New Keynesian DSGE

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Introduction

New Keynesian DSGE

- the workhorse model for monetary policy analysis
  - backbone of medium-scale DSGE models (Smet and Woutes, 2007)
  - widely used by central banks (FED, Bank of England, PBoC ?, ...)
- same methodology to RBC:
  - dynamic stochastic general equilibrium (DSGE)
  - rational expectation equilibrium (REE)
- key features that depart from RBC
  - nominal rigidities $\rightsquigarrow$ price is not flexible $\rightarrow$ MP is more effective
  - monopolistic competition $\rightsquigarrow$ market power of price setting $\rightarrow$ GE is distorted

Outline

- DSGE + money via cash-in-advance (CIA) constraint

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- DSGE + money + nominal rigidity via Calvo-type price stickiness $\rightsquigarrow$ NK Philips Curve

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- DSGE + money + nominal rigidity + monetary policy

Cash-in-advance Model

Setup

- a final good with price P_t
- a representative household
  - use money to buy consumption $\rightsquigarrow$ money demand
- competitive firms $\rightsquigarrow$ no distortions
- a central bank
  - inject money to households $\rightsquigarrow$ money supply

Household

- beginning of t: holds wealth H_t (state variable) $+$ gov. injects new money X_t
- given $H_t + X_t$, chooses money/cash M_t (no interest) and bond B_t (earns $R_{bt} > 0$)

$$M_t + B_t \leq H_t + X_t \tag{1}$$

- use cash to buy consumption c_t, facing CIA constraint

$$P_t c_t \leq M_t \tag{2}$$

  - **Q**: household has no incentive to hold cash without CIA, why?
- chooses labor n_t and capital investment $k_{t+1} = (1-\delta)k_t + i_t$
- end of t: receives wage and capital income: $W_t n_t + R_t k_t$
- next-period wealth H_{t+1}:

$$H_{t+1} = M_t + (1 + R_{bt}) B_t + W_t n_t + R_t k_t - P_t (c_t + i_t) \tag{3}$$

Household's problem

- maximizes life-time utility $\sum_{t=0}^{\infty} \beta^t \left(\log c_t - \psi n_t\right)$
  - subject to constraints (1), (2), (3)
- define real variables:

$$
\begin{aligned}
z_t &\equiv Z_t / P_t, \text{for } Z = \{M, B, X, W, R\} \\
h_{t+1} &\equiv H_{t+1} / P_t
\end{aligned}
$$

Household's problem

- rewrite household's optimization problem as

$$\max \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \psi n_t\right)$$

- subject to

$$\begin{aligned}
m_t + b_t &\leq \frac{h_t}{1+\pi_t} + x_t \\
c_t &\leq m_t \\
h_{t+1} &= m_t + \left(1 + R_{bt}\right) b_t + w_t n_t + r_t k_t \\
&\quad - \left[c_t + k_{t+1} - \left(1 - \delta\right) k_t\right]
\end{aligned}$$

Household's problem

- rewrite household's optimization problem as

$$\max \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \psi n_t\right)$$

- subject to

$$m_t + b_t \leq \frac{h_t}{1+\pi_t} + x_t \rightsquigarrow \mu_t \quad (4)$$

$$c_t \leq m_t \rightsquigarrow \eta_t \quad (5)$$

$$h_{t+1} = m_t + (1 + R_{bt})\, b_t + w_t n_t + r_t k_t - \left[c_t + k_{t+1} - (1-\delta)\, k_t\right] \rightsquigarrow \lambda_t \quad (6)$$

- Lagrangian multipliers: $\{\mu_t, \eta_t, \lambda_t\}$

Household's optimal decisions

- optimal conditions for $\{m_t, b_t, h_{t+1}, c_t, n_t, k_{t+1}\}$

$$
\begin{aligned}
\mu_t &= \eta_t + \lambda_t \rightsquigarrow \text{money demand} && (7)\\
\mu_t &= \lambda_t (1 + R_{bt}) \rightsquigarrow \text{bond demand} && (8)\\
\lambda_t &= \beta \mathbb{E}_t \left(\frac{1}{1+\pi_{t+1}} \mu_{t+1} \right) \rightsquigarrow \text{intertemporal wealth} && (9)\\
1/c_t &= \lambda_t + \eta_t \rightsquigarrow \text{consumption} && (10)\\
\psi &= \lambda_t w_t \rightsquigarrow \text{labor supply} && (11)\\
\lambda_t &= \beta \mathbb{E}_t \left[\lambda_{t+1} \left(r_{t+1} + 1 - \delta \right) \right] \rightsquigarrow \text{capital supply} && (12)
\end{aligned}
$$

- (7) & (10) $\rightarrow \mu_t > 0, \lambda_t > 0$ (household fully utilizes wealth)
- (7) & (8) $\rightarrow \eta_t > 0$ if $R_{bt} > 0$ (CIA binds: household will not hold idle cash)
- CIA may not bind if $R_{bt} \leq 0$ (precautionary motive, Wen, 2018, EER; Xu and Liu, 2019, ERJ)

Firm's problem

- production function $y_t = a_t k_t^{\alpha} n_t^{1-\alpha}$
- profit maximization: $\max_{\{k_t, n_t\}} y_t - w_t n_t - r_t k_t$
- demand for labor and capital

$$
\begin{aligned}
r_t &= \alpha y_t / k_t \rightsquigarrow \text{capital demand} && (13) \\
w_t &= (1-\alpha)\, y_t / n_t \rightsquigarrow \text{labor demand} && (14)
\end{aligned}
$$

General equilibrium

- given prices, individual achieves the optimization
- all markets clear
  - final good market:
    $$c_t + k_{t+1} - (1-\delta)\, k_t = y_t$$
  - bond market:
    $$b_t = b_{t-1} = 0 \rightsquigarrow \text{close economy} \tag{15}$$
  - money market:
    $$m_t = \frac{m_{t-1}}{1+\pi_t} + x_t \rightsquigarrow x_t \text{ captures monetary policy} \tag{16}$$
- two markets clear $\rightarrow$ all three markets clear, why? (Walras's law)

Full system

- 13 endogenous variables

$$\{\mu_t, \eta_t, \lambda_t, m_t, b_t, h_{t+1}, c_t, n_t, k_{t+1}, r_t, w_t, \pi_t, R_{bt}\} .$$

- 13 equations: Eq. (5) to (12), Eq. (13) and (14), Eq. (15) and (16)
- neutrality of money: stock of money does not matter for dynamics
  - m_t can be determined but M_t cannot

Steady-State

- money mrkt equi.(16) $\rightarrow$ long-run inflation π depends on money growth

$$\frac{\pi}{1+\pi} = \frac{x}{m} = g$$

- bond demand (8) & Euler Eq. (9) $\rightarrow$

$$R_b = \frac{1+\pi}{\beta} - 1$$

  - zero cost for printing money $\rightarrow$ negative optimal inflation (Friedman's law)
  - incomplete market $\rightarrow$ holding cash has extra value (Wen, 2018, EER)

$$R_b = \left(\frac{1+\pi}{\beta} - 1\right) \times \underbrace{\Phi}_{\text{liquidity premium}}$$

Basic New Keynesian DSGE Model

CIA model with New Keyesnian features

- CIA model plus two basic features
- monopolistic competition
  - $\rightarrow$ positive profit and endogenous price setting
- nominal rigidities
  - $\rightarrow$ price is not flexible $\rightarrow$ MP has persistent effects

Basic environment

- a representative household $\rightsquigarrow$ similar to CIA model
- competitive final good producers
  - combines intermediate goods to produce $\rightsquigarrow$ demand for intermediate goods
- one unit measure of intermediate goods producers
  - sets price given final-good demand function
  - Calvo-type sticky price: can only adjust price with some probability

Household's problem

- similar to the optimization problem in the CIA model:

$$\max \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \psi n_t\right)$$

subject to

$$\begin{aligned}
M_t + B_t &\leq H_t + X_t \\
P_t c_t &\leq M_t \\
H_{t+1} &= M_t + \left(1 + R_{bt}\right) B_t + W_t n_t + R_t k_t \\
&\quad - P_t \left[c_t + k_{t+1} - \left(1 - \delta\right) k_t\right] + \Pi_t
\end{aligned}$$

  - $\Pi_t > 0$ total profit distributed from the intermediate goods sector

Final good producer

- uses intermediate goods y_{jt} to produce y_t via CES function

$$y_t = \left[\int_0^1 y_{it}^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right]^{\frac{\sigma}{\sigma-1}}$$

- profit maximization in competitive market

$$\max_{\{y_{it}\}} P_t y_t - \int_0^1 P_{it} y_{it} \mathrm{d}i$$

- demand for j-th intermediate good y_{jt}

$$y_{it} = \left(P_{it}/P_t\right)^{-\sigma} y_t \tag{17}$$

- price indexation equation:

$$P_t^{1-\sigma} = \int_0^1 P_{it}^{1-\sigma} \mathrm{d}i \tag{18}$$

$\rightsquigarrow$ zero profit, why?

Intermediate goods sector

- monopolistic competition + sticky price
- firm i produces y_{it} via C-D function $y_{it} = k_{it}^{\alpha} n_{it}^{1-\alpha}$
- real profit π_{it} is defined as

$$\pi_{it} = \frac{P_{it}}{P_t} y_{it} - w_t n_{it} - r_t k_{it}$$

facing demand: $y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\sigma} y_t$

Intermediate goods sector

- rewrite profit through a cost minimization problem

$$\pi_{it} = \left(\frac{P_{it}}{P_t} - \phi_t\right)\left(\frac{P_{it}}{P_t}\right)^{-\sigma} y_t \leftarrow \min_{\{n_{it}, k_{it}\}} w_t n_{it} - r_t k_{it} + \phi_t (y_{it} - k_{it}^{\alpha} n_{it}^{1-\alpha})$$

  - marginal cost ϕ_t satisfies

$$\phi_t = \frac{1}{A_t}\left(\frac{w_t}{\alpha}\right)^{\alpha}\left(\frac{r_t}{1-\alpha}\right)^{1-\alpha}$$

- monopolistic competition: can adjust P_{it} but cannot affect aggregate y_t
  - Q: why? only has market power on its own product

Sticky price setting

- firm i sets price P_{it} to maximize discounted profit flows

$$V_t \equiv \max_{\{P_{it}\}} \mathbb{E}_t \sum_{j=0} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \pi_{it+j}$$

- Calvo-type sticky price
  - can adjust price with prob. $1 - \theta$
  - cannot adjust with prob. θ $\rightsquigarrow$ large θ, more severe stickiness

- value function
  - $V_{0,t}$: value of active firm (can adjust price)
  - $V_{j,t}$: value of inactive firm adjusted price j periods ago

Optimal pricing

- value functions

$$V_{0,t}(P_{it}) = \max_{\{P_{it}\}} \pi_{it} + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\theta) V_{0,t+1} + \theta V_{1,t+1}(P_{it}) \right]$$

$$V_{j,t}(P_{it-j}) = \pi_{it}(P_{it-j}) + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\theta) V_{0,t+1} + \theta V_{j+1,t+1} \right]$$

- only active firm needs to set optimal price intertemporally

$$\frac{\partial \pi_{it}}{\partial P_{it}} + \beta\theta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{\partial V_{1,t+1}}{\partial P_{it}} \right] = 0 \Rightarrow \mathbb{E}_t \sum_{\tau=0} (\beta\theta)^\tau \left[\frac{\lambda_{t+\tau}}{\lambda_t} \frac{\partial \pi_{it+\tau}(P_{it})}{\partial P_{it}} \right] = 0 \quad (19)$$

- proof
  - $\frac{\partial \pi_{it+\tau}(P_{it})}{\partial P_{it}} = \frac{1}{P_{t+\tau}} \left(\frac{1}{\frac{P_{it}}{P_{t+\tau}} - \phi_{t+\tau}} - \frac{\sigma}{\frac{P_{it}}{P_{t+\tau}}} \right) \left(\frac{P_{it}}{P_{t+\tau}} - \phi_{t+\tau} \right) \left(\frac{P_{it}}{P_{t+\tau}} \right)^{-\sigma} y_{t+\tau}$
  - $\frac{\partial V_{j,t}}{\partial P_{it-j}} = \frac{\partial \pi_{it}(P_{it-j})}{\partial P_{it-j}} + \beta\theta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\partial V_{j+1,t+1}}{\partial P_{it-j}} = \sum_{\tau=0} (\beta\theta)^\tau \mathbb{E}_t \left[\frac{\lambda_{t+\tau}}{\lambda_t} \frac{\partial \pi_{it+\tau}(P_{it-j})}{\partial P_{it-j}} \right] \rightsquigarrow \frac{\partial V_{1,t+1}}{\partial P_{it}}$

Optimal pricing

- derive optimal price P_{it}^* from Eq. (19)

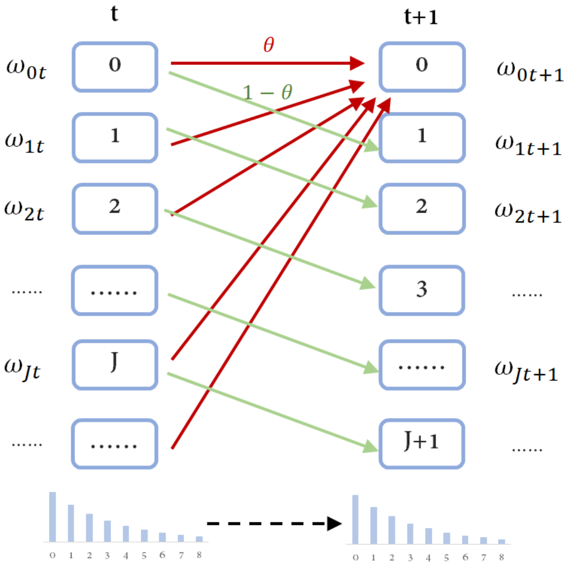
$$P_{it}^* = P_t^* = \frac{\sigma}{\sigma - 1} \frac{\mathbb{E}_t \sum_{\tau=0} (\beta\theta)^{\tau} \lambda_{t+\tau} P_{t+\tau}^{\sigma} y_{t+\tau} \phi_{t+\tau}}{\mathbb{E}_t \sum_{\tau=0} (\beta\theta)^{\tau} \lambda_{t+\tau} P_{t+\tau}^{\sigma-1} y_{t+\tau}} \tag{20}$$

  - every firm sets the same optimal price, i.e., P_{it}^* only depends on aggregate states
  - w/o sticky price ($\theta = 0$) $\rightarrow P_t^* = \frac{\sigma}{\sigma-1}\phi_t$

- price indexation equation (18) becomes:

$$P_t^{1-\sigma} = \theta P_{t-1}^{1-\sigma} + (1-\theta)\left(P_t^*\right)^{1-\sigma} \tag{21}$$

Price distribution evolution



New Keynesian Philips Curve

- NKPC is derived from the log-linearized (around steady-state) (20) and (21)
- log-linearizing (20) and (21) gives

$$
\begin{aligned}
\hat{P}_t^* &= \mathbb{E}_t \sum_{\tau=0} (1-\beta\theta)(\beta\theta)^\tau \left(\hat{\lambda}_{t+\tau} + \sigma \hat{P}_{t+\tau} + \hat{y}_{t+\tau} + \hat{\phi}_{t+\tau}\right) - \\
&\quad \mathbb{E}_t \sum_{\tau=0} (1-\beta\theta)(\beta\theta)^\tau \left(\hat{\lambda}_{t+\tau} + (\sigma-1)\hat{P}_{t+\tau} + \hat{y}_{t+\tau}\right) \\
&= \mathbb{E}_t \sum_{\tau=0} (1-\beta\theta)(\beta\theta)^\tau \left(\hat{P}_{t+\tau} + \hat{\phi}_{t+\tau}\right) \\
&= (1-\beta\theta) \sum_{\tau=0} \left(\beta\theta\mathbb{L}^{-1}\right)^\tau \left(\hat{P}_t + \hat{\phi}_t\right) \rightsquigarrow \text{define operator } \mathbb{L}^{-1} x_t = \mathbb{E}_t(x_{t+1}) \\
&= \frac{1-\beta\theta}{1-\beta\theta\mathbb{L}^{-1}} \left(\hat{P}_t + \hat{\phi}_t\right) \qquad (22) \\
\hat{P}_t &= \theta \hat{P}_{t-1} + (1-\theta)\hat{P}_t^* \qquad (23)
\end{aligned}
$$

New Keynesian Philips Curve

- (22) and (23) imply

$$\hat{P}_t = \theta \hat{P}_{t-1} + (1-\theta) \frac{1-\beta\theta}{1-\beta\theta\mathbb{L}^{-1}} \left(\hat{P}_t + \hat{\phi}_t\right)$$

$\rightarrow$

$$\hat{P}_t - \beta\theta\mathbb{E}_t\hat{P}_{t+1} = \theta\hat{P}_{t-1} - \beta\theta^2\hat{P}_t + (1-\theta)(1-\beta\theta)(\hat{P}_t + \hat{\phi}_t)$$

$\rightarrow$ NKPC

$$\hat{\pi}_t = \beta\mathbb{E}_t\hat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\hat{\phi}_t$$

where inflation rate satisfies $\hat{\pi}_t \equiv \hat{P}_t - \hat{P}_{t-1}$

New Keynesian Philips Curve

- NKPC

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{\phi}_t$$

- two factors drive inflation dynamics:
    - expectation: $\mathbb{E}_t \hat{\pi}_{t+1}$
    - cost push: $\hat{\phi}_t$ $\rightsquigarrow$ in a simple case w/o capital k_t, $\hat{\phi}_t \propto \hat{y}_t$
- traditional Philips curve doesn't consider expectation
    - $\rightarrow$ positive correlation between inflation and output
    - cannot fit the data

Full Dynamic System

Production side

- input demand

$$w_t = \phi_t (1-\alpha) \frac{y_t}{n_t}, \;\; r_t = \phi_t \alpha \frac{y_t}{k_t} \rightsquigarrow \text{derive from cost minimization problem}$$

- aggregate production

$$\tilde{y}_t = \int_0^1 y_{it} \mathrm{d}i = k_t^{\alpha} n_t^{1-\alpha} \rightsquigarrow \tilde{y}_t \text{ is total amount of intermediate goods, } k_t = \int_0^1 k_{it} \mathrm{d}i, \; n_t = \int_0^1 n_{it} \mathrm{d}i$$

- aggregate output

$$y_t = \tilde{y}_t \Delta_t$$

  - where $\Delta_t = \int \left(\frac{P_{it}}{P_t}\right)^{-\sigma} \mathrm{d}i = \frac{\theta P_{t-1}^{-\sigma} + (1-\theta)(P_t^*)^{-\sigma}}{P_t^{-\sigma}}$ $\rightsquigarrow$ log-linearizing $\Delta_t \rightarrow \hat{\Delta}_t = 0$

- NKPC

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{\phi}_t$$

Household side

- refer to CIA model, first order conditions:

$$\begin{aligned} 1/c_t &= \lambda_t + \eta_t, \\ \psi &= \lambda_t w_t, \\ \lambda_t &= \beta \mathbb{E}_t \left(\lambda_{t+1} / \pi_{t+1} \right) + \eta_t \\ \lambda_t &= \beta R_{bt} \mathbb{E}_t \left(\lambda_{t+1} / \pi_{t+1} \right), \\ \lambda_t &= \beta \mathbb{E}_t \lambda_{t+1} \left(r_{t+1} + 1 - \delta \right). \end{aligned}$$

- CIA constraint: $c_t = m_t$
- budget constraint:

$$c_t + m_t + k_{t+1} - (1 - \delta) k_t + \frac{b_t}{R_{bt}} \leq w_t n_t + r_t k_t + \frac{b_{t-1}}{\pi_t} + div_t + \frac{m_{t-1} + x_t}{\pi_t}$$

  - div_t: profit distributed from firms

Monetary authority

- consider two different types of monetary policies
- exogenous growth rate rule, $g_{mt} = \frac{M_t}{M_{t-1}} = 1 + \frac{X_t}{M_{t-1}}$,

$$\ln g_{mt} = \rho \ln g_{mt-1} + \varepsilon_t,$$

- Taylor rule

$$\frac{R_{bt}}{R_b} = \left(\frac{y_t}{y^{ss}}\right)^{\rho_y} \left(\frac{\pi_t}{\pi^{ss}}\right)^{\rho_\pi} \exp(\varepsilon_t) \longrightarrow \hat{R}_{bt} = \rho_y \hat{y}_t + \rho_\pi \hat{\pi}_t + \varepsilon_t$$

where $\rho_y > 0$ and $\rho_\pi > 1$ $\rightsquigarrow \rho_\pi < 1$ leads to self-fulfilling equilibrium (indeterminancy), why?

Market clearing

- money market:

$$M_t = M_{t-1} + X_t, \text{ or } m_t = g_{mt} m_{t-1} / \pi_t$$

- bond market:

$$B_t = B_{t-1} = 0$$

- two input markets (k, n) automatically clear, why?
- final good market automatically clears, i.e., $c_t + i_t = y_t$, why?
  - Walras's law
  - household's budget constraint is equivalent to $c_t + i_t = y_t$

Monetary authority

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- exogenous growth rate rule, $g_{mt} = \frac{M_t}{M_{t-1}} = 1 + \frac{X_t}{M_{t-1}}$,

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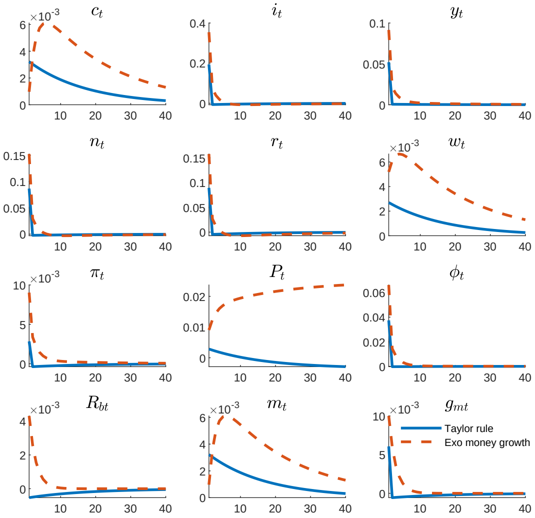
Dynamics under expansionary monetary policy



Note: quantitative note and dynare code can be downloaded at https://xuzhiwei09.wixsite.com/econ/lectures