partial \log b} \log \left(\frac{bC_t}{\Omega_t(1 - u_t + u_t b)} \right) \\
&= 1 - \frac{u_t b}{1 - u_t + u_t b}
\end{aligned}
\tag{52}
$$

where the partial derivative on the right hand side of (52) is with respect to b alone.[29] So, we have:

$$
\left.\frac{\partial W_t}{\partial b}\right|_{M,q,h} = u_t \left(\frac{1}{b} - 1 \right) \left.\frac{\partial \log (b\tilde{c}_t)}{\partial \log b}\right|_{u,h}
\tag{53}
$$

Turning to the incentives terms

$$
\begin{aligned}
\left.\frac{\partial W_t}{\partial q_t}\right|_M &= \left.\frac{\partial W_t}{\partial u_t} \frac{\partial u_t}{\partial q_t}\right| - q_t^\kappa \\
\frac{\partial W_t}{\partial u_t} &= \frac{-1}{C_t} \left(A_t h_t - \psi_1 M_t^{\psi_2} \right) + \frac{1 - b}{1 - u_t + u_t b} + \log b + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi + \frac{d\mathcal{R}_t}{du_t}
\end{aligned}
\tag{54}
$$

and combining these terms along with (20) we have

$$
\left.\frac{\partial W_t}{\partial q_t}\right|_M = \left(\frac{-1}{C_t} \left(A_t h_t - \psi_1 M_t^{\psi_2} \right) + \frac{1 - b}{1 - u_t + u_t b} + \frac{d\mathcal{R}_t}{du_t} \right) \left.\frac{\partial u_t}{\partial q_t}\right|_M.
$$

Now note that

$$
\begin{aligned}
\tilde{c}_t &\equiv \frac{C_t}{\Omega_t(1 - u_t + u_t b)} = \frac{A_t h_t(1 - u_t) - \psi_1 M_t^{\psi_2}(1 - u_t) - G_t}{\Omega_t(1 - u_t + u_t b)} \\
\left.\frac{\partial \log \tilde{c}_t}{\partial u_t}\right|_{M,\Omega,G} &= \frac{-1}{C_t} \left(A_t h_t - \psi_1 M_t^{\psi_2} \right) + \frac{1 - b}{1 - u_t + u_t b}
\end{aligned}
$$

---

[29] As $\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right]$ is an endogenous state that depends on the history of u_t, we are taking the partial derivative holding fixed this history.

so

$$\left.\frac{\partial W_t}{\partial q_t}\right|_M = \left(\left.\frac{\partial \log \tilde{c}_t}{\partial u_t}\right|_{M,\Omega,G} + \frac{d\mathcal{R}_t}{du_t}\right)\left.\frac{\partial u_t}{\partial q_t}\right|_M$$

and

$$\left.\frac{\partial W_t}{\partial q_t}\right|_M \left.\frac{\partial q_t}{\partial b}\right|_M = \left(\left.\frac{\partial \log \tilde{c}_t}{\partial u_t}\right|_{M,\Omega,G} + \frac{d\mathcal{R}_t}{du_t}\right)\left.\frac{\partial u_t}{\partial b}\right|_M.$$

Now for the incentives for the intensive margin of labor supply

$$\left.\frac{\partial W_t}{\partial h_t}\right|_M \left.\frac{\partial h_t}{\partial b}\right|_M = (1 - u_t)\left(\frac{A_t}{C_t} - h_t^\gamma\right)\left.\frac{\partial h_t}{\partial b}\right|_M.$$

Finally, note that if $\theta \in (0,1)$ then we also need to differentiate $S(b, \tau, M_t, \eta_t^A)$ with respect to b, which adds a term to $\left.\frac{\partial W_t}{\partial b}\right|_{M,q,h}$. The additional term is

$$-\frac{1+\chi}{Y_t - J_t} \frac{\eta_t^A h_t(1 - u_t)}{S_t^2} \left.\frac{\partial S_t}{\partial b}\right|_M = -\frac{Y_t}{C_t S_t} \left.\frac{\partial S_t}{\partial b}\right|_M.$$

$\square$

***Proof of Proposition* 2.** Conceptually, the proof follows the same steps as Proposition 1. The first order condition with respect to τ is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{dW_t}{d\tau} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{\left.\frac{\partial W_t}{\partial \tau}\right|_{M,q,h} + \left.\frac{\partial W_t}{\partial q_t}\right|_M \left.\frac{\partial q_t}{\partial \tau}\right|_M + \left.\frac{\partial W_t}{\partial h_t}\right|_M \left.\frac{\partial h_t}{\partial \tau}\right|_M + \frac{dW_t}{dM_t}\frac{dM_t}{d\tau}\right\} = 0.$$

For the insurance term we use Lemmas 4 and 6 to arrive at

$$\left.\frac{\partial W_t}{\partial \tau}\right|_{M,q,h} = \frac{\text{Cov}\left(\alpha_{i,0}^{1-\tau}, \log \alpha_{i,0}\right)}{\mathbb{E}\left[\alpha_{i,0}^{1-\tau}\right]} + \left.\frac{\partial \mathcal{R}_t}{\partial \tau}\right|_u.$$

For the insurance term we proceed as in the proof of Proposition 1 to write

$$\left.\frac{\partial W_t}{\partial q_t}\right|_M \left.\frac{\partial q_t}{\partial \tau}\right|_M = \left(\left.\frac{\partial \log \tilde{c}_t}{\partial u_t}\right|_{M,\Omega,G} + \frac{d\mathcal{R}_t}{du_t}\right)\left.\frac{\partial u_t}{\partial \tau}\right|_M$$

and

$$\frac{\partial W_t}{\partial h_t}\bigg|_M \frac{\partial h_t}{\partial \tau}\bigg|_M = (1 - u_t)\left(\frac{A_t}{C_t} - h_t^\gamma\right)\frac{\partial h_t}{\partial \tau}\bigg|_M.$$

$\square$

C   Proofs for section 5

Proof of Proposition 3. Differentiating (51) with respect to M_t yields

$$\frac{dW_t}{dM_t} = \frac{\partial W_t}{\partial u_t}\frac{du_t}{dM_t} + \frac{1}{C_t}\left[-\frac{Y_t}{S_t}\frac{dS_t}{dM_t} + A_t(1 - u_t)\frac{dh_t}{dM_t} - \psi_1\psi_2 M_t^{\psi_2 - 1}(1 - u_t)\right] - (1 - u_t)\frac{dh_t}{dM_t} - q_t^\kappa \frac{dq_t}{dM_t}.$$

Now use $\frac{du_t}{dM_t} = \frac{\partial u_t}{\partial M_t}\big|_q - M_t\frac{dq_t}{dM_t}$, and equations (54) and (20) to get:

$$\begin{aligned}
\frac{dW_t}{dM_t} &= -\left[\xi - \log b - \frac{h_t^{1+\gamma}}{1+\gamma}\right]\frac{\partial u_t}{\partial M_t}\bigg|_q \\
&\quad + \left[\frac{-1}{C_t}\left(A_t h_t - \psi_1 M_t^{\psi_2}\right) + \frac{1-b}{1 - u_t + u_t b} + \frac{d\mathcal{R}_t}{du_t}\right]\frac{du_t}{dM_t} \\
&\quad + \frac{1}{C_t}\left[-\frac{Y_t}{S_t}\frac{dS_t}{dM_t} + A_t(1 - u_t)\frac{dh_t}{dM_t} - \psi_1\psi_2 M_t^{\psi_2 - 1}(1 - u_t)\right] - (1 - u_t)h_t^\gamma\frac{dh_t}{dM_t}.
\end{aligned} \tag{55}$$

Using the resource constraint $C_t = A_t h_t(1 - u_t) - \psi_1 M_t^{\psi_2}(1 - u_t) - G_t$ and the definition $J_t = \psi_1 M_t^{\psi_2}(1 - u_t)$ we have

$$\frac{\partial C_t}{\partial u_t}\bigg|_{M,G} = -A_t h_t - \psi_1 M_t^{\psi_2} \tag{56}$$

$$\frac{\partial J_t}{\partial M_t}\bigg|_u = \psi_1\psi_2 M_t^{\psi_2 - 1}(1 - u_t). \tag{57}$$

Rearranging (55)-(57) yields the result. $\square$

D   Description of methods for section 6

D.1   Estimated income process

The material in this appendix describes the estimation of the time-varying skill risk process following McKay (2017). The income process is as follows: $\alpha_{i,t}$ evolves as in (2). Earnings are given by $\alpha_{i,t} w_t$ when employed and zero when unemployed. Notice that here we normalize $h_t = 1$ and subsume all movements in h_t into w_t. While this gives a different interpretation to w_t it does not affect the distribution of earnings growth rates apart from a constant term. The innovation distribution is given by

$$\epsilon_{i,t+1} \sim F(\epsilon; x_t) = \begin{cases} N(\mu_{1,t}, \sigma_1) & \text{with prob. } P_1, \\ N(\mu_{2,t}, \sigma_2) & \text{with prob. } P_2, \\ N(\mu_{3,t}, \sigma_3) & \text{with prob. } P_3 \\ N(\mu_{4,t}, \sigma_4) & \text{with prob. } P_4 \end{cases}$$

The tails of F move over time as driven by the latent variable x_t such that

$$\mu_{1,t} = \bar{\mu}_t,$$
$$\mu_{2,t} = \bar{\mu}_t + \mu_2 - x_t,$$
$$\mu_{3,t} = \bar{\mu}_t + \mu_3 - x_t,$$
$$\mu_{4,t} = \bar{\mu}_t.$$

where $\bar{\mu}_t$ is a normalization such that $\mathbb{E}_i[exp\{\epsilon_{i,t+1}\}] = 1$ in all periods.

The model period is one quarter. The parameters are selected to match the median earnings growth, the dispersion in the right tail (P90 - P50), and the dispersion in the left-tail (P50-P10) for one, three, and five year earnings growth rates computed each year using data from 1978 to 2011. In addition we target the kurtosis of one-year and five year earnings growth rates and the increase in cross-sectional variance over the life-cycle. The moments are computed from the Social Security Administration earnings data as reported by Guvenen et al. (2014) and Guvenen et al. (2015). Our objective function is a weighted sum of the squared difference between the model-implied and

empirical moments.

The estimation procedure simulates quarterly data using the observed job-finding and -separation rates and then aggregates to annual income and computes these moments. To simulate the income process, we require time series for x_t and w_t. We assume that these series are linearly related to observable labor market indicators (for details see McKay, 2017). Call the weights in these linear relationships β. We then search over the parameters P, μ, σ, and β subject to the restrictions $P_2 = P_3$ and $\sigma_2 = \sigma_3$.

Guvenen et al. (2014) emphasize the pro-cyclicality in the skewness of earnings growth rates. The estimated income process does an excellent job capturing this as shown in the top panel of figure 6. The estimated β implies a time-series for x_t which shifts the tails of the earnings distribution and gives rise the pro-cyclical skewness shown in figure 6. We regress this time-series on the unemployment rate and find a coefficient of 16.7. The fourth component of the mixture distribution occurs with very low probability, and in our baseline specification we set it to zero. This choice is not innocuous, however, because the standard deviation σ_4 is estimated to be very large and this contributes to the high kurtosis of the earnings growth distribution. In particular, omitting this component leads to a substantially smaller τ as a result of having less risk in the economy. We prefer to omit this from our baseline calibration because the interpretation of these high-kurtosis terms is unclear and we are not entirely satisfied with modeling them as permanent shocks to skill.

The resulting income process that we use in our computations is as follows: The innovation distribution is given by

$$
\epsilon_{i,t+1} \sim F(\epsilon; u_t) = \begin{cases} N(\mu_{1,t}, 0.0403) & \text{with prob. } 0.9855, \\ N(\mu_{2,t}, 0.0966) & \text{with prob. } 0.00727, \\ N(\mu_{3,t}, 0.0966) & \text{with prob. } 0.00727 \end{cases}
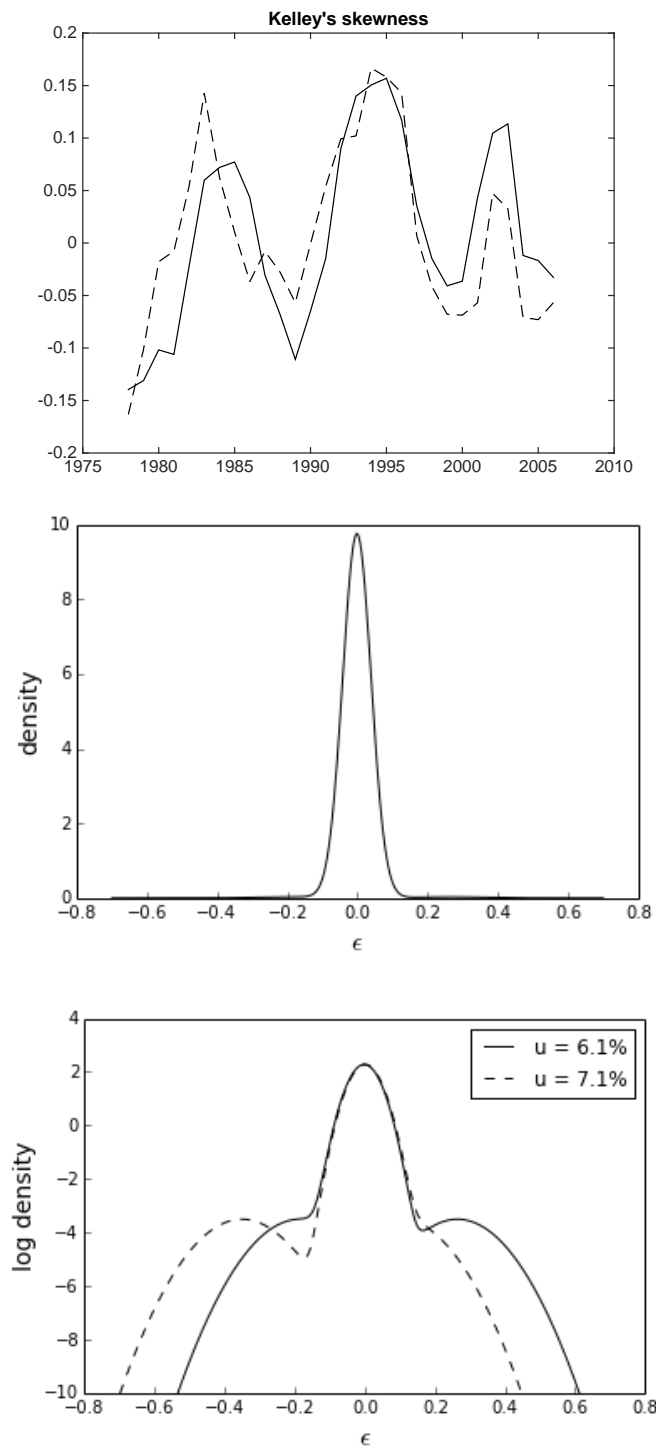$$

Figure 6: Properties of $F(\epsilon)$.

with

$$\mu_{1,t} = \bar{\mu}_t,$$

$$\mu_{2,t} = \bar{\mu}_t + 0.266 - 16.73(u_t - u^*),$$

$$\mu_{3,t} = \bar{\mu}_t - 0.184 - 16.73(u_t - u^*),$$

where u^* is the steady state is unemployment rate in our baseline calibration. The bottom panels of figure 6 show the density of ϵ and how it changes with an increase in the unemployment rate.

D.2 Global solution method

As a first step, we need to rewrite the Calvo-pricing first-order condition recursively:

$$\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \ell_s}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s},$$

where

$$\ell_s \equiv \frac{h_s w_s + \psi_1 M_s^{\psi_2} - R_{s+1}^{-1}(1-\theta)(1-\upsilon)\mathbb{E}_s \psi_1 M_{s+1}^{\psi_2}}{\eta_s^A h_s}$$

is a measure of real marginal cost. Define p_t^A as

$$p_t^A = \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \ell_s$$

and p_t^B as

$$p_t^B = \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s$$

such that

$$\frac{p_t^*}{p_t} = \frac{p_t^A}{p_t^B}.$$

p_t^A and p_t^B can be rewritten as

$$p_t^A = \mu Y_t \ell_t + (1-\theta)\mathbb{E}_t\left[\left(\frac{I_t}{\pi_{t+1}}\right)^{-1}\pi_{t+1}^{-\mu/(1-\mu)}p_{t+1}^A\right] \tag{58}$$

$$p_t^B = Y_t + (1-\theta)\mathbb{E}_t\left[\left(\frac{I_t}{\pi_{t+1}}\right)^{-1}\pi_{t+1}^{-1/(1-\mu)}p_{t+1}^B\right]. \tag{59}$$

The procedure we use builds on the method proposed by Maliar and Maliar (2015) and their application to solving a New Keynesian model. We first describe how we solve the model for a given grid of aggregate state variables and then describe how we construct the grid.

There are seven state variables that evolve according to

$$\mathbb{E}_i\left[\alpha_{i,t+1}^{1-\tau}\right] = (1-\delta)\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau}u_t\right] + \delta$$

$$\mathbb{E}_i\left[\log \alpha_{i,t+1}\right] = (1-\delta)\left[\mathbb{E}_i\left[\log \alpha_{i,t}\right] + \mathbb{E}_i\left[\log \epsilon i, t+1|u_t\right]\right]$$

$$S_{t+1}^A = S_t$$

$$u_{t+1}^{\text{Lag}} = u_t$$

$$\log \eta_{t+1}^A = \rho^A \log \eta_t^A + \varepsilon_{t+1}^A$$

$$\log \eta_{t+1}^G = \rho^G \log \eta_t^G + \varepsilon_{t+1}^G$$

$$\log \eta_{t+1}^I = \rho^I \log \eta_t^I + \varepsilon_{t+1}^I,$$

where S^A is the level of price dispersion in the previous period and the ε terms are i.i.d. normal innovations.

There are six variables that we approximate with complete second-order polynomials in the state: $(1/C_t)$, p_t^A, p_t^B, J_t, V^n and V_t, where V_t^n is the value of being employed and V_t is the value of the social welfare function. We use (17) and (18) to write the Euler equation in terms of C_t and this equation pins down $1/C_t$. p_t^A and p_t^B satisfy (58) and (59). V_t^n satisfies (35). V_t satisfies

$$V_t = W_t + \beta \mathbb{E}_t\left[V_{t+1}\right].$$

J_t satisfies $J_t = \psi_1 M_t^{\psi_2}(v - u_t)$. Abusing language slightly, we will refer to these variables that we approximate with polynomials as "forward-looking variables."

The remaining variables in the equilibrium definition can be calculated from the remaining equations and all of which only involve variables dated t. We call these the "static" variables.

To summarize, let $\mathcal{S}_t$ be the state variables, $\mathcal{X}_t$ be the forward-looking variables, and $\mathcal{Y}_t$ be the static variables. The three blocks of equations are

$$\mathcal{S}' = \mathcal{G}^{\mathcal{S}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \varepsilon')$$
$$\mathcal{X} = \mathbb{E}\,\mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$$
$$\mathcal{Y} = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}, \mathcal{X})$$

where $\mathcal{G}^{\mathcal{S}}$ are the state-transition equations, $\mathcal{G}^{\mathcal{X}}$ are the forward-looking equations and $\mathcal{G}^{\mathcal{Y}}$ are the state equations. Let $\mathcal{X} \approx \mathcal{F}(\mathcal{S}, \Omega)$ be the approximated solution for the forward-looking equations for which we use a complete second-order polynomial with coefficients given by Ω. We then operationalize the equations as follows: given a value for $\mathcal{S}$, we calculate $\mathcal{X} = \mathcal{F}(\mathcal{S}, \Omega)$ and $\mathcal{Y} = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}, \mathcal{X})$. We then take an expectation over ε' using Gaussian quadrature. For each value of ε' in the quadrature grid, we compute $\mathcal{S}' = G^{\mathcal{S}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \varepsilon')$, $\mathcal{X}' = \mathcal{F}(\mathcal{S}', \Omega)$ and $\mathcal{Y}' = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}', \mathcal{X}')$. We can now evaluate $\mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$ for this value of ε' and looping over all the values in the quadrature grid we can compute $\hat{\mathcal{X}} = \mathbb{E}\,\mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$. $\hat{\mathcal{X}}$ will differ from the value of $\mathcal{X}$ that was obtained initially from $\mathcal{F}(\mathcal{S}, \Omega)$. We repeat these steps for all the values of $\mathcal{S}$ in our grid for the aggregate state space. We then adjust the coefficients Ω part of the way towards those implied by the solutions $\hat{\mathcal{X}}$. We then iterate this procedure to convergence of Ω.

Evaluating some of the equations of the model involves taking integrals against the distribution of idiosyncratic skill risk $\epsilon_{i,t+1} \sim F(\epsilon_{i,t+1}, u_t)$. We do this using Gaussian quadrature within each of the components of the mixture distribution.

We use a two-step procedure to construct the grid on the aggregate state space. We have seven aggregate states so we choose the grid to lie in the region of the aggregate state space that is visited by simulations of the solution. We create a box of policy parameters $[b_L, b_H] \times [\tau_L, \tau_H]$. We then create a grid of twelve Sobol points on this box and for each pair (b, τ) we use the procedure of Maliar and Maliar (2015) to construct a grid on the aggregate state space and solve the model. This procedure iterates between solving the model and simulating the solution and constructing a grid in the part of the state space visited by the simulation. This gives us twelve grids, which we

then merge and eliminate nearby points using the techniques of Maliar and Maliar (2015). This leaves us with one grid that we use to solve the model when we evaluate policies. Each of the grids that we construct have 100 points.

D.3   The policy trade-offs in the quantitative model

We now explain how the policy trade-offs documented in Propositions 1, 2, and 3 can be calculated in the richer quantitative model.

The social welfare function is

$$
\begin{aligned}
V &\left(\mathbb{E}_i \left[\alpha_i^{1-\tau} \right], S_{-1}, \eta, u_{-1}, \mathbb{E}_i \log \left(\alpha_{i,t} \right) \right) \\
&= (1-\tau) \mathbb{E}_i \log \left(\alpha_i \right) - \log \left(\mathbb{E}_i \left[\alpha_i^{1-\tau} \right] \right) + u \log b - \log(1 - u + ub) \\
&+ (1+\chi) \log \left(\frac{\eta^A}{S} h(1-u) - J \right) - (1-u) \frac{h^{1+\gamma}}{1+\gamma} - (u_{-1} + \upsilon (1 - u_{-1})) \frac{q^{1+\kappa}}{1+\kappa} - u\xi \\
&+ \beta \mathbb{E} \left[V \left(\mathbb{E}_i \left[\alpha_i^{1-\tau} \right]', S, \eta', u, \mathbb{E}_i \log \left(\alpha_i \right)' \right) \right].
\end{aligned}
$$

In addition we will use the following equations of the model

$$
\begin{aligned}
u &= \left[u_{-1} + \upsilon \left(1 - u_{-1} \right) \right] \left(1 - qM \right) \\
h^{1+\gamma} &= (1-\tau)\bar{w} \left(1 - \frac{J}{Y} \right)^{-1} S \left(\frac{1-u}{1-\bar{u}} \right)^\zeta \\
q^\kappa &= MV^n \left(\mathbb{E}_i \left[\alpha_i^{1-\tau} \right], A, S_{-1}, \eta^I, \eta^G, u_{-1}, \mathbb{E}_i \log \left(\alpha_{i,t} \right) \right) \\
J &= \psi_1 M^{\psi_2} \left[1 - u - (1-\upsilon)(1 - u_{-1}) \right] \\
V^n (\cdots) &= \left[-\log \left(b \right) - \frac{h^{1+\gamma_1}}{1+\gamma_1} + \xi \right] \\
&+ \beta(1-\upsilon) \mathbb{E} \left[\left(1 - \frac{\kappa}{1+\kappa} q'M' \right) V^n \left(\mathbb{E}_i \left[\alpha_i^{1-\tau} \right]', A', S, \eta^{I'}, \eta^{G'}, u, \mathbb{E}_i \log \left(\alpha_i \right)' \right) \right]
\end{aligned}
$$

Insurance term (*b*)   Take the derivative of V with respect to b taking q, h, and M as given

$$
V_{\text{Insur}} = \frac{u}{b} - \frac{u}{1 - u + ub} + \beta \mathbb{E} \left[V'_{\text{Insur}} \right].
$$

Incentives term (b)   First, take the derivative of V with respect to q and multiply it by dq/db taking M as given:

$$V_{\text{Incen-q}} = \frac{\partial W}{\partial u}\frac{\partial u}{\partial q}\frac{\partial q}{\partial b}$$

$$- \left(u_{-1} + v\left(1 - u_{-1}\right)\right)q^{\kappa}\frac{\partial q}{\partial b} + \beta\mathbb{E}\left[V'_{\text{Incen-q}}\right] + \beta\mathbb{E}\left[V'_{u_{-1}}\right]\frac{\partial u}{\partial q}\frac{\partial q}{\partial b}$$

where

$$\frac{\partial W}{\partial u} = \left(\log b + \frac{1 - b}{1 - u + ub}\right) - \frac{1}{C}\frac{1 + \chi}{1 + \chi\eta^{G}}\left(\frac{\eta^{A}}{S}h + \frac{\partial J}{\partial u}\right) + \frac{h^{1+\gamma}}{1 + \gamma} - \xi + \frac{d\mathcal{R}}{du}$$

$$\frac{\partial u}{\partial q} = -M\left[u_{-1} + v\left(1 - u_{-1}\right)\right]$$

$$\frac{\partial q}{\partial b} = \frac{1}{\kappa}\left(MV^{n}\right)^{\frac{1}{\kappa}-1}M\frac{\partial V^{n}}{\partial b} = \frac{1}{\kappa}\frac{q}{V^{n}}\frac{\partial V^{n}}{\partial b}$$

$$\frac{\partial h}{\partial b} \approx \frac{\partial h}{\partial u}\frac{\partial u}{\partial q}\frac{\partial q}{\partial b} + \frac{\partial h}{\partial\bar{u}}\frac{\partial\bar{u}}{\partial q}\frac{\partial\bar{q}}{\partial b} = \frac{\partial h}{\partial q}\frac{1}{\kappa}\frac{q}{V^{n}}\frac{\partial V^{n}}{\partial b} + \frac{\partial h}{\partial\bar{u}}\frac{\partial\bar{u}}{\partial b}$$

$$\frac{\partial V^{n}}{\partial b} = -\frac{1}{b} - h^{\gamma}\frac{\partial h}{\partial b} + \beta(1 - v)\mathbb{E}\left[\left(1 - \frac{\kappa}{1 + \kappa}q'M'\right)\frac{\partial V^{n}}{\partial b}' - \frac{\kappa}{1 + \kappa}M'\frac{\partial q'}{\partial b}V^{n'}\right]$$

$$= -\frac{1}{b} - h^{\gamma}\frac{\partial h}{\partial b} + \beta(1 - v)\mathbb{E}\left[\left(1 - \frac{\kappa}{1 + \kappa}q'M'\right)\frac{\partial V^{n}}{\partial b}' - \frac{1}{1 + \kappa}M'^{1/\kappa+1}\left(V^{n'}\right)^{\frac{1}{\kappa}}\frac{\partial V^{n'}}{\partial b}\right]$$

$$= -\frac{1}{b} - h^{\gamma}\frac{\partial h}{\partial b} + \beta(1 - v)\mathbb{E}\left[\left(1 - \frac{\kappa}{1 + \kappa}q'M'\right)\frac{\partial V^{n'}}{\partial b} - \frac{1}{1 + \kappa}M'q'\frac{\partial V^{n'}}{\partial b}\right]$$

$$= -\frac{1}{b} - h^{\gamma}\frac{\partial h}{\partial b} + \beta(1 - v)\mathbb{E}\left[\left(1 - q'M'\right)\frac{\partial V^{n'}}{\partial b}\right]$$

$$= -\frac{1}{b} - h^{\gamma}\left(\frac{\partial h}{\partial q}\frac{1}{\kappa}\frac{q}{V^{n}}\frac{\partial V^{n}}{\partial b} + \frac{\partial h}{\partial\bar{u}}\frac{\partial\bar{u}}{\partial b}\right) + \beta(1 - v)\mathbb{E}\left[\left(1 - q'M'\right)\frac{\partial V^{n'}}{\partial b}\right]$$

$$= \left(1 + h^{\gamma}\frac{\partial h}{\partial q}\frac{1}{\kappa}\frac{q}{V^{n}}\right)^{-1}\left[-\frac{1}{b} - h^{\gamma}\frac{\partial h}{\partial\bar{u}}\frac{\partial\bar{u}}{\partial b} + \beta(1 - v)\mathbb{E}\left[\left(1 - q'M'\right)\frac{\partial V^{n'}}{\partial b}\right]\right]$$

$$\frac{\partial h}{\partial u} = -\frac{\zeta}{1 + \gamma}\frac{h}{1 - u}$$

$$\frac{\partial h}{\partial\bar{u}} = \frac{\zeta}{1 + \gamma}\frac{h}{1 - \bar{u}}$$

In forming $\frac{\partial h}{\partial b}$ we ignore changes in J/Y as this ratio is always small. Second, take the derivative of V with respect to h and multiply it by dh/db taking M as given:

$$V_{\text{Incen-h}} = (1 - u)\left[\frac{A}{C}\frac{1 + \chi}{1 + \chi\eta^{G}} - h^{\gamma}\right]\frac{\partial h}{\partial b}.$$

Note

$$\frac{d\mathcal{R}}{du} = \beta \frac{d\mathbb{E}\left[V'\right]}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}\frac{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}{du} + \beta \frac{d\mathbb{E}\left[V'\right]}{d\mathbb{E}_i\log\left(\alpha_i\right)}\frac{d\mathbb{E}_i\log\left(\alpha_i\right)}{du}$$

Macro-stabilization term (b) Take the derivative of V with respect to M and multiply by the derivative of M with respect to b

$$
\begin{aligned}
V_M =& \left[\left(\log b + \frac{1-b}{1-u+ub}\right) + \frac{h^{1+\gamma}}{1+\gamma} - \xi\right]\frac{du}{dM}\frac{dM}{db} - (u_{-1} + \upsilon\,(1-u_{-1}))\,q^\kappa \frac{dq}{dM}\frac{dM}{db}\\
&+ \left[-\frac{1}{C}\frac{1+\chi}{1+\chi\eta^G}\left(\frac{A}{S}h + \frac{\partial J}{\partial u}|_M\right)\right]\frac{du}{dM}\frac{dM}{db}\\
&- \frac{1}{C}\frac{1+\chi}{1+\chi\eta^G}\psi_1\psi_2 M^{\psi_2-1}\left[1-u-(1-\upsilon)(1-u_{-1})\right]\frac{dM}{db}\\
&+ \beta\mathbb{E}\left[V'_{u_{-1}}\right]\frac{du}{dM}\frac{dM}{db}\\
&+ \left[\frac{1}{C}\frac{1+\chi}{1+\chi\eta^G}\frac{A}{S}(1-u) - (1-u)h^\gamma\right]\frac{dh}{dM}\frac{dM}{db}\\
&- \frac{1}{C}\frac{1+\chi}{1+\chi\eta^G}\frac{A}{S^2}h(1-u)\frac{dS}{dM}\frac{dM}{db} + \beta\mathbb{E}\left[V'_{S_{-1}}\right]\frac{dS}{dM}\frac{dM}{db}\\
&+ \frac{d\mathcal{R}}{du}\frac{du}{dM}\frac{dM}{db} + \beta\mathbb{E}\left[V'_M\right]
\end{aligned}
$$

The first line is unemployment risk; the second, third, and fourth lines are the Hosios terms as they reflect the gain in resources from reducing unemployment and the loss from tightening the labor market; the fifth line is the labor wedge; the sixth line is price dispersion; the seventh line is skill risk.

We can compute $\frac{du}{dM}$, $\frac{dq}{dM}$, $\frac{dh}{dM}$, and $\frac{dS}{dM}$ numerically from a monetary shock. We compute $\frac{dM}{db}$ from the finite difference across b.

The summary statistics of skill dispersion evolve according to:

$$
\begin{aligned}
\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right] &= (1-\delta)\mathbb{E}_i\left[\alpha_{i,t-1}^{1-\tau}\right]\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right] + \delta\\
\frac{d}{dM}\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right] &= (1-\delta)\mathbb{E}_i\left[\alpha_{i,t-1}^{1-\tau}\right]\frac{d}{dM}\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]\\
\mathbb{E}_i\left[\log\left(\alpha_{i,t}\right)\right] &= (1-\delta)\mathbb{E}_i\left[\log\left(\alpha_{i,t-1}\right)\right] + (1-\delta)\mathbb{E}_i\left[\log\left(\epsilon_{i,t}\right)\right]\\
\frac{d}{dM}\mathbb{E}_i\left[\log\left(\alpha_{i,t}\right)\right] &= (1-\delta)\frac{d}{dM}\mathbb{E}_i\left[\log\left(\epsilon_{i,t}\right)\right].
\end{aligned}
$$

V_M reflects in part the change in value from how M affects future state variables, which can be calculated from the envelope conditions:

$$V_{u_{-1}} = -(1-v)\frac{q^{1+\kappa}}{1+\kappa}$$

$$\left\{ \left(\log b + \frac{1-b}{1-u+ub} \right) - \frac{1+\chi}{C}\frac{A}{S}h + \frac{h^{1+\gamma}}{1+\gamma} - \xi \right\}\frac{du}{du_{-1}} - \frac{1+\chi}{C}\frac{dJ}{du_{-1}}$$

$$+ \left(\frac{1+\chi}{C}\frac{A}{S} - h^\gamma \right)(1-u)\frac{dh}{du_{-1}} - (u_{-1} + v(1-u_{-1}))q^\kappa \frac{dq}{du_{-1}}$$

$$\beta\,\mathbb{E}\left[V'_{\mathbb{E}_i[\alpha_i^{1-\tau}]}\frac{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]'}{du_{-1}} + V'_S\frac{dS}{du_{-1}} + V'_{u_{-1}}\frac{du}{du_{-1}} \right]$$

$$V_{\mathbb{E}_i[\alpha_i^{1-\tau}]} = -\frac{1}{\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}$$

$$+ \left\{ \left(\log b + \frac{1-b}{1-u+ub} \right) - \frac{1+\chi}{C}\frac{A}{S}h + \frac{h^{1+\gamma}}{1+\gamma} - \xi \right\}\frac{du}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} - \frac{1+\chi}{C}\frac{dJ}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}$$

$$+ \left(\frac{1+\chi}{C}\frac{A}{S} - h^\gamma \right)(1-u)\frac{dh}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} - (u_{-1} + v(1-u_{-1}))q^\kappa \frac{dq}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]}$$

$$\beta\,\mathbb{E}\left[V'_{\mathbb{E}_i[\alpha_i^{1-\tau}]}\frac{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]'}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} + V'_S\frac{dS}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} + V'_{u_{-1}}\frac{du}{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]} \right]$$

$$V_{\mathbb{E}_i\log(\alpha_{i,t})} = 1 - \tau$$

$$+ \left\{ \left(\log b + \frac{1-b}{1-u+ub} \right) - \frac{1+\chi}{C}\frac{A}{S}h + \frac{h^{1+\gamma}}{1+\gamma} - \xi \right\}\frac{du}{d\mathbb{E}_i\log(\alpha_{i,t})} - \frac{1+\chi}{C}\frac{dJ}{d\mathbb{E}_i\log(\alpha_{i,t})}$$

$$+ \left(\frac{1+\chi}{C}\frac{A}{S} - h^\gamma \right)(1-u)\frac{dh}{d\mathbb{E}_i\log(\alpha_{i,t})} - (u_{-1} + v(1-u_{-1}))q^\kappa \frac{dq}{d\mathbb{E}_i\log(\alpha_{i,t})}$$

$$\beta\,\mathbb{E}\left[V'_{\mathbb{E}_i[\alpha_i^{1-\tau}]}\frac{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]'}{d\mathbb{E}_i\log(\alpha_{i,t})} + V'_S\frac{dS}{d\mathbb{E}_i\log(\alpha_{i,t})} + V'_{u_{-1}}\frac{du}{d\mathbb{E}_i\log(\alpha_{i,t})} \right]$$

$$V_{S_{-1}} = -\frac{1+\chi}{C}\frac{Y}{S}\frac{dS}{S_{t-1}}$$
$$+ \left\{\left(\log b + \frac{1-b}{1-u+ub}\right) - \frac{1+\chi}{C}\frac{A}{S}h + \frac{h^{1+\gamma}}{1+\gamma} - \xi\right\}\frac{du}{dS_{-1}} - \frac{1+\chi}{C}\frac{dJ}{dS_{-1}}$$
$$+ \left(\frac{1+\chi}{C}\frac{A}{S} - h^\gamma\right)(1-u)\frac{dh}{dS_{-1}} - (u_{-1} + v(1-u_{-1}))q^\kappa\frac{dq}{dS_{-1}}$$
$$\beta\,\mathbb{E}\left[V'_{\mathbb{E}_i[\alpha_i^{1-\tau}]}\frac{d\mathbb{E}_i\left[\alpha_i^{1-\tau}\right]'}{dS_{-1}} + V'_S\frac{dS}{dS_{-1}} + V'_{u_{-1}}\frac{du}{dS_{-1}}\right].$$

Insurance term (τ)   Take the derivative of V with respect to τ taking q, h, and M as given

$$V_{\text{Insur}} = \frac{\text{Cov}(\alpha_i^{1-\tau}, \log\alpha_i)}{\mathbb{E}[\alpha_i^{1-\tau}]} + \beta\,\mathbb{E}\left[V_{\text{Insur}}\right]$$

Incentives term (τ)   The structure of this term parallels that for b. In this case we need

$$\frac{\partial q}{\partial\tau} = \frac{1}{\kappa}\frac{q}{V^n}\frac{\partial V^n}{\partial\tau}$$
$$\frac{\partial h}{\partial\tau} \approx -\frac{h}{1+\gamma}\frac{1}{1-\tau} + \frac{\partial h}{\partial q}\frac{\partial q}{\partial\tau} + \frac{\partial h}{\partial\bar{u}}\frac{\partial\bar{u}}{\partial\tau}$$
$$= -\frac{h}{1+\gamma}\frac{1}{1-\tau} + \frac{\partial h}{\partial q}\frac{1}{\kappa}\frac{q}{V^n}\frac{\partial V^n}{\partial\tau} + \frac{\partial h}{\partial\bar{u}}\frac{\partial\bar{u}}{\partial\tau}$$
$$\frac{\partial V^n}{\partial\tau} = -h^\gamma\frac{\partial h}{\partial\tau} + \beta(1-v)\,\mathbb{E}\left[-\frac{\kappa}{1+\kappa}M'V^{n\prime}\frac{\partial q'}{\partial\tau} + \left(1-\frac{\kappa}{1+\kappa}q'M'\right)\frac{\partial V^{n\prime}}{\partial\tau}\right]$$
$$= -h^\gamma\frac{\partial h}{\partial\tau} + \beta(1-v)\,\mathbb{E}\left[-\frac{1}{1+\kappa}M'q'\frac{\partial V^{n\prime}}{\partial\tau} + \left(1-\frac{\kappa}{1+\kappa}q'M'\right)\frac{\partial V^{n\prime}}{\partial\tau}\right]$$
$$= -h^\gamma\frac{\partial h}{\partial\tau} + \beta(1-v)\,\mathbb{E}\left[(1-q'M')\frac{\partial V^{n\prime}}{\partial\tau}\right]$$
$$= \left(1 + h^\gamma\frac{\partial h}{\partial q}\frac{1}{\kappa}\frac{q}{V^n}\right)^{-1}\left\{\frac{h^{1+\gamma}}{1+\gamma}\frac{1}{1-\tau} - h^\gamma\frac{\partial h}{\partial\bar{u}}\frac{\partial\bar{u}}{\partial\tau} + \beta(1-v)\,\mathbb{E}\left[(1-q'M')\frac{\partial V^{n\prime}}{\partial\tau}\right]\right\}$$

Macro-stabilization term (τ)   As above, but with $dM/d\tau$ in place of $dM/d\tau$.

D.4   Extended model with savings

Define $V(a,n,\epsilon)$ as the value of being employed ($n=1$) or unemployed($n=0$) with assets a and in group ϵ. We omit aggregate states for simplicity of notation. The value of searching, V^s, satisfies

$$V^s(a,\epsilon) = \max_q\left\{MqV(a,1,\epsilon) + (1-Mq)V(a,0,\epsilon) - \frac{q^{1+\kappa}}{1+\kappa}\right\}. \tag{60}$$

The decision problem of the employed household is

$$V(a, 1, \epsilon) = \max_{c,h,a'} \left\{ \log(c) - \frac{h^{1+\gamma}}{1+\gamma} + \beta_\epsilon (1 - v_\epsilon) V(a', 1, \epsilon') + \beta_\epsilon v_\epsilon V^s(a', \epsilon') \right\} \quad (61)$$

subject to the budget constraint

$$a' + c = \lambda \left(wh + d \right)^{1-\tau} \alpha_\epsilon^{1-\tau} + Ra.$$

The decision problem of the unemployed household is

$$V(a, 0, \epsilon) = \max_{c,a'} \left\{ \log(c) - \xi + \beta_\epsilon V^s(a', \epsilon') \right\} \quad (62)$$

subject to the budget constraint

$$a' + c = b\lambda \left(wh(a, \epsilon) + d \right)^{1-\tau} \alpha_\epsilon^{1-\tau} + Ra$$

where $h(a, \epsilon)$ is the hours the household would have worked had they been employed.

The optimal q solves

$$q(a, \epsilon)^\kappa = M \left(V(a, 1, \epsilon) - V(a, 0, \epsilon) \right).$$

And substituting this into the definition of V_t^s

$$V^s(a, \epsilon) = V(a, 0, \epsilon) + \frac{\kappa}{1+\kappa} \left(M \left[V(a, 1, \epsilon) - V(a, 0, \epsilon) \right] \right)^{1+1/\kappa}.$$

The Euler equation for a household is

$$\frac{1}{c} \geq \beta R_t \mathbb{E} \left[\frac{1}{c'} \right].$$

The labor supply optimality condition is

$$h^\gamma = \frac{1}{c} \lambda (1 - \tau) w \left(wh + d \right)^{-\tau} \alpha_\epsilon^{1-\tau}.$$

We track the distribution of wealth before the employment and group transitions occur at the start of the period, call the distribution $\Gamma_t(a, n, \epsilon)$. For convenience, let $\tilde{\Gamma}_t(a, n, \epsilon)$ be the distribution

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of wealth after transitions have occurred. The two are related according to

$$\tilde{\Gamma}_t(a, 1, \epsilon') = \sum_\epsilon \Pr(\epsilon'|\epsilon) \left\{ (1 - \upsilon_\epsilon)\Gamma_t(a, 1, \epsilon) + M_t q_t(a, \epsilon) \left[\Gamma_t(a, 0, \epsilon) + \upsilon_\epsilon \Gamma_t(a, 1, \epsilon) \right] \right\}.$$

We then have average labor supply among workers of

$$\mathcal{H}_t = \int h_t(a)\alpha_\epsilon d\tilde{\Gamma}_t(a, 1, \epsilon) / \int d\tilde{\Gamma}_t(a, 1, \epsilon).$$

and aggregate consumption is given by

$$\sum_n \int c_t(a, n, \epsilon) d\tilde{\Gamma}_t(a, n, \epsilon).$$

The government's receipts are from labor income taxes

$$\int \alpha_\epsilon \left[w_t h_t(a, \epsilon) \right] + d_t - \lambda_t \left(w_t h_t(a, \epsilon) + d_t \right)^{1-\tau} \alpha_\epsilon^{1-\tau} d\tilde{\Gamma}_t(a, 1, \epsilon)$$

and it's outlays are G_t, interest $(R_t - 1)B$ where R_t is the ex post real return on bonds $R_t = (1 + i_{t-1})/\pi_t$, and UI payments

$$b\lambda_t \int \alpha_\epsilon^{1-\tau} \left(w_t h_t(a, \epsilon) + d_t \right)^{1-\tau} d\tilde{\Gamma}_t(a, 0, \epsilon).$$

The firm's problem is the same as in the baseline economy with the exception that we replace h_t with the skill-weighted average work effort among employed households, denoted $\mathcal{H}_t$.

An equilibrium of the economy can be calculated from a system equations that is similar to the baseline economy, but with aggregate work effort and consumption replaced by the equations above. As those equations depend on the policy rules and distribution of wealth, we also require equations that dictate how the distribution of wealth evolves, and how the policy rules are determined. The Reiter (2009) method the Euler equation, labor supply condition, and search effort first-order condition, and Bellman equations, at many points for the individual states and interpolates the policy rules between them. The distribution of wealth is approximated as a histogram that evolves according to the idiosyncratic shocks and the policy rules.

To calibrate the model, we think of an age group as approximately 12 years of life and set the

probability of stochastically aging to 1/48. We use the 2001 SCF and divide the sample into age groups 25 to 36, 37 to 48 and 49 to 60. For each group we compute the median liquid assets, median earnings, and unemployment rate. We then set the values of β_ϵ, α_ϵ, and υ_ϵ to target these moments. Liquid assets are defined as the sum of liquid accounts ("liq" in the SCF extracts sums checking, savings, and money market accounts), directly held mutual funds, stocks, and bonds less revolving debt. Following Kaplan et al. (2014), liquid account holdings are scaled by 1.05 to reflect cash holdings.