Digitalization through Fiscal Competition

Guangzhong Li[a], Guanchun Liu[b,c], Yan Liu[a], Dandan Zhu[b]

[a] School of Business, Sun Yat-sen University, Guangzhou 510275, China

[b] Lingnan College, Sun Yat-sen University, Guangzhou 510275, China

[c] Laboratory of Mezzoeconomics and Regional Industrial Coordinated Development, Sun Yat-sen University, Guangzhou 510275, China

Emails:

Guangzhong Li: liguangzhong@mail.sysu.edu.cn
Guanchun Liu: liuguanchun1@126.com
Yan Liu: liuy2696@mail.sysu.edu.cn
Dandan Zhu: zhudd5@mail.sysu.edu.cn

Corresponding author: Dandan Zhu
Address: No.135 Xingang Xi Road, Haizhu District, Guangzhou, Guangdong Province, 510275, China.

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Abstract

This paper studies how fiscal competition among local governments shapes digitalization by developing a dynamic two-region spatial general equilibrium model. The digital services and manufacturing sectors are linked through bidirectional externalities. Frontier digital technology spills over to manufacturing and raises its productivity, while manufacturing activity generates data that feeds back into digital innovation. The bidirectional externalities create a self-reinforcing growth loop whose strength is increasing in the manufacturing sector's digital adaptability. However, since private agents fail to internalize these externalities, laissez-faire under-allocates resources to the digital services sector and is Pareto-inefficient. A centralized coordinator that finances digital output subsidies through manufacturing taxes can correct the bidirectional externalities and deliver efficiency gains over laissez-faire. We further introduce tournament competition into the decentralized economy, in which local governments set digital subsidies to attract mobile digital talent and maximize relative regional growth. Aggregate welfare responds non-monotonically to tournament intensity. Moderate competition partially corrects the externalities and raises allocative efficiency above the laissez-faire level, while excessive competition drives subsidies beyond the social optimum into over-subsidization. At the welfare peak, decentralized competition achieves the centralized coordinator's optimum. This implies that well-designed fiscal competition can alleviate the externalities inherent in digitalization and enhance aggregate welfare.

Keywords: Digitalization, Technology spillover, Data feedback, Fiscal competition, Tournament intensity

1. Introduction

Digital economy promotion has rapidly ascended to the forefront of intergovernmental fiscal competition worldwide. Regional jurisdictions across diverse institutional settings increasingly deploy competitive fiscal instruments to attract digital industries. In China, provincial and municipal governments vie to establish state-led digital industry clusters, channeling public resources into computing infrastructure and talent subsidies (Aghion et al., 2015; Lu et al., 2019). Across the European Union, member states compete through divergent corporate tax regimes, inducing digital multinationals to concentrate profits in low-tax jurisdictions (Devereux et al., 2008; Tørsløv et al., 2023). In the United States, states engage in escalating subsidy wars, committing billions in grants and tax credits to lure digital technology firms (Slattery and Zidar, 2020; Slattery, 2025). A fundamental welfare question arises as to whether intergovernmental competition and the fiscal incentives it engenders genuinely accelerate digital development and enhance aggregate welfare, or predominantly generate allocative distortions that impose a net social cost.

Addressing this question requires confronting several structural features that distinguish digital economy competition from competition over traditional industries. First, specialized digital talent constitutes the primary productive factor. The core inputs to digital production, such as the code developed by software engineers, the algorithmic models built by data scientists, and the product architectures designed by technology managers, are embodied in mobile workers rather than in fixed physical capital. Digital talent is therefore both geographically footloose and intensely contested across jurisdictions (Moretti and Wilson, 2017; Akcigit et al., 2016). Second, digital technologies generate cross-industry spillovers that operate through multiple channels. Digital knowledge diffuses across sectors, raising adopting firms' total factor productivity, while consumption activity generates data that feeds back into the digital innovation process, creating dynamic complementarities between the digital frontier and the broader economy (Bresnahan and Trajtenberg, 1995; Bloom et al., 2013). Third, digital markets exhibit pronounced winner-take-all dynamics, in which the leading search engine, social network, or e-commerce platform captures the dominant share of its market (Autor et al., 2020; Goldfarb and Tucker, 2019). These features jointly imply that the welfare consequences of competitive fiscal policies in the digital economy are qualitatively richer than those arising in conventional factor-competition contexts.

We develop a dynamic two-region spatial general equilibrium model in which each region comprises a digital services sector and a manufacturing sector. The digital services sector employs mobile specialized talent whose innovation effort, combined with data generated by economic activity, drives technological progress along a balanced growth path. Manufacturing technology draws on both the firm's own accumulated knowledge and frontier digital technology that spills over from the digital services sector. A digital adaptability parameter governs the mixing weight between these two sources. The two sectors are coupled through bidirectional externalities. Frontier digital technology raises the technology frontier available to manufacturing, while manufacturing activity generates data that feeds back into digital innovation. The bidirectional externalities create a self-reinforcing growth loop where digital innovation raises manufacturing productivity, which in turn generates more data that accelerates digital innovation further. However, since private agents fail to internalize these externalities, laissez-faire under-allocates resources to the digital services sector and is Pareto-inefficient. This result motivates the introduction of fiscal policy instruments to reallocate resources toward the digital services sector.

Building on this framework, we introduce government fiscal instruments. Each regional government sets a digital output subsidy financed by a manufacturing tax under a balanced budget constraint. A centralized coordinated planner who jointly controls both regions' subsidies can correct the bidirectional externalities and deliver the second-best outcome with substantial welfare gains over laissez-faire. Rather than relying on centralized coordination, we next consider whether decentralized tournament competition can deliver comparable efficiency gains. The government's objective combines own-region welfare with relative growth performance against the rival region. This formulation reflects the promotion tournament mechanism widely documented in China's intergovernmental governance. Because digital talent is mobile across regions, fiscal competition motivates each region to raise subsidies to attract scarce digital talent from rivals. The welfare consequences of this competition exhibit an inverted-U pattern in tournament intensity, driven by a trade-off between externality correction and over-subsidization. As tournament intensity rises, intergovernmental competitive pressure induces larger subsidies that partially correct the bidirectional externalities, pushing welfare above the laissez-faire level. At the welfare peak, decentralized competition achieves the centralized coordinator's optimum. Beyond this threshold, however, the fiscal cost of financing ever-larger subsidies through manufacturing taxes outweighs the marginal externality correction, and welfare declines. This non-monotonic relationship implies the existence of an optimal tournament design in which moderate competition serves as an effective instrument for externality correction.

Complementing the theory, we provide empirical evidence using Chinese city-level and firm-level panel data spanning 2011 to 2019. At the city level, digitalization in neighboring cities within the same province is significantly and robustly negatively associated with local digitalization. At the firm level, we decompose local spillover effects from cross-city competition effects. Local digitalization is positively associated with firm-level digital patenting, consistent with beneficial within-city knowledge spillovers. In contrast, digitalization in other cities exerts a negative effect on firm digital patenting, consistent with competitive reallocation of mobile resources across cities. The magnitude of the negative cross-city competition effect dominates the positive local spillover effect. These patterns are consistent with the model's mechanism that fiscal competition reallocates mobile resources across regions and ultimately shapes local digitalization outcomes.

This paper contributes to four strands of the economics literature. The first is the theory of fiscal competition among decentralized governments. The canonical race-to-the-bottom result establishes that interjurisdictional competition for mobile capital depresses tax rates and public good provision below efficient levels (Zodrow and Mieszkowski, 1986; Wilson, 1986; Wildasin, 1988; Agrawal et al., 2022). Jurisdictional asymmetries amplify these distortions, as smaller or capital-scarce regions set systematically lower tax rates (Bucovetsky, 1991; Kanbur and Keen, 1993). The objects of fiscal competition have broadened beyond physical capital to encompass mobile skilled workers and high-tech firms. Tax differentials drive the location choices of superstar inventors (Akcigit et al., 2016) and star scientists (Moretti and Wilson, 2017; Kleven et al., 2013). Governments have increasingly shifted from tax competition toward direct subsidies that predominantly redistribute rather than create economic activity (Slattery and Zidar, 2020; Ferrari and Ossa, 2023; Slattery, 2025). Cooperative tax policies can generate Pareto improvements by internalizing cross-border fiscal externalities (Edwards and Keen, 1996; Keen and Konrad, 2013; Keen and Wildasin, 2004), and harmonized R&D subsidies correct both strategic distortions and underinvestment in innovation (Borota Milicevic et al., 2026).

The second strand examines tournament competition among local governments. The theory of rank-order tournaments shows that compensation based on ordinal rank can elicit efficient effort when absolute performance is hard to measure (Lazear and Rosen, 1981). Yardstick competition extends this logic to the public sector, where voters or central authorities use peer performance as a benchmark for evaluating incumbents (Shleifer, 1985; Besley and Case, 1995). China's regionally decentralized governance provides a leading application. The multi-divisional organizational structure of the Chinese state promotes yardstick competition among provincial governments (Maskin et al., 2000). Empirical evidence confirms that the turnover and promotion of Chinese provincial leaders depend on GDP growth relative to predecessors and neighbors (Li and Zhou, 2005; Chen et al., 2005). Local officials maximize a composite objective that combines own-region welfare with relative growth performance against rival regions (Li et al., 2019; Fang et al., 2025). However, tournament incentives also generate distortions, including local protectionism, duplicative investment, and biased resource allocation against competing regions (Xu, 2011; Fang et al., 2025).

The third strand concerns the growth effects of digital technology and cross-sector technology spillovers. Growth accounting established that information technology was a major driver of productivity acceleration across advanced economies (Oliner and Sichel, 2000; Jorgenson and Stiroh, 2000; Stiroh, 2002; Jorgenson et al., 2008), though realizing these gains requires complementary organizational and human capital investments (Brynjolfsson and Hitt, 2000; Bloom et al., 2012). These growth effects extend well beyond IT-producing sectors, as broadband diffusion and automation technologies have been shown to raise productivity across a wide range of industries (Czernich et al., 2011; Akerman et al., 2015; Graetz and Michaels, 2018). At the cross-country level, R&D investments generate substantial productivity spillovers through trade and knowledge linkages (Coe and Helpman, 1995). More recently, artificial intelligence has emerged as a general-purpose technology that may sustain exponential growth if it enters the production of ideas (Aghion et al., 2019). Task-based models show that the net effect of AI depends on the balance between automation displacement and new task creation (Acemoglu and Restrepo, 2018, 2019, 2022). Field experiments document substantial individual productivity gains from the adoption of generative AI (Brynjolfsson et al., 2025; Noy and Zhang, 2023).

The fourth strand addresses data as an economic input and its role in growth. Growth models incorporating data as a nonrival input show that data sharing generates increasing returns and sustains long-run growth through enhanced product variety and knowledge spillovers (Jones and Tonetti, 2020; Cong et al., 2021, 2022). However, data accumulation alone exhibits diminishing returns because prediction errors can at most be reduced to zero (Farboodi and Veldkamp, 2020). Data generates externalities through multiple channels. Users reveal information about others when sharing data (Acemoglu et al., 2022). The social dimension of data shapes intermediary pricing (Bergemann et al., 2022). Consumers over-disclose when they do not internalize how their data helps sellers extract surplus (Ichihashi, 2020). These externalities interact with market structure. Data-intensive intangible assets underpin the rise of high-markup superstar firms (Autor et al., 2020; Crouzet et al., 2022), and data-enabled learning creates persistent competitive advantages (Bergemann and Bonatti, 2024; Goldfarb and Tucker, 2019). Data as a macroeconomic factor challenges standard growth frameworks through its nonrivalry, externalities, and increasing returns (Veldkamp and Chung, 2024).

This paper makes three contributions to the literature. First, we extend the digital technology, data, and growth literatures by characterizing two externalities specific to the digital economy, a digital technology spillover externality and a manufacturing data feedback externality. Our framework formalizes bidirectional externalities whose interaction generates a self-reinforcing growth mechanism. This also creates a new source of Pareto inefficiency rooted in the failure of private agents to internalize cross-sector externalities. Second, we contribute to the fiscal competition and tournament competition literatures by uncovering a non-monotonic welfare consequence of decentralized competition. Our analysis shows that competition can instead serve a corrective function when the underlying market failures are sectoral externalities rather than conventional underprovision of public goods. Third, we bridge the digital economy and fiscal competition literatures. The existing literature lacks an analytical framework that jointly captures digital externalities and strategic interaction among local governments. Our model fills this gap by analyzing digital market failures and intergovernmental fiscal competition within a unified welfare analysis.

The remainder of the paper is organized as follows. Section 2 presents motivating evidence on regional and firm digitalization. Section 3 develops the laissez-faire model and establishes the two externalities that render it Pareto-inefficient. Section 4 compares centralized coordination with decentralized tournament competition in welfare terms. Section 5 provides quantitative analyses. Section 6 explores extensions that relax the benchmark assumptions. Section 7 concludes.

2. Motivating Evidence

In this section, we present stylized facts on the economic effects of regional digitalization, focusing on two key dimensions: the impact of digitalization in other cities on local digitalization at the city level, and the influence of both local and other-city digitalization on firm-level digital activity. In particular, our empirical tests exploit two-dimensional variation in digitalization, both across cities and over time.

2.1. Empirical Specification

We construct econometric specifications at both the city and firm levels. To investigate the influence of digitalization in other cities on local digitalization, we establish a city-level specification of the form

$$Digit\_local_{ct} = \beta Digit\_other_{ct} + \gamma \mathbf{X}_{ct} + \vartheta_c + \delta_t + \zeta Trend_{pt} + \xi_{ct},$$

where the subscripts c, p, and t refer to city, province, and year, respectively. The dependent variable, $Digit\_local_{ct}$, captures the digitalization of city c in province p during year t. The explanatory variable of interest, $Digit\_other_{ct}$, measures the average digitalization of other cities in province p during year t. $\mathbf{X}_{ct}$ is a vector of city-specific covariates covering economic development, population density, and industrial structure. ϑ_c represents city fixed effects that absorb time-invariant unobserved characteristics across cities. δ_t denotes year fixed effects that capture common temporal shocks to digitalization. To account for time-varying province-level factors, we further include a province-specific time trend $Trend_{pt}$. ξ_{ct} is the error term, with standard errors clustered at the city level.

To examine how local and external digitalization shape firm-level digitalization, we estimate a firm-level specification of the form

$$Digit\_firm_{it} = \beta_1 Digit\_local_{ct} + \beta_2 Digit\_other_{ct} + \eta \mathbf{W}_{it} + \gamma \mathbf{X}_{ct} + \theta_i + \delta_t + \zeta Trend_{pt} + \xi_{it},$$

where the subscript i denotes the firm, while the remaining subscripts are as defined in the city-level specification. The dependent variable, $Digit\_firm_{it}$, captures the digitalization of firm i in year t. The two explanatory variables of interest, $Digit\_local_{ct}$ and $Digit\_other_{ct}$, measure the digitalization level of the city where the firm is registered and the average digitalization of other cities within the same province, respectively. In addition to city-level covariates, we control for a vector of firm characteristics, $\mathbf{W}_{it}$, including firm size, leverage, profitability, asset tangibility, cash holdings, firm age, and ownership concentration. This specification additionally includes firm fixed effects, θ_i, capturing all time-invariant firm attributes, such as unobserved corporate culture. As in the city-level specification, standard errors are clustered at the city level.

The parameters of interest in these specifications are β, β_1, and β_2. At the city level, β measures the association between other-city digitalization and local digitalization. At the firm level, β_1 and β_2 capture the effects of local and other-city digitalization on firm digitalization, respectively. To account for potential within-city correlation, we follow the approach in Matray (2021) and cluster standard errors at the city level in both specifications.

2.2. Data and Variables

2.2.1. Data

We employ two distinct panel datasets, one at the city level and the other at the firm level. The core digitalization data is drawn from the Key Digital Technology Patent Database (KDTD) of the China National Research Data Service Platform (CNRDS). This database focuses on key technological innovations aligned with national strategic priorities and targets emerging digital industries and frontier technologies. It strictly follows the 2023 Key Digital Technology Patent Classification System and covers seven major fields:[1] artificial intelligence, advanced semiconductors, quantum information, Internet of Things, blockchain, industrial internet, and metaverse technologies. As digital technologies constitute the foundation of digitalization, patent data in these domains serves as an appropriate proxy for measuring the degree of digitalization.

Coverage of cities and firms in the KDTD enables us to construct city- and firm-level panels. Both datasets cover the period 2011–2019. We choose 2011 as the starting year because China's 12th Five-Year Plan (2011–2015) elevated informatization to a national strategic priority. This marked the beginning of a sustained policy effort to promote the digital economy through infrastructure investment, industrial upgrading programs, and targeted fiscal support at both central and local government levels. The sample ends in 2019 to avoid potential distortions associated with the COVID-19 pandemic.

[1] An official document on the classification for key digital technology patents in China, source: `https://www.cnipa.gov.cn/art/2023/9/25/art_75_187769.html`.

First, we compile a panel of Chinese cities. Economic and demographic statistics originate from the China Statistical Yearbook and the China City Statistical Yearbook. We process the data in two steps: (i) observations with missing values for key variables are removed; (ii) to reduce the influence of outliers, all continuous variables are winsorized at the 1% level on both tails. The final sample comprises 280 cities, with a total of 2,511 city-year observations.

Second, we construct a panel of Chinese A-share listed firms. Firm-level operational and financial data is sourced from the China Stock Market & Accounting Research (CSMAR) Database. The sample is constructed as follows: (i) firms in the financial and real estate sectors are excluded; (ii) firms designated as ST, ST*, or PT are dropped; (iii) firms whose registered city changes during the sample period are removed; (iv) firms with leverage below 0 or above 1 are removed; (v) observations with missing values for key variables are excluded; and (vi) all continuous variables are winsorized at the 1% level on both tails. The final sample consists of 2,516 unique firms, corresponding to 17,436 firm-year observations.

2.2.2. Variable Definition

Measures of city-level digitalization. Following Wan et al. (2026), we adopt patent applications in key digital technologies for each city from the KDTD to measure city-level digitalization. Patent applications reflect both a city's technological engagement and its commitment to digitalization. We utilize two types of patent data: one is the total of invention patent applications and utility model patent applications, reflecting overall digitalization levels; the other focuses on invention patent applications only, which are subject to stricter inventive-step requirements than utility model patents and better capture substantive progress in digital capabilities.

Specifically, we construct two sets of city-level digitalization measures. First, we capture local digitalization with $Digit\_local1$ and $Digit\_local2$. $Digit\_local1$ is defined as the sum of invention patents and utility model patents in key digital technologies applied by city c in year t, scaled by the local population (per 10,000 persons). $Digit\_local2$ measures only the number of invention patent applications in key digital technologies per 10,000 persons for city c in year t. Second, we compute the average digitalization levels of other cities within the same province during the same year, denoted as $Digit\_other1$ and $Digit\_other2$. $Digit\_other1$ corresponds to the provincial average of $Digit\_local1$, while $Digit\_other2$ corresponds to the provincial average of $Digit\_local2$.

Measures of firm-level digitalization. We measure firm-level digitalization using key digital technology patents jointly applied by firms from the KDTD. Compared with alternative proxies such as text-based keyword frequency (He et al., 2025), patent data offers a more objective, output-oriented indicator of a firm's engagement in digitalization. Similar to the city-level measures, we use both total patent applications and invention patent applications.

We define two indicators of firm-level digitalization, $Digit\_firm1$ and $Digit\_firm2$. $Digit\_firm1$ is defined as the sum of invention patents and utility model patents in key digital technologies jointly applied by firm i in year t, scaled by the number of employees (per 10,000 employees). $Digit\_firm2$ is measured as the number of invention patents in key digital technologies per 10,000 employees jointly applied by firm i in year t. Therefore, $Digit\_firm1$ captures a firm's total digitalization efforts, whereas $Digit\_firm2$ emphasizes its substantive digitalization, together providing a nuanced view of firm digitalization.

Control variables. Inspired by earlier studies (Gaspar et al., 2024; Feng and Yuan, 2025), we include a set of city- and firm-level control variables to isolate the effect of digitalization. At the city level, we control for economic development (*Economic development*), measured as the natural logarithm of real per capita GDP [yuan/person] deflated to 2003 prices; population density (*Population density*), measured as the natural logarithm of population per square kilometer [persons/km^2]; and industrial structure (*Industrial structure*), defined as the share of secondary industry in GDP [%].

At the firm level, we include firm size (*Size*), measured as the natural logarithm of total assets; leverage (*Lev*), defined as total liabilities divided by total assets; profitability (*Roa*), measured as net profit scaled by total assets; asset tangibility (*Tang*), defined as net fixed assets scaled by total assets; cash holdings (*Cash*), measured as net operating cash flow scaled by total assets; firm age (*Age*), measured as the natural logarithm of years since the firm's establishment; and ownership concentration ($Top1$), defined as the ownership share of the largest shareholder [%].

Descriptive statistics for all variables are reported in Table A.1.

2.3. Empirical Results

2.3.1. City-Level Evidence

Table 1 reports the city-level results. We use $Digit\_local1$ and $Digit\_local2$ as alternative dependent variables and progressively add city-level covariates to assess robustness. In the baseline specification with city fixed effects, year fixed effects, and province-specific time trends but without additional controls, columns (1)–(2) yield significantly negative coefficients on $Digit\_other1$ and $Digit\_other2$. This indicates that local digitalization tends to be lower when other cities within the same province exhibit higher digitalization. Columns (3)–(4) augment the specification with controls for economic development, population density, and industrial structure. The negative coefficients remain stable in both magnitude and statistical significance, suggesting that the estimated relationship is not driven by observable city characteristics. Across all specifications, city-level panel estimates reveal a significant negative relationship between digitalization in other cities and local digitalization, with coefficients ranging from −0.7968 to −0.4741.

Table 1: City-level Evidence: The Impact of Digitalization in Other Cities on Local Digitalization

	(1) $Digit\_local1$	(2) $Digit\_local2$	(3) $Digit\_local1$	(4) $Digit\_local2$
$Digit\_other1$	−0.7968***		−0.7604***	
	(0.1856)		(0.1782)	
$Digit\_other2$		−0.5227***		−0.4741***
		(0.1847)		(0.1779)
City controls	NO	NO	YES	YES
City FE	YES	YES	YES	YES
Year FE	YES	YES	YES	YES
Province FE × Time trends	YES	YES	YES	YES
Observations	2,511	2,511	2,511	2,511
R-squared	0.8892	0.8688	0.8903	0.8703

Notes: $Digit\_local1$ and $Digit\_local2$ are dependent variables. $Digit\_local1$ is measured by the sum of invention patents and utility model patents of key digital technology, while $Digit\_local2$ is measured by invention patents of key digital technology; both are for the city in that year and are scaled by the population (per 10,000 persons). Columns (1)–(2) do not include city controls, whereas columns (3)–(4) do. Standard errors in parentheses are clustered at the city level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

2.3.2. Firm-Level Evidence

Table 2 reports the firm-level results, with $Digit\_firm1$ and $Digit\_firm2$ as alternative dependent variables. All regressions include the full set of fixed effects specified in the firm-level equation. Panel A uses $Digit\_local1$ and $Digit\_other1$ as the key explanatory variables, while Panel B uses the alternatives $Digit\_local2$ and $Digit\_other2$. In Panel A, the coefficients on $Digit\_local1$ are significantly positive across all columns, whereas those on $Digit\_other1$ are significantly negative, with results robust to the flexible inclusion of firm-level controls. This indicates that local digitalization is positively associated with firm digitalization, while digitalization in other cities is negatively associated with firm outcomes. Panel B yields qualitatively identical patterns, with both coefficients gaining in magnitude and significance when attention is restricted to invention patents. Taken together, the firm-level estimates reveal that local digitalization significantly enhances firm digitalization, whereas digitalization in other cities exerts an inhibitory effect.

2.3.3. Summary and Discussion

Before summarizing the empirical findings, we briefly discuss the empirical strategy underlying the regression specifications. The regressions are designed to establish conditional

Table 2: Firm-level Evidence: The Impact of City Digitalization on Firm Digitalization

Panel A: Using Digit_local1 and Digit_other1

	(1) $Digit\_firm1$	(2) $Digit\_firm2$	(3) $Digit\_firm1$	(4) $Digit\_firm2$
$Digit\_local1$	0.1461**	0.1347***	0.1417**	0.1315***
	(0.0592)	(0.0421)	(0.0601)	(0.0429)
$Digit\_other1$	−0.5388*	−0.3533**	−0.5469**	−0.3616**
	(0.2777)	(0.1712)	(0.2742)	(0.1693)
Firm controls	NO	NO	YES	YES
City controls	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES
Year FE	YES	YES	YES	YES
Province FE × Time trends	YES	YES	YES	YES
Observations	17,436	17,436	17,436	17,436
R-squared	0.5942	0.5959	0.5948	0.5965

Panel B: Using Digit_local2 and Digit_other2

	(5) $Digit\_firm1$	(6) $Digit\_firm2$	(7) $Digit\_firm1$	(8) $Digit\_firm2$
$Digit\_local2$	0.2476***	0.2148***	0.2432***	0.2116***
	(0.0845)	(0.0579)	(0.0860)	(0.0590)
$Digit\_other2$	−1.2099***	−0.8670***	−1.2183***	−0.8766***
	(0.3726)	(0.2320)	(0.3720)	(0.2310)
Firm controls	NO	NO	YES	YES
City controls	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES
Year FE	YES	YES	YES	YES
Province FE × Time trends	YES	YES	YES	YES
Observations	17,436	17,436	17,436	17,436
R-squared	0.5945	0.5963	0.5952	0.5969

Notes: $Digit\_firm1$ and $Digit\_firm2$ are dependent variables. $Digit\_firm1$ is measured by the sum of invention patents and utility model patents of key digital technology, while $Digit\_firm2$ is measured by invention patents of key digital technology only; both refer to patents co-applied by the firm in that year and are scaled by the number of employees (per 10,000 employees). Columns (1)–(2) and (5)–(6) do not include firm controls, whereas columns (3)–(4) and (7)–(8) do. Standard errors in parentheses are clustered at the city level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

correlations that are informative about two mechanisms central to the theoretical model, that is, within-region digitalization spillovers and cross-region competition for contestable resources. Both the city-level and firm-level specifications include city (or firm) fixed effects and year fixed effects, absorbing time-invariant heterogeneity and common temporal shocks. Province-specific time trends further control for province-wide trajectories in digitalization policy or institutional reform that could simultaneously drive digitalization across cities. After conditioning on these controls, for the city-level regressions, the remaining variation in local digitalization comes from differential changes in peer-city digitalization. For the firm-level regressions, the remaining variation comes from differential changes in local and other-city digitalization conditional on firm attributes.

The empirical analysis establishes three stylized facts. The first is the cross-city crowding out of digital innovation. Digitalization in neighboring cities is negatively and significantly associated with local digitalization. The second stylized fact is the dual pattern observed at the firm level. Local digitalization is positively associated with firm-level digital patenting, while digitalization in neighboring cities is negatively associated with firm patenting. This dual pattern is consistent with firm-level responses to two distinct forces: beneficial local spillovers that enhance firms' digital innovation capacity, and cross-city competition that may reallocate rivalrous digital resources. The third stylized fact is that the negative cross-city competition effect dominates the positive local spillover effect in magnitude. In the firm-level estimates, when each coefficient is scaled by the standard deviation of its corresponding variable, the standardized effect of other-city digitalization exceeds that of local digitalization across all specifications. This asymmetry motivates the theoretical model below, which formalizes the interplay between local spillovers and cross-city competition and characterizes its welfare consequences under alternative fiscal arrangements.

These three stylized facts align closely with the structural mechanisms formalized in the theoretical model. First, the negative cross-city competition effect resonates with the inter-regional competition of mobile digital talent in the model. Since digital talent is the key rivalrous input to digital innovation, its reallocation toward one region reduces the other region's innovation capacity, consistent with the empirical findings at both levels. Second, the positive local spillover effect corresponds to the bidirectional externality mechanism in the model. The digital technology spillover raises the manufacturing sector's technology, while manufacturing activity generates data that feeds back into the digital sector's innovation. Together, these two channels form a self-reinforcing loop between digital progress and manufacturing activity. These within-city externalities generate positive co-movement between local digital activity and firm-level digital patenting. Third, the dominance of the negative cross-city competition effect over the positive local spillover effect provides empirical motivation for the policy analysis below. The ensuing analysis therefore examines the potential for moderate fiscal competition to mitigate the inefficiency of the laissez-faire digital economy and ultimately improve social welfare.

3. Benchmark Model: The Laissez-Faire Economy

This section develops a two-region spatial general equilibrium model of digital transformation as the benchmark without government intervention. The framework features bidirectional externalities between the digital services and manufacturing sectors through technology spillovers and data feedback, shaping the economy's balanced growth path.

3.1. Environment

Consider an economy with infinite time. The economy comprises two symmetric regions, indexed by $i \in \{A, B\}$, with $j \neq i$ denoting the other region. Each region contains a digital services sector, indexed by D, and a manufacturing sector, indexed by M. There is a fixed supply of homogeneous labor $\bar{L}$, primarily employed in goods production in both sectors. Labor is perfectly mobile across regions and sectors, allocated as L_i^D and L_i^M in region i. There is also a fixed supply of specialized digital talent $\bar{H}$, allocated as H_i^D exclusively to the digital sector's innovation process. Digital talent is perfectly mobile across regions.

Consumers in each region purchase goods from both local and remote producers. Manufacturing goods shipped between regions incur iceberg transport costs $\tau \geq 1$ (one unit shipped, $1/\tau$ units arrive), while digital services are delivered at zero marginal transport cost. Digital services, such as artificial intelligence, cloud computing, Internet of Things, and big data analytics, can be transmitted across regions at negligible cost, unlike physical goods that require transportation. The consumption of both types of goods generates data as a natural byproduct of economic activity (Jones and Tonetti, 2020; Veldkamp and Chung, 2024; Farboodi and Veldkamp, 2025). In the process of purchasing and using products, consumers automatically leave digital traces, including browsing histories, preference data, transaction records, location data, and sensor readings.

The digital services sector in each region consists of a representative firm producing digital services. Advances in digital technology are driven by the combination of digital talent, technology-based knowledge, and data-based knowledge. This frontier digital technology is nonrival and diffuses to the manufacturing firm through intersectoral technology spillovers. Specifically, digital technology reduces the costs of search, replication, transportation, tracking, and verification, thereby broadly benefiting economic activity across industries (Goldfarb and Tucker, 2019). Moreover, as a general-purpose technology, digital technology is pervasive across sectors and its advances raise the R&D productivity of application sectors through innovational complementarities (Bresnahan and Trajtenberg, 1995; Brynjolfsson et al., 2021). The spillover mechanism reflects the widely recognized phenomenon whereby digital technologies permeate sectoral boundaries and enhance manufacturing productivity. For example, cloud enterprise resource planning enables small and midsize manufacturers to access computational capabilities previously exclusive to large enterprises. Artificial intelligence algorithms enable automated quality inspection that significantly improves production accuracy. Internet-of-Things sensor networks combined with big data analytics permit real-time optimization of supply chain logistics across the manufacturing spectrum.

In the benchmark model, the manufacturing sector in each region consists of a representative firm producing manufactured goods. The firm operates a continuum of production lines, ranked by the difficulty of digital transformation on a $[0, 1]$ uniform distribution. Production lines near zero are the easiest to digitalize (e.g., electronic components, precision instruments, information-intensive processes), while those near one are the hardest (e.g., traditional heavy manufacturing, basic materials processing, textile and garment production). The parameter $m \in (0, 1)$ denotes the manufacturing sector's digital adaptability, where a high m indicates that more production lines can absorb and integrate digital spillovers from the digital services sector. The first m fraction of production lines adopt digital technology in the process of technology upgrading, while the remaining $(1 - m)$ fraction still rely on traditional manufacturing technology. Beyond technology spillovers from the digital to the manufacturing sector, the two sectors are also linked through a data feedback channel. Manufacturing activity generates data that feeds back into the digital sector's knowledge production process. This establishes a bidirectional externality relationship that amplifies the economy's dynamic growth path, as discussed below.

3.2. Consumer Preference

In period t, the representative consumer in region i has a period utility function that takes the logarithmic Cobb-Douglas form:[2]

$$u_i(t) = \beta \ln C_i^D(t) + (1 - \beta) \ln C_i^M(t),$$

where the parameter $\beta \in (0, 1)$ is the expenditure share allocated to digital services. The consumer maximizes discounted lifetime utility:

$$\mathcal{V}_i = \sum_{t=1}^{\infty} \frac{1}{(1+\rho)^{t-1}} u_i(t) = \sum_{t=1}^{\infty} \frac{1}{(1+\rho)^{t-1}} \left[\beta \ln C_i^D(t) + (1 - \beta) \ln C_i^M(t)\right],$$

[2]In the structural transformation literature, Ngai and Pissarides (2007) and Desmet and Rossi-Hansberg (2014) adopt CES preferences over manufacturing and services with an elasticity of substitution below one. Accordingly, labor reallocates from the fast-growing manufacturing sector to the slow-growing service sector. In our paper, however, the digital services sector exhibits faster productivity growth driven by data-augmented innovation. This distinguishes it from the traditional low-growth service sector. Since our core interest is the growth effects of bidirectional spillovers rather than the sectoral reallocation driven by complementarity, we adopt the Cobb-Douglas form for analytical tractability without loss of qualitative generality. Section 6.2 extends the analysis to CES preferences and shows that all main results are qualitatively preserved.

where the parameter ρ is the rate of time preference.

Within each sector, consumption is a CES aggregate of local and imported regional varieties with elasticity of substitution $\sigma > 1$ and home-bias parameter $\alpha \in (1/2, 1)$:

$$C_i^D(t) = \left[\alpha^{1/\sigma}\big(c_{ii}^D(t)\big)^{(\sigma-1)/\sigma} + (1-\alpha)^{1/\sigma}\big(c_{ji}^D(t)\big)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)},$$

$$C_i^M(t) = \left[\alpha^{1/\sigma}\big(c_{ii}^M(t)\big)^{(\sigma-1)/\sigma} + (1-\alpha)^{1/\sigma}\big(c_{ji}^M(t)\big)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)},$$

where $c_{ii}^k(t)$ denotes consumption of region i's own variety in sector k, and $c_{ji}^k(t)$ denotes consumption of region j's variety. Let $p_i^D(t)$ and $p_i^M(t)$ denote the producer prices of region i's digital and manufacturing varieties, respectively. The corresponding price indices are:

$$P_i^D(t) = \left[\alpha\big(p_i^D(t)\big)^{1-\sigma} + (1-\alpha)\big(p_j^D(t)\big)^{1-\sigma}\right]^{1/(1-\sigma)},$$

$$P_i^M(t) = \left[\alpha\big(p_i^M(t)\big)^{1-\sigma} + (1-\alpha)\big(\tau p_j^M(t)\big)^{1-\sigma}\right]^{1/(1-\sigma)}.$$

3.3. Digital Services Sector

For the representative digital services firm in region i, the production technology is concave in labor:

$$Y_i^D(t) = Z_i^D(t) \cdot \big(L_i^D(t)\big)^{\eta},$$

where $Z_i^D(t)$ denotes digital productivity and $\eta \in (0,1)$ is the labor output elasticity. Given CES demand with elasticity σ, the firm charges a constant markup $\sigma/(\sigma-1) > 1$ over marginal cost. The firm chooses labor $L_i^D(t)$ to maximize profit:

$$\max_{L_i^D(t)} \quad \pi_i^D(t) = p_i^D(t) Y_i^D(t) - w_i^L(t) L_i^D(t),$$

where $w_i^L(t)$ denotes the labor wage. The first-order condition yields $w_i^L(t)L_i^D(t) = \eta(\sigma - 1)/\sigma \cdot p_i^D(t)Y_i^D(t)$, so that labor receives a fraction $\eta(\sigma-1)/\sigma$ of revenue. We assume that digital talent owns the patents and firms in the digital sector. The residual profit goes to talent as compensation $w_i^H(t)H_i^D(t)$:

$$w_i^H(t)H_i^D(t) = p_i^D(t)Y_i^D(t) - w_i^L(t)L_i^D(t) = \left[1 - \frac{\eta(\sigma-1)}{\sigma}\right] p_i^D(t)Y_i^D(t).$$

Following Jones and Tonetti (2020), Cong et al. (2021), and Abis and Veldkamp (2024), digital technology evolves as digital talent combines technology-based and data-based knowledge to produce innovation:

$$Z_i^D(t) = \kappa^D \left(Z_i^D(t-1)\right)^{\xi} \left(\mathcal{D}_i(t-1)\right)^{1-\xi} H_i^D(t), \tag{1}$$

where the parameter $\kappa^D > 0$ is the innovation intensity. Technology-based knowledge $Z_i^D(t-1)$ and data-based knowledge $\mathcal{D}_i(t-1)$ are accumulated from last period $t-1$, while digital talent $H_i^D(t)$ applies these inputs in period t to generate new technology. The parameter $\xi \in (0,1)$ governs the relative contribution of the two knowledge sources. The data input $\mathcal{D}_i(t-1)$ will be specified in Section 3.5 after both sectors have been introduced.

3.4. Manufacturing Sector

For the representative manufacturing firm in region i, the production technology is concave in labor:

$$Y_i^M(t) = Z_i^M(t) \cdot \left(L_i^M(t)\right)^{\eta},$$

where $Z_i^M(t)$ denotes manufacturing productivity.[3]

The manufacturing firm's technology combines frontier technology spillovers from the digital services sector with the firm's own technology accumulated through past R&D. The baseline technology before the current-period R&D decision is a linear combination of the last-period digital technology spillover and the last-period firm's own technology:

$$Z_i^{M-}(t) = m \cdot Z_i^D(t-1) + (1-m) \cdot Z_i^M(t-1), \tag{2}$$

where the weight $m \in (0,1)$ is the digital adaptability introduced above, so that a higher m tilts the baseline technology toward the digital spillover component.[4]

[3]Both sectors share the same labor output elasticity η, which simplifies the algebra. Allowing sector-specific elasticities would modify the sectoral labor ratio but would not alter the qualitative results, as the core mechanisms operate through TFP levels and policy instruments rather than through the labor elasticity.

[4]The digital technology spillover $Z_i^D(t-1)$ enters the manufacturing firm's baseline technology as a non-priced externality, meaning that manufacturing firms benefit without compensating the digital sector. This follows the standard treatment of knowledge spillovers in endogenous growth theory (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992). Frontier knowledge is inherently non-rivalrous and only partially excludable, so it raises other agents' productivity without requiring market transactions. Innovators' discoveries and ideas diffuse freely through the economy via publication, professional networks, labor mobility, and informal communication. This allows subsequent producers to build on the existing technological frontier at no direct cost. In our setting, frontier digital technology plays an analogous role. Open-source software libraries, publicly available algorithms, and standardized digital protocols enable manufacturing firms to absorb digital advances through adoption without directly purchasing digital services. If instead manufacturing firms were required to purchase digital services to access the spillover, the cost of adoption would partially internalize the bidirectional externality and reduce the scope for welfare-improving subsidies. The welfare gap between laissez-faire and the First-Best would narrow. The Second-Best subsidy would be smaller, because the market price of digital services already conveys part of the return back to the

Given the baseline technology $Z_i^{M-}(t)$, the firm invests in R&D to obtain effective productivity:

$$Z_i^M(t) = \kappa^M \cdot \left(\psi_i^M(t)\right)^{\phi} \cdot \left(Z_i^{M-}(t)\right)^{1-\phi},$$

where $\psi_i^M(t)$ is R&D investment measured in units of the manufacturing good, the parameter $\kappa^M > 0$ is the R&D intensity, and the parameter $\phi \in (0,1)$ is the R&D technology elasticity.

As in the digital sector, the firm charges a constant markup $\sigma/(\sigma-1)$ over marginal cost given CES demand. The firm jointly chooses labor $L_i^M(t)$ and R&D investment $\psi_i^M(t)$ to maximize profit:

$$\max_{L_i^M(t),\psi_i^M(t)} \quad \pi_i^M(t) = p_i^M(t)Y_i^M(t) - w_i^L(t)L_i^M(t) - p_i^M(t)\psi_i^M(t).$$

The first-order conditions imply $w_i^L(t)L_i^M(t) = \eta(\sigma-1)/\sigma \cdot p_i^M(t)Y_i^M(t)$ and $p_i^M(t)\psi_i^M(t) = \varphi^{LF} p_i^M(t)Y_i^M(t)$, where $\varphi^{LF} \equiv \phi(\sigma-1)/\sigma$ is the manufacturing R&D expenditure share. The residual profit accrues to the representative consumer as the owner of the manufacturing firm.

3.5. Data Generation

Data is generated as a byproduct of economic activity. In the digital services sector, consumption generates data directly for innovation. For example, e-commerce platforms record payment and logistics data, and streaming services collect viewing behavior data. In the manufacturing sector, consumption and R&D activities also generate data. Smart home appliances upload usage data, connected vehicles transmit driving data, R&D laboratories produce experimental and testing data, and pharmaceutical firms accumulate clinical trial records. All such data flows into the producing region's digital sector.[5] Critically, we assume that only consumption within the producing region generates usable data for that region's

digital sector. Nevertheless, the under-provision of digital innovation and the welfare gains from coordination would be preserved as long as the market price does not fully reflect the social value of the spillover.

[5]We abstract from cross-regional and cross-sector data trading. In practice, data can be sold or licensed across jurisdictions and across sectors. A growing literature studies the economics of such transactions (Arrieta-Ibarra et al., 2018; Jones and Tonetti, 2020; Acemoglu et al., 2022; Bergemann et al., 2022; Bergemann and Bonatti, 2024; Liu et al., 2025). This literature highlights several features that distinguish data markets from conventional goods markets. First, data is nonrival. The same dataset can be sold to multiple buyers without being depleted. This undermines sellers' commitment power and depresses data prices through a "data dilution" effect. Second, individual data contains social information that is predictive of other individuals' behavior. This social dimension generates externalities that distort both pricing and trading volume. Third, data transactions involve substantial frictions. It is difficult to assign property rights over data, to verify data quality before purchase, and to prevent unauthorized resale. Fourth, platform market power shapes how data is collected, governed, and distributed, with implications for consumer welfare. Section 6.4 relaxes this abstraction by introducing data ownership and trading frictions—including privacy costs and creative-destruction costs—under two polar ownership structures, and shows that the three propositions continue to hold. The additional trading frictions reduce data usage and weaken the growth effect of the data feedback externality channel, but do not alter the qualitative conclusions of the paper.

digital sector. Goods exported to and consumed in the other region do not contribute to the home region's data stock. This assumption reflects the growing prevalence of cross-border data protection regulations, including the European Union's (EU) General Data Protection Regulation (GDPR) and China's Personal Information Protection Law (PIPL), which impose substantial restrictions on the cross-jurisdictional transfer and use of consumer data (Acquisti et al., 2016; Chang et al., 2023; Sun and Trefler, 2023; Goldberg et al., 2024). As a conservative benchmark, we therefore exclude cross-regional consumption data.

The data generation equation is given by

$$\mathcal{D}_i(t-1) = c_{ii}^D(t-1) + \frac{m\chi_i(t-1)}{m\chi_i(t-1) + (1-m)} \cdot \left[c_{ii}^M(t-1) + \psi_i^M(t-1)\right], \tag{3}$$

where $\chi_i(t-1) \equiv Z_i^D(t-1)/Z_i^M(t-1)$ is the technology ratio, with $\chi_i > 1$ reflecting the digital sector's technological lead. The first term on the right-hand side, c_{ii}^D, captures data from home-region digital consumption. The second term captures data from home-region manufacturing consumption c_{ii}^M and R&D investment ψ_i^M, weighted by the digital spillover share $m\chi_i/[m\chi_i + (1-m)]$. This weight equals the share of the digital spillover component $m \cdot Z_i^D(t-1)$ in the baseline technology Eq. (2), which means only the digitally sourced component of manufacturing activity generates data useful for digital innovation.

The data generation mechanism distinguishes digital innovation from traditional innovation. Digital innovation benefits from data generated as a byproduct of all economic activity, including both production for consumption and R&D investment. Moreover, as the manufacturing sector deepens its digital transformation, a larger share of its activity becomes data-generating, so the volume of usable data increases. Without loss of generality, we assume data fully depreciates each period, so sustained innovation requires ongoing activity.

3.6. General Equilibrium

Figure 1 summarizes the two-sector framework and the bidirectional externality linkages. The solid arrow between the two sectors represents the digital technology spillover, through which digital sector technology spills over to upgrade manufacturing baseline technology. The dashed arrow between the two sectors represents the manufacturing data feedback, through which manufacturing activity generates data that feeds back into digital innovation. Together, these two channels create a self-reinforcing growth loop: higher digital technology improves manufacturing technology, which raises consumption and generates more data, which in turn accelerates digital innovation and further increases digital technology.

Definition 1 (Laissez-Faire Equilibrium)**.** The equilibrium in the laissez-faire economy consists of sequences of allocations $\{L_i^D(t),\, L_i^M(t),\, H_i^D(t),\, c_{ii}^D(t),\, c_{ij}^D(t),\, c_{ii}^M(t),\, c_{ij}^M(t),\, \psi_i^M(t)\}_{t=1}^{\infty}$

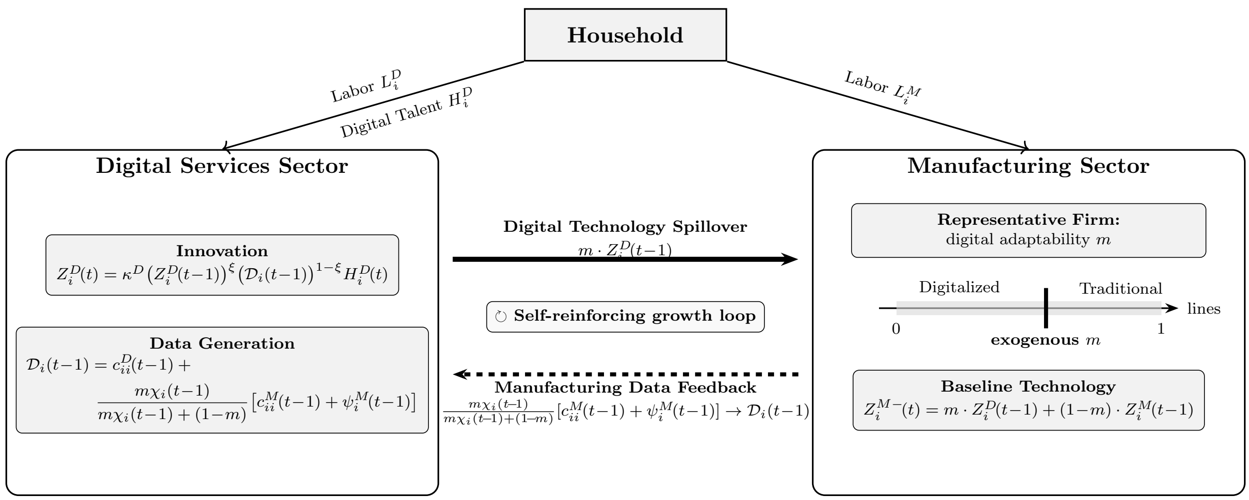

Figure 1: Two-sector framework with bidirectional externalities.

Notes: The solid arrow between the two sectors represents the digital technology spillover from the digital services sector to manufacturing. The dashed arrow between the two sectors represents the manufacturing data feedback from both manufacturing consumption and R&D activity to digital innovation.

and prices $\{w_i^L(t), w_i^H(t), p_i^D(t), p_i^M(t)\}_{t=1}^{\infty}$ such that consumers maximize utility subject to their budget constraints, firms in both sectors maximize profits, and the following market clearing conditions hold:

(i) **Factor Market Clearing:** Both types of labor are perfectly mobile, so their wages are equalized across all sectors and regions:

$$w_A^L = w_B^L \equiv w^L, \quad w_A^H = w_B^H \equiv w^H.$$

Regular labor clears as $\sum_i (L_i^D + L_i^M) = \bar{L}$. Digital talent clears as $\sum_i H_i^D = \bar{H}$, with talent flowing to the region offering higher returns until w^H is equalized.

(ii) **Goods Market Clearing:** Digital output satisfies $Y_i^D = c_{ii}^D + c_{ij}^D$ (local consumption plus exports). Manufacturing output satisfies $Y_i^M = c_{ii}^M + \tau c_{ij}^M + \psi_i^M$ (local consumption, iceberg-adjusted exports, and R&D investment).

(iii) **Trade Balance:** Each region's export revenue equals its import expenditure, so that $p_i^D c_{ij}^D + p_i^M \tau c_{ij}^M = p_j^D c_{ji}^D + p_j^M \tau c_{ji}^M$.

The labor market clearing condition determines the laissez-faire (LF) digital labor share:

$$\Theta_L^{LF} \equiv \frac{L_i^D}{L_i^D + L_i^M} = \frac{\beta(1-\varphi^{LF})}{1-\varphi^{LF}\beta}. \tag{4}$$

The digital sector attracts more labor when consumers' preference for digital services

(β) is stronger. A higher R&D share φ^{LF} diverts part of manufacturing revenue to R&D investment, reducing the labor demand in manufacturing and thereby raising Θ_L^{LF}.

On the balanced growth path (BGP), output, consumption, and technology levels in both sectors grow at the same rate. The steady-state growth rate satisfies

$$g^{*,LF} = \kappa^D (\Phi^{*,LF})^{1-\xi} \left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)} \frac{\bar{H}}{2} - 1. \tag{5}$$

Eq. (5) embeds an endogenous growth engine following Romer (1990), with the steady-state growth rate increasing in the talent stock $\bar{H}/2$. In addition, the cross-sector bidirectional externalities between the digital and manufacturing sectors generate a self-reinforcing growth effect, captured by the effective data intensity $\Phi^{*,LF}$. This composite term aggregates both the digital sector's own data and the manufacturing sector's data feedback:

$$\Phi^{*,LF} = \alpha(\Theta_L^{LF})^\eta + \underbrace{\frac{m}{m\chi^{*,LF} + (1-m)}}_{\text{digital spillover intensity}} \cdot \underbrace{\left[\frac{\alpha(1-\varphi^{LF})}{\alpha + (1-\alpha)\tau^{1-\sigma}} + \varphi^{LF}\right]}_{\text{manufacturing data feedback rate}} \cdot (1-\Theta_L^{LF})^\eta, \tag{6}$$

$$m\chi^{*,LF} + (1-m) = \frac{\kappa^D (\Phi^{*,LF})^{1-\xi} \left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)} \frac{\bar{H}}{2}}{(\kappa^M)^{\frac{1}{1-\phi}} (\varphi^{LF})^{\frac{\phi}{1-\phi}} \left(\frac{(1-\Theta_L^{LF})\bar{L}}{2}\right)^{\frac{\phi\eta}{1-\phi}}}. \tag{7}$$

In Eq. (6), the first term on the right-hand side captures data generated directly by digital consumption. The second term embodies the two bidirectional spillovers jointly. The first underbrace, $m/[m\chi^{*,LF} + (1-m)]$, captures the digital spillover intensity: it equals the digital technology share in manufacturing's composite technology base, $m\chi^{*,LF}/[m\chi^{*,LF} + (1-m)]$, divided by the technology ratio $\chi^{*,LF}$, reflecting the conversion from manufacturing output units to digital-sector units. The second underbrace captures the manufacturing data feedback rate, aggregating home consumption data $\alpha(1-\varphi^{LF})/[\alpha + (1-\alpha)\tau^{1-\sigma}]$ and local R&D data φ^{LF}. Home consumption is attenuated by the CES trade structure, as lower iceberg costs reduce the locally consumed share. When no production line adopts digital technology ($m \to 0$), both spillover channels shut down and growth depends solely on the digital sector's own data. Eq. (7) determines the equilibrium technology ratio $\chi^{*,LF}$. A faster digital innovation rate widens the technology gap between the two sectors.

The two-sector economy with digital and manufacturing sectors reveals the growth effect of the bidirectional spillover mechanism, which yields the following result.

Proposition 1 (Bidirectional Spillovers and Endogenous Growth)**.** *The digital and manufacturing sectors are linked by bidirectional spillovers, the digital technology spillover and the* *manufacturing data feedback. Define the growth premium*

$$\Delta g^{LF}(m) \equiv g^{*,LF}(m) - \lim_{m\to 0} g^{*,LF}(m),$$

where $\lim_{m\to 0} g^{*,LF}(m) = \kappa^D \alpha^{1-\xi}(\Theta_L^{LF})^{\eta(1-\xi)}(\bar{L}/2)^{\eta(1-\xi)}\bar{H}/2 - 1$ *is the growth rate at which the two sectors become technologically independent. For all* $m \in (0,1)$*,* $\Delta g^{LF}(m) > 0$ *and* $d\Delta g^{LF}/dm > 0$*. The bidirectional spillovers generate a positive growth premium that is increasing in the manufacturing sector's digital adaptability.*

Proof. See Appendix B.

On the BGP, both $C_i^D(t)$ and $C_i^M(t)$ grow at rate $g^{*,LF}$, so $u_i(t) = u_i(t=1) + (t-1)\ln(1+g^{*,LF})$. The lifetime utility of the representative consumer in region i decomposes into a level effect and a growth effect:

$$\mathcal{V}_i^{LF} = \frac{1+\rho}{\rho} u_i^{LF}(t=1) + \frac{1+\rho}{\rho^2}\ln(1+g^{*,LF}),$$

where initial-period utility is given by

$$u_i^{LF}(t=1) = \ln\Big[Z_i^M(t=1)(\chi^{*,LF})^{\beta}(\Theta_L^{LF})^{\eta\beta}(1-\Theta_L^{LF})^{\eta(1-\beta)}(\bar{L}/2)^{\eta} \times\big[\alpha + (1-\alpha)\tau^{1-\sigma}\big]^{-\frac{1-\beta}{1-\sigma}}(1-\varphi^{LF})^{1-\beta}\Big]. \quad (8)$$

The level effect captures the welfare contribution of the initial consumption allocation, determined by the technology ratio $\chi^{*,LF}$ and the sectoral labor allocation Θ_L^{LF}. The growth effect captures the welfare contribution of sustained consumption growth, amplified by an additional factor of $1/\rho$ relative to the level effect. Aggregate lifetime welfare is:

$$\mathcal{W}^{LF} \equiv \sum_{i\in\{A,B\}} \mathcal{V}_i^{LF} = \frac{2(1+\rho)}{\rho} u^{LF}(t=1) + \frac{2(1+\rho)}{\rho^2}\ln(1+g^{*,LF}).$$

4. Fiscal Competition and Efficiency

This section introduces government fiscal policy into the framework. It begins with the efficiency of the laissez-faire equilibrium, then characterizes coordinated fiscal intervention, and finally analyzes decentralized fiscal competition between local governments.

4.1. First-Best Optimality

The bidirectional spillovers generate externalities that private agents do not internalize. To evaluate the efficiency of the laissez-faire equilibrium, we introduce a First-Best planner who directly assigns resources across sectors.

Definition 2 (First-Best Allocation)**.** The First-Best (FB) allocation is a symmetric pair $(\Theta_L^{FB}, \varphi^{FB})$ that maximizes aggregate lifetime welfare

$$\max_{\Theta_L,\varphi} \quad \mathcal{W}^{FB}(\Theta_L,\varphi) = \frac{2(1+\rho)}{\rho} u(t=1;\Theta_L,\varphi) + \frac{2(1+\rho)}{\rho^2} \ln\bigl(1+g^*(\Theta_L,\varphi)\bigr),$$

where $\Theta_L \equiv L_i^D/(L_i^D + L_i^M) \in (0,1)$ is the digital labor share and $\varphi \equiv \psi_i^M / Y_i^M \in (0,1)$ is the manufacturing R&D expenditure share, subject to:

(i) **Symmetry:** Both regions receive identical (Θ_L,φ), with $L_A = L_B = \bar{L}/2$ and $H_A^D = H_B^D = \bar{H}/2$.

(ii) **Equilibrium:** Given (Θ_L,φ), the equilibrium conditions Eq. (5)–Eq. (7) determine (Φ^*, χ^*, g^*), and the initial-period utility $u(t=1;\Theta_L,\varphi)$ follows from Eq. (8).

The FB planner's first-order conditions are:

$$\frac{\partial \mathcal{W}^{FB}}{\partial \Theta_L} = \frac{2(1+\rho)}{\rho} \left\{ \underbrace{\frac{\eta\beta}{\Theta_L} - \frac{\eta(1-\beta)}{1-\Theta_L} + \frac{\beta[m\chi^* + (1-m)]}{m\chi^*} \cdot \left[\frac{\eta\phi}{(1-\Theta_L)(1-\phi)} + \frac{1-\xi}{\Phi^*}\frac{\partial \Phi^*}{\partial \Theta_L} \right]}_{\text{level effect}} + \underbrace{\frac{1-\xi}{\rho\Phi^*}\frac{\partial \Phi^*}{\partial \Theta_L}}_{\text{growth effect}} \right\} = 0, \tag{9}$$

$$\frac{\partial \mathcal{W}^{FB}}{\partial \varphi} = \frac{2(1+\rho)}{\rho} \left\{ \underbrace{-\frac{1-\beta}{1-\varphi} + \frac{\beta[m\chi^* + (1-m)]}{m\chi^*} \cdot \left[\frac{1-\xi}{\Phi^*}\frac{\partial \Phi^*}{\partial \varphi} - \frac{\phi}{(1-\phi)\varphi} \right]}_{\text{level effect}} + \underbrace{\frac{1-\xi}{\rho\Phi^*}\frac{\partial \Phi^*}{\partial \varphi}}_{\text{growth effect}} \right\} = 0. \tag{10}$$

Eq. (9) decomposes the social return to digital labor into a level effect and a growth effect. The level effect captures the welfare impact through initial-period utility and combines two components. The first two terms $\eta[\beta/\Theta_L - (1-\beta)/(1-\Theta_L)]$ measure the net marginal return of reallocating labor from manufacturing to the digital sector, reflecting the standard wage-equalization trade-off. The product term $\beta[m\chi^* + (1-m)]/(m\chi^*)$ multiplied by the χ^*-elasticity term captures externalities operating through the technology ratio χ^*, which private agents do not internalize. The first element of the χ^*-elasticity term, $\eta\phi/[(1-\Theta_L)(1-\phi)]$, reflects the direct output-level channel. Expanding digital labor strengthens the digital sector's technological position relative to manufacturing, raising χ^* and thereby elevating the initial consumption level. The second element, $(1-\xi)/\Phi^* \cdot \partial\Phi^*/\partial\Theta_L$, reflects the equilibrium feedback channel. More digital labor raises the effective data intensity Φ^* by expanding data generation and strengthening the digital spillover, which further increases χ^* and amplifies the level gain. Both elements are strictly positive, confirming that the χ^*-elasticity term favors a larger digital sector. The growth effect $(1-\xi)/(\rho\Phi^*) \cdot \partial\Phi^*/\partial\Theta_L > 0$ captures the dynamic welfare gain. A higher Φ^* raises the balanced growth rate g^*, generating a permanent growth dividend that fuels future innovation through richer data accumulation.

Eq. (10) decomposes the social return to manufacturing R&D investment. The consumption cost $-(1-\beta)/(1-\varphi) < 0$ represents the direct resource diversion, as allocating more manufacturing revenue to R&D reduces consumption. The χ^*-elasticity term combines two opposing channels. The first element $(1-\xi)/\Phi^* \cdot \partial\Phi^*/\partial\varphi > 0$ reflects that higher R&D raises the effective data intensity Φ^*, which in turn increases χ^* and lifts the initial consumption level. The second element $-\phi/[(1-\phi)\varphi] < 0$ reflects that R&D strengthens manufacturing's technological position, narrowing χ^* and depressing the initial consumption level. The sign of the χ^*-elasticity term is therefore ambiguous, depending on whether the data-enrichment channel or the technology-catching-up channel dominates. The growth effect is strictly positive. Higher R&D raises Φ^* and hence g^*, as the additional manufacturing data feeds back into digital innovation, permanently raising the growth rate.

Under laissez-faire, private agents equalize wages across sectors and equate the private marginal product of R&D to its marginal cost:

$$\frac{\beta}{\Theta_L^{LF}} = \frac{1-\beta}{(1-\Theta_L^{LF})(1-\varphi^{LF})}, \tag{11}$$

$$\varphi^{LF} = \frac{\phi(\sigma-1)}{\sigma}. \tag{12}$$

Substituting Eq. (11) into Eq. (9), the first two terms reduce to $\eta(1-\beta)\varphi/[(1-\Theta_L)(1-\varphi)] > 0$. Hence at the LF allocation the labor reallocation margin is strictly positive, indicating that the digital sector remains under-staffed even after private optimization. The product term multiplied by the χ^*-elasticity term and the growth effect are entirely absent from private decision-making; in Eq. (10), only the consumption cost is reflected in the private calculus Eq. (12). These uninternalized externalities render the laissez-faire equilibrium generically inefficient. This leads to the following result.

Proposition 2 (Externalities Invalidate Laissez-Faire Efficiency)**.** *The laissez-faire equilibrium is Pareto-inefficient. The digital technology spillover externality and the manufacturing data feedback externality jointly imply:*

(i) $\Theta_L^{LF} < \Theta_L^{FB}$*: laissez-faire under-allocates labor to the digital sector relative to the First-Best;*

(ii) the sign of $\varphi^{FB} - \varphi^{LF}$ *is ambiguous: laissez-faire may over- or under-invest in manufacturing R&D relative to the First-Best, depending on whether the data-feedback externality (raising* Φ^**) outweighs the technology-ratio externality (narrowing* χ^**);*

(iii) $\mathcal{W}^{LF} < \mathcal{W}^{FB}$*: laissez-faire lifetime welfare is strictly below the First-Best.*

Proof. See Appendix C.

Proposition 2 identifies two sources of inefficiency rooted in the bidirectional spillovers between sectors. The digital sector generates data that raises the effective data intensity Φ^*, benefiting both sectors through faster innovation. The manufacturing sector, by conducting R&D and accumulating production experience, feeds data back into digital innovation, reinforcing the cycle. Private agents, however, fail to capture these social returns and therefore under-value digital labor. Expanding the digital sector would raise Φ^* and χ^*, lifting both the consumption level and the growth rate. Laissez-faire consequently under-allocates labor to the digital sector.

For manufacturing R&D, two opposing externalities arise. On the one hand, higher R&D enriches the data feedback loop, raising Φ^* and boosting long-run growth. On the other hand, it strengthens manufacturing's technological position relative to the digital sector, narrowing χ^* and depressing the short-run consumption level. Whether the First-Best requires more or less R&D than laissez-faire depends on which externality dominates at the margin.

Taken together, welfare under laissez-faire is strictly below the First-Best. This inefficiency provides the rationale for government intervention analyzed below.

4.2. Government Instruments and the Second-Best Allocation

Given that laissez-faire is Pareto-inefficient, we now introduce government fiscal intervention. Regional governments deploy two fiscal instruments to influence digital transformation, a digital output subsidy $s_i^D \geq 0$ that promotes digital sector expansion, and a manufacturing tax rate $t_i \in [0, 1)$ that finances subsidy expenditures. The subsidy augments digital producers' revenue, so that a digital firm in region i receives $(1 + s_i^D)p_i^D$ per unit of output. The manufacturing tax is levied on producers at rate t_i on output, so the firm retains only the fraction $(1 - t_i)$ of gross revenue.

Given the government instruments, the balanced growth path equilibrium, including the effective data intensity Φ^*, the technology ratio χ^*, the balanced growth rate g^*, and aggregate welfare $\mathcal{W}$, follows from the equilibrium conditions. The government budget constraint pins down the equilibrium tax rate as a function of the subsidy s^D.

$$t_i = \frac{s_i^D \beta[1 - \phi(\sigma - 1)/\sigma]}{(1 - \beta) - s_i^D \beta \phi(\sigma - 1)/\sigma}. \tag{13}$$

The manufacturing tax reduces the R&D share to the policy-adjusted level

$$\varphi = \frac{\phi(\sigma - 1)(1 - t_i)}{\sigma}, \tag{14}$$

and the policy-adjusted digital labor share becomes

$$\Theta_L = \frac{(1+s_i^D)\beta[1-\phi(\sigma-1)/\sigma]}{1-\phi(\sigma-1)\beta(1+s_i^D)/\sigma}. \tag{15}$$

We now define the Second-Best allocation as the optimal outcome achievable when a centralized planner coordinates both regions' fiscal instruments (s_i^D, t_i) to maximize aggregate welfare. The centralized coordinator can only influence allocations indirectly through fiscal instruments, unlike the First-Best planner who directly selects the optimal (Θ_L, φ). As a result, the resulting welfare improves upon laissez-faire but does not reach the First-Best. The welfare gap $\mathcal{W}^{SB} - \mathcal{W}^{LF}$ therefore represents the maximum welfare improvement that the government's digital subsidy instrument can achieve.

Definition 3 (Second-Best Allocation)**.** The Second-Best (SB) allocation is a symmetric digital output subsidy $s^{D,SB}$ that maximizes aggregate lifetime welfare

$$\max_{s^D} \quad \mathcal{W}^{SB}(s^D) = \frac{2(1+\rho)}{\rho} u(t=1; s^D) + \frac{2(1+\rho)}{\rho^2} \ln\bigl(1+g^*(s^D)\bigr),$$

where $s^D \geq 0$ is the common digital output subsidy applied to both regions, subject to:

(i) **Symmetry:** Both regions receive identical s^D, with $L_A = L_B = \bar{L}/2$ and $H_A^D = H_B^D = \bar{H}/2$.

(ii) **Budget balance:** The manufacturing tax rate $t_i \in [0,1)$ finances the subsidy period by period, $t_i\, p_i^M Y_i^M = s^D p_i^D Y_i^D$, yielding Eq. (13).

(iii) **Equilibrium:** Given (s^D, t), the policy-induced allocations (φ, Θ_L) follow from Eq. (14)–Eq. (15); private agents then optimize as in Definition 1, and the equilibrium quantities (Φ^*, χ^*, g^*) satisfy the structural system Eq. (5)–Eq. (7).

The digital spillover externality and the data feedback externality jointly ensure that the SB-optimal subsidy is strictly positive. At $s^D = 0$, the marginal welfare gain is positive through both channels. Through the level effect, a higher subsidy expands the digital sector, raising the technology ratio and initial consumption in both regions. Through the growth effect, digital sector expansion raises the effective data intensity via both labor reallocation and amplified data feedback, thereby raising g^*. These gains are offset by a rising marginal welfare cost, as the manufacturing tax rate required to balance the budget rises, compressing manufacturing output and R&D investment. The SB-optimal subsidy $s^{D,SB}$ is determined where the marginal welfare benefit equals the marginal welfare cost, yielding the ranking $\mathcal{W}^{FB} > \mathcal{W}^{SB} > \mathcal{W}^{LF}$.

4.3. Tournament Competition and Nash Equilibrium

The preceding subsection shows that centralized coordination delivers substantial welfare gains. This subsection asks whether decentralized tournament competition can serve as an alternative mechanism to reach the SB optimum. A large literature on China's promotion tournament establishes that, within a system combining political centralization with economic decentralization, local officials are evaluated and promoted on the basis of relative economic performance (Maskin et al., 2000; Li and Zhou, 2005; Xu, 2011). The existing literature typically models this as a weighted combination of the official's own-region welfare and a yardstick term that rewards outperforming peer jurisdictions in GDP growth (Li et al., 2019; Fang et al., 2025). The growth rate serves as the central tournament metric because, as a relative performance measure, it filters out common external shocks and thereby isolates individual officials' policy effort (Maskin et al., 2000).

Following this approach, each regional government serves a finite tenure of T periods and chooses its digital output subsidy to maximize a composite objective. This objective combines own-region welfare with relative growth performance against the rival region.

Definition 4 (Nash Equilibrium)**.** The Nash Equilibrium (NE) is a symmetric digital output subsidy $s^{D,NE}$ such that each regional government maximizes

$$\begin{aligned} \max_{s_i^D} \quad & (1-\lambda)\sum_{t=1}^{T}\frac{u_i(t)}{(1+\rho)^{t-1}} + \lambda\big[g_i^* - g_j^*\big] \\ &= (1-\lambda)\big[\Gamma_1(T)u_i(t=1) + \Gamma_2(T)\ln(1+g_i^*)\big] + \lambda\big[g_i^* - g_j^*\big], \end{aligned} \tag{16}$$

where $\lambda \in (0,1)$ is the tournament competition intensity,

$$\Gamma_1(T) \equiv \sum_{t=1}^{T}\frac{1}{(1+\rho)^{t-1}}, \qquad \Gamma_2(T) \equiv \sum_{t=1}^{T}\frac{t-1}{(1+\rho)^{t-1}},$$

subject to:

(i) **Budget balance:** The manufacturing tax rate $t_i \in [0,1)$ finances the subsidy period by period, yielding Eq. (13).

(ii) **Equilibrium:** Given (s_i^D, t_i), the policy-induced allocations (φ, Θ_L) follow from Eq. (14)–Eq. (15); private agents then optimize as in Definition 1, and the equilibrium quantities (Φ^*, χ^*, g^*) satisfy the structural system Eq. (5)–Eq. (7).

(iii) **Best response:** Each regional government simultaneously chooses s_i^D to maximize Eq. (16), taking the rival's subsidy s_j^D as given.

At $\lambda = 0$ the regional government cares only about local welfare over its finite tenure, and at $\lambda = 1$ only about relative performance. The tournament term $\lambda(g_i^* - g_j^*)$ captures the competition in output growth rates, measuring the gap between the local region's growth rate and that of its rival. The total effect of s_i^D on region i's growth rate decomposes into two channels:

$$\frac{dg_i^*}{ds_i^D} = \underbrace{\left.\frac{\partial g_i^*}{\partial s_i^D}\right|_{L_i, H_i^D}}_{\text{intra-regional policy}} + \underbrace{\underbrace{\frac{\partial g_i^*}{\partial L_i} \cdot \frac{\partial L_i}{\partial s_i^D}}_{\text{labor grabbing}} + \underbrace{\frac{\partial g_i^*}{\partial H_i^D} \cdot \frac{\partial H_i^D}{\partial s_i^D}}_{\text{talent grabbing}}}_{\text{inter-regional factor}},$$

where the intra-regional policy channel operates through Θ_L, φ, and χ^*, holding the inter-regional factor allocation (L_i, H_i^D) fixed. The inter-regional factor channel captures the inflow of labor and talent attracted by the subsidy. At the symmetric equilibrium, the talent-grabbing elasticity and the labor-grabbing elasticity are:

$$\left.\frac{\partial H_i^D}{\partial s_i^D}\right|_{\text{sym}} = \frac{\bar{H}}{4(1-\eta)(1+s^D)},$$

$$\left.\frac{\partial L_i}{\partial s_i^D}\right|_{\text{sym}} = \frac{\bar{L}}{4(1+s^D)} \left[\frac{1}{1-\eta} - \frac{1}{1-\phi(\sigma-1)\beta(1+s^D)/\sigma}\right],$$

where the talent-grabbing elasticity is quantitatively larger, making talent competition the dominant strategic force.

The first-order condition of the Nash problem is:

$$(1-\lambda)\left[\Gamma_1(T)\left.\frac{\partial u_i(t=1)}{\partial s_i^D}\right|_{L_i, H_i^D} + \frac{\Gamma_2(T)}{1+g_i^*}\left.\frac{\partial g_i^*}{\partial s_i^D}\right|_{L_i, H_i^D}\right] + \lambda\left[\frac{dg_i^*}{ds_i^D} - \frac{\partial g_j^*}{\partial s_i^D}\right] = 0.$$

The second bracket captures the tournament incentive. Because talent-grabbing raises the subsidizing region's growth rate while lowering the rival's, it widens the growth differential that determines tournament outcomes. A higher λ therefore induces each regional government to raise its subsidy, creating a race-to-the-top dynamic. Comparing the Nash equilibrium with the SB optimum yields the following result.

Proposition 3 (Inverted-U Welfare in Tournament Intensity)**.**

(i) The Nash equilibrium subsidy $s^{D*,Nash}(\lambda)$ is strictly increasing in λ on $(0, 1)$.

(ii) Nash welfare $\mathcal{W}^{Nash}(\lambda)$ is inverted-U-shaped in λ: $d\mathcal{W}^{Nash}/d\lambda > 0$ for $\lambda \in (0, \lambda^{peak})$ ("good competition") and $d\mathcal{W}^{Nash}/d\lambda < 0$ for $\lambda \in (\lambda^{peak}, 1)$ ("bad competition").

(iii) At the peak, the Nash subsidy coincides with the SB optimum: $s^{D*,Nash}(\lambda^{peak}) = s^{D,SB}$, so $\mathcal{W}^{Nash}(\lambda^{peak}) = \mathcal{W}^{SB}$.

Proof. See Appendix D.

The inverted-U pattern reflects a trade-off between externality correction and over-subsidization. Because laissez-faire under-allocates resources to the digital sector, the digital spillover externality and the data feedback externality remain uncorrected. Competition-induced subsidies expand the digital sector, generating stronger technology spillovers to manufacturing and more data that feeds back into digital innovation. This partially corrects both externalities. When tournament intensity is low, regional governments set subsidies too conservatively, leaving much of the externality uncorrected. At the peak intensity λ^{peak}, the Nash subsidy coincides with the Second-Best optimum, and decentralized competition replicates the centralized coordinator's outcome. Beyond this threshold, the economy moves into over-subsidization. The fiscal cost of financing ever-larger subsidies through the manufacturing tax outweighs the marginal externality correction, and welfare declines.

5. Quantitative Analyses

This section calibrates the model to Chinese and EU economies and compares welfare across the laissez-faire, First-Best, and Second-Best allocations. The China calibration, which features tournament competition, additionally illustrates the inverted-U welfare pattern as tournament intensity varies.

5.1. Calibration

Table 3 reports baseline parameter values for two settings, a China calibration treating cities as regions and an EU calibration treating member states as regions. Each externally set parameter is grounded in the empirical literature. The two scale parameters (κ^D, κ^M) are calibrated by targeting a balanced growth rate and a technology ratio under laissez-faire. The discussion below presents China and EU values jointly for each parameter group.

Preferences and expenditure. The annual discount factor in calibrated macroeconomic models typically falls in the range 0.96–0.99, corresponding to a discount rate of 0.01–0.04 (Kydland and Prescott, 1982; Smets and Wouters, 2003; Christiano et al., 2005; Song et al., 2011). We set $\rho = 0.03$ for both calibrations, a moderate choice within this range.

The government tenure is $T = 5$ years for China, corresponding to the de jure term of local government officials. This parameter determines the finite-horizon coefficients $\Gamma_1(T)$ and $\Gamma_2(T)$ in the Nash objective. At $T = 5$ and $\rho = 0.03$, $\Gamma_1(5) = 4.72$ and $\Gamma_2(5) = 9.16$, yielding a myopia ratio $\Gamma_2(5)/\Gamma_1(5) = 1.94$ compared with the infinite-horizon ratio $1/\rho = 33.3$. Because the EU setting does not involve tournament competition, the SB outcome is interpreted as the welfare level attainable through centralized coordination rather than through decentralized tournament competition.

Table 3: Baseline Parameter Calibration

Symbol	Description	China	EU
Panel A: Preferences and expenditure			
ρ	Discount rate	0.03	0.03
β	Digital expenditure share	0.20	0.25
T	Government tenure (years)	5	—
Panel B: Production technology and innovation			
η	Labor output elasticity	0.50	0.67
ϕ	R&D technology elasticity	0.15	0.15
ξ	Knowledge persistence	0.80	0.80
Panel C: Trade and market structure			
σ	CES elasticity of substitution	5.0	5.0
α	Home-bias parameter	0.70	0.65
τ	Iceberg trade cost	1.71	2.70
Panel D: Internally calibrated and normalization			
κ^D	Digital innovation intensity	$g^{*,LF} = 2\%$	$g^{*,LF} = 2\%$
κ^M	Manufacturing R&D efficiency	$\chi^{*,LF} = 1.50$	$\chi^{*,LF} = 1.20$
$\bar{L}$	Total labor supply	2.0	2.0
$\bar{H}$	Total digital talent	0.02	0.02

Notes: The China calibration treats cities as regions and the EU calibration treats member states as regions. Parameters in Panels A–C are set externally from the literature (see main text). The tenure T enters the Nash objective only under the China calibration; the EU calibration does not involve tournament competition, so no tenure value is required. In Panel D, κ^D and κ^M are jointly calibrated by targeting the laissez-faire growth rate $g^{*,LF}$ and technology ratio $\chi^{*,LF}$. The normalization $\bar{L} = 2$ implies $\bar{L}/2 = 1$, which drops from all equilibrium expressions.

The digital expenditure share β governs the ratio of consumer spending on digital services relative to manufactured goods ($\beta : 1 - \beta$). We set $\beta = 0.20$ for China and $\beta = 0.25$ for the EU. Zhang and Chen (2019) report that China's digital economy, defined as the ICT sector based on the OECD framework, accounts for approximately 6% of GDP; Eurostat reports approximately 5.5% for the EU. With manufacturing value added at approximately 25% (China, World Bank) and 15% (EU, World Bank), the digital-to-manufacturing ratios are roughly 1:4 and 1:3, yielding $\beta = 0.20$ and 0.25.

Production technology and innovation. The labor output elasticity is $\eta = 0.50$ for China and $\eta = 0.67$ for the EU. For China, we adopt the estimate of Bai and Qian (2010), who document that China's labor share is approximately 0.50. The EU value of 0.67 is consistent with the higher labor share observed in advanced economies.

The R&D technology elasticity is $\phi = 0.15$ for both calibrations. Hall and Mairesse (1995) and Kancs and Siliverstovs (2016) estimate that the elasticity of productivity with respect to R&D is approximately 0.15, and we adopt this value directly. Our $\phi = 0.15$

implies that 15% of manufacturing technology improvement comes from endogenous R&D.

The knowledge persistence parameter is $\xi = 0.80$ for both calibrations, implying a data elasticity of $1 - \xi = 0.20$. Existing estimates span a wide range. Jones and Tonetti (2020) calibrate the data elasticity at 0.06 in their benchmark exercise. Cong et al. (2021) specify separate exponents of 0.85 for knowledge spillovers and 0.50 for data in their innovation possibility frontier, implying a normalized data share of $0.50/(0.50 + 0.85) \approx 0.37$. Our value of 0.20 is a moderate choice between these estimates.

Trade and market structure. The CES elasticity of substitution is $\sigma = 5.0$ for both calibrations, implying a constant markup of $5/4 = 1.25$. Simonovska and Waugh (2014) use cross-country price data to estimate a trade elasticity of approximately 4, implying $\sigma \approx 5$ under the standard Armington framework where the trade elasticity equals $\sigma - 1$. Similarly, for China, Tombe and Zhu (2019) adopt the same trade elasticity of 4 in their study of inter-provincial trade, and we follow this convention for inter-city trade in the China calibration.

The home-bias parameter and iceberg trade cost are $(\alpha, \tau) = (0.70, 1.71)$ for China and $(0.65, 2.70)$ for the EU. For China, Tombe and Zhu (2019) estimate inter-provincial home spending shares in the range 0.623–0.787 and iceberg trade costs in the range 1.71–2.89 for 2007. We set $\alpha = 0.70$ and adopt the lower bound $\tau = 1.71$, given the greater openness of inter-city trade relative to inter-provincial trade. For the EU, Anderson and Van Wincoop (2004) estimate that comprehensive trade costs among developed countries are approximately 170% ad valorem, corresponding to $\tau = 2.70$. World Bank data indicate that domestic expenditure shares of major EU member states fall in the range 0.60–0.70, and we set $\alpha = 0.65$ accordingly.

Factor endowments. Total labor supply is normalized to $\bar{L} = 2.0$ for both calibrations, so that each region's labor endowment is $\bar{L}/2 = 1$. Under this normalization, all equilibrium quantities can be interpreted on a per capita basis. Total digital talent is set to $\bar{H} = 0.02$, implying that digital specialists account for 1% of the total labor force.

Joint calibration of κ^D and κ^M. The two scale parameters κ^D (digital innovation intensity) and κ^M (manufacturing R&D efficiency) have no direct empirical counterparts. We jointly calibrate them by targeting $g^{*,LF} = 2\%$, matching the long-run average real GDP per capita growth rate, and the technology ratio $\chi^{*,LF} = 1.5$ (China) and 1.2 (EU). These targets are conservative relative to available data. China's NBS reports ICT-to-manufacturing wage ratios near 2, and García-Herrero and Xu (2018) estimate ICT labor productivity at 1.8 times the economy-wide average. EU Eurostat data indicate a comparable ratio of 1.5–1.6. Because labor productivity differentials overstate TFP gaps when the ICT sector is more skill-intensive, we adopt the lower calibration values.

5.2. Quantitative Results

5.2.1. Welfare Costs and Gains from Coordination

This subsection compares equilibrium outcomes under three allocations, namely laissez-faire (LF), the First-Best (FB), and the Second-Best (SB), to quantify the welfare cost of the externalities identified in Section 3 and the scope for centralized coordination.

To measure welfare gaps, we define the consumption equivalent welfare of an allocation alloc $\in \{LF, SB, Nash\}$ relative to the First-Best as the scalar $\Omega^{\text{alloc}} \in (0,1]$ that solves $\mathcal{W}^{\text{alloc}} = \mathcal{W}^{FB} + \frac{2(1+\rho)}{\rho} \ln \Omega^{\text{alloc}}$, i.e.,

$$\Omega^{\text{alloc}} \equiv \exp\left(\frac{\rho}{2(1+\rho)}(\mathcal{W}^{\text{alloc}} - \mathcal{W}^{FB})\right),$$

so Ω^{alloc} is the fraction by which FB aggregate consumption must be permanently scaled down to deliver the same welfare as the chosen allocation, and $1 - \Omega^{\text{alloc}}$ is the welfare loss expressed as a share of consumption. By construction $\Omega^{FB} = 1$. Because $\mathcal{W}$ decomposes additively into a level component $\frac{2(1+\rho)}{\rho} u(1)$ and a growth component $\frac{2(1+\rho)}{\rho^2} \ln(1+g^*)$, the welfare gap between any two allocations can be traced to differences in initial consumption (level) and in the balanced growth rate (growth).

Figure 2 extends the comparison across the full range of digital adaptability $m \in (0,1)$ for both calibrations. Panels (a)–(b) plot Ω^{SB} and Ω^{LF} against m. In both calibrations, Ω^{LF} (purple dash-dotted) rises monotonically with m because when manufacturing is more digitally adaptable, the digital technology spillover translates more effectively into manufacturing productivity gains, narrowing the consumption-equivalent welfare loss $1 - \Omega^{LF}$. The SB welfare Ω^{SB} (dark orange solid) exhibits a hump-shaped pattern that reflects the tension inherent in the SB instrument. At low m, an increase in digital adaptability amplifies the returns to the digital subsidy, so the SB planner can more effectively internalize the cross-sector externality. Beyond the peak, however, further increases in m raise the opportunity cost of the manufacturing tax that finances the subsidy, causing the net welfare gain from coordination to shrink. Despite this hump shape, Ω^{SB} exceeds Ω^{LF} throughout, confirming that centralized coordination dominates laissez-faire for all m, consistent with the inefficiency of laissez-faire established in Proposition 2. The gap between Ω^{SB} and Ω^{LF} narrows as m increases, indicating that the welfare gains from coordination diminish as manufacturing becomes more digitally adaptable.

Panels (c)–(d) decompose SB welfare into level and growth effects and plot total welfare $\mathcal{W}^{SB}$. The growth component (teal dashed line, right axis) increases monotonically with m, consistent with the increasing growth premium established in Proposition 1, and shifts the welfare composition toward the growth channel. The level component (blue solid line, left axis) exhibits a U-shaped pattern. At low m, the SB-optimal subsidy reallocates substantial labor from manufacturing to the digital sector, depressing manufacturing output and reducing initial consumption. However, at high m, the resulting digital productivity gains feed back into manufacturing output through stronger spillovers, partially restoring initial consumption.[6] Total welfare $\mathcal{W}^{SB}$ (red solid line, right axis) traces the sum of the two components and inherits its shape from their interaction. The level component dominates at low m, while the growth component drives welfare at high m. The two calibrations exhibit similar patterns, confirming that the welfare costs of the two externalities and the gains from centralized coordination are robust across the two economies.

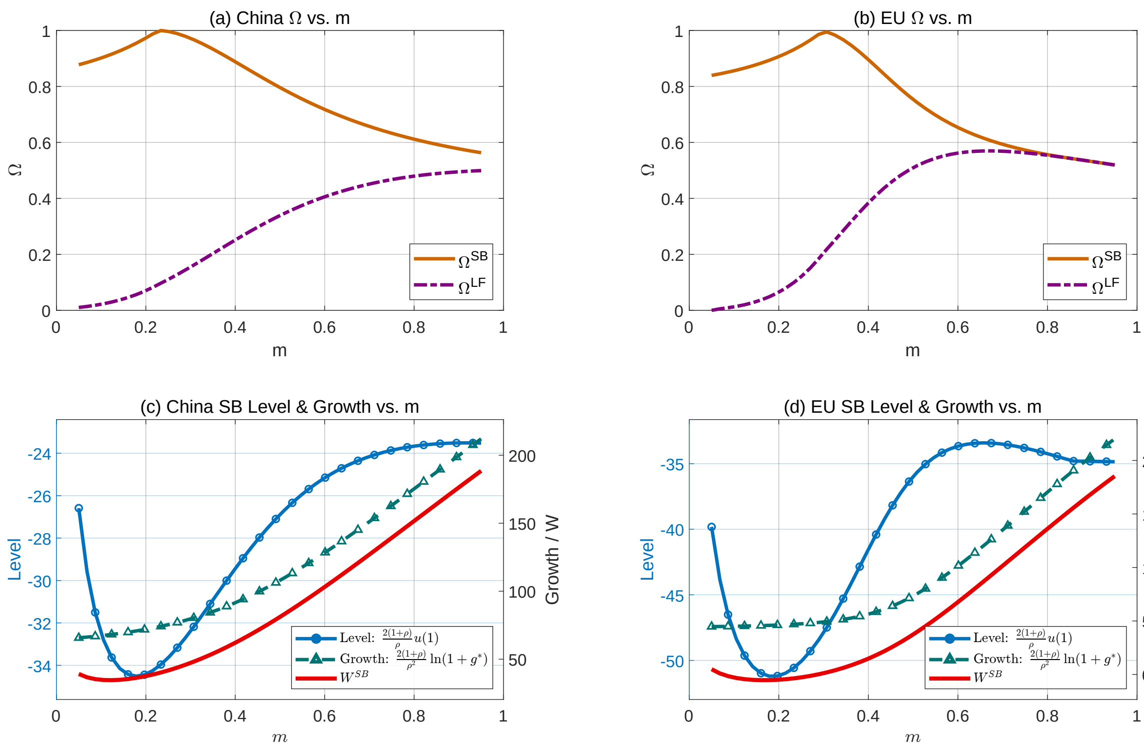

Figure 2: Welfare costs of laissez-faire and gains from coordination.

Notes: Left column: China calibration (city = region); right column: EU calibration (country = region). Panels (a)–(b): consumption equivalent welfare Ω^{SB} (dark orange solid) and Ω^{LF} (purple dash-dotted) against m. A higher Ω^{alloc} indicates a smaller welfare gap relative to the First-Best, with $1-\Omega^{\text{alloc}}$ measuring the permanent consumption loss. Panels (c)–(d): decomposition of SB welfare into the level component $\frac{2(1+\rho)}{\rho}u(t=1)$ (blue solid, left axis) and the growth component $\frac{2(1+\rho)}{\rho^2}\ln(1+g^*)$ (teal dashed, right axis), together with total welfare $\mathcal{W}^{SB} = \text{Level} + \text{Growth}$ (red solid, right axis), as functions of m.

[6]In the EU calibration, the level component declines further at high m rather than recovering. Two features of the EU parameterization contribute to this difference. First, the higher labor elasticity ($\eta = 0.67$ versus 0.50 for China) amplifies the effect of labor reallocation on initial-period utility, so the consumption loss from shifting labor into the digital sector is larger. Second, the higher iceberg trade cost ($\tau = 2.70$ versus 1.71) makes consumers more reliant on local manufacturing output, so the consumption loss from reducing local manufacturing labor is harder to offset through inter-regional trade.

5.2.2. Tournament Competition

The preceding analysis shows that the laissez-faire equilibrium entails substantial welfare losses in both calibrations, and that centralized coordination can significantly reduce these losses. Rather than relying on centralized coordination, this subsection asks whether decentralized tournament competition can deliver comparable efficiency gains. The analysis focuses on the China calibration, where local government officials face promotion tournaments that map directly to the Nash competition analyzed in Section 4.

Because the digital adaptability parameter m has no precise empirical counterpart, we set $m = 0.50$ as baseline. Figure 3 presents the results. Panels (a)–(c) fix $m = 0.50$ and vary tournament intensity λ, while Panel (d) extends the analysis over $m \in (0, 1)$. Panel (a) traces the Nash equilibrium subsidy ratio $s^{D,Nash}(\lambda)/s^{D,SB}$. The ratio rises monotonically in λ, crossing unity at $\lambda^{peak} = 0.44$. For $\lambda < \lambda^{peak}$, the economy is under-subsidized relative to the coordinated Second-Best optimum, and for $\lambda > \lambda^{peak}$, it is over-subsidized.

Panel (b) decomposes Nash welfare into its level and growth components and plots total welfare $\mathcal{W}^{Nash}$ as functions of λ. The level component (blue solid line, left axis) declines monotonically because stronger tournament competition raises subsidies and reallocates resources from manufacturing to the digital sector, depressing initial consumption. The growth component (teal dashed line, right axis) exhibits an inverted-U pattern. At low λ, moderate subsidies partially correct the under-provision of digital innovation and raise the balanced growth rate. Beyond a threshold, however, over-subsidization drains too many resources from the manufacturing sector, reducing data feedback and R&D activity and causing the growth component to decline. Total welfare $\mathcal{W}^{Nash}$ (red solid line, right axis) inherits an inverted-U shape from the interaction of these two components.

Panel (c) plots the consumption equivalent welfare $\Omega^{Nash}(\lambda)$ as λ increases from 0 to 1, with horizontal reference lines marking Ω^{SB} and Ω^{LF}. The welfare measure peaks at $\lambda^{peak} = 0.44$, where $\Omega^{Nash}(\lambda^{peak}) \approx \Omega^{SB} = 0.80$, implying that the welfare loss relative to the First-Best is equivalent to a permanent 20% consumption shortfall. For $\lambda < \lambda^{peak}$ ("good competition"), moderate competition partially corrects the bidirectional externalities and pushes welfare above the laissez-faire level. For $\lambda > \lambda^{peak}$ ("bad competition"), excessive competition drives subsidies beyond the coordinated optimum into over-subsidization, and welfare declines. At the peak, decentralized tournament competition achieves the coordinator's Second-Best welfare. These results are consistent with Proposition 3.

Panel (d) plots the optimal tournament intensity λ^{peak} against digital adaptability m. The optimal intensity is monotonically decreasing in m. When m is low, a large subsidy is needed to reach the Second-Best because the welfare gap between laissez-faire and the Second-Best is wide (as shown in Figure 2(a)), so strong tournament incentives are required.

As m rises, the digital spillover translates more effectively into manufacturing productivity, the laissez-faire allocation moves closer to the Second-Best, and a smaller subsidy suffices. Consequently, even mild tournament pressure can push the economy into over-subsidization. This monotonic relationship implies that as an economy's manufacturing sector becomes more digitally adaptable, the institutional design of intergovernmental competition must be recalibrated. Tournament incentives that were welfare-improving at an early stage of digital development become welfare-destroying at a later stage.

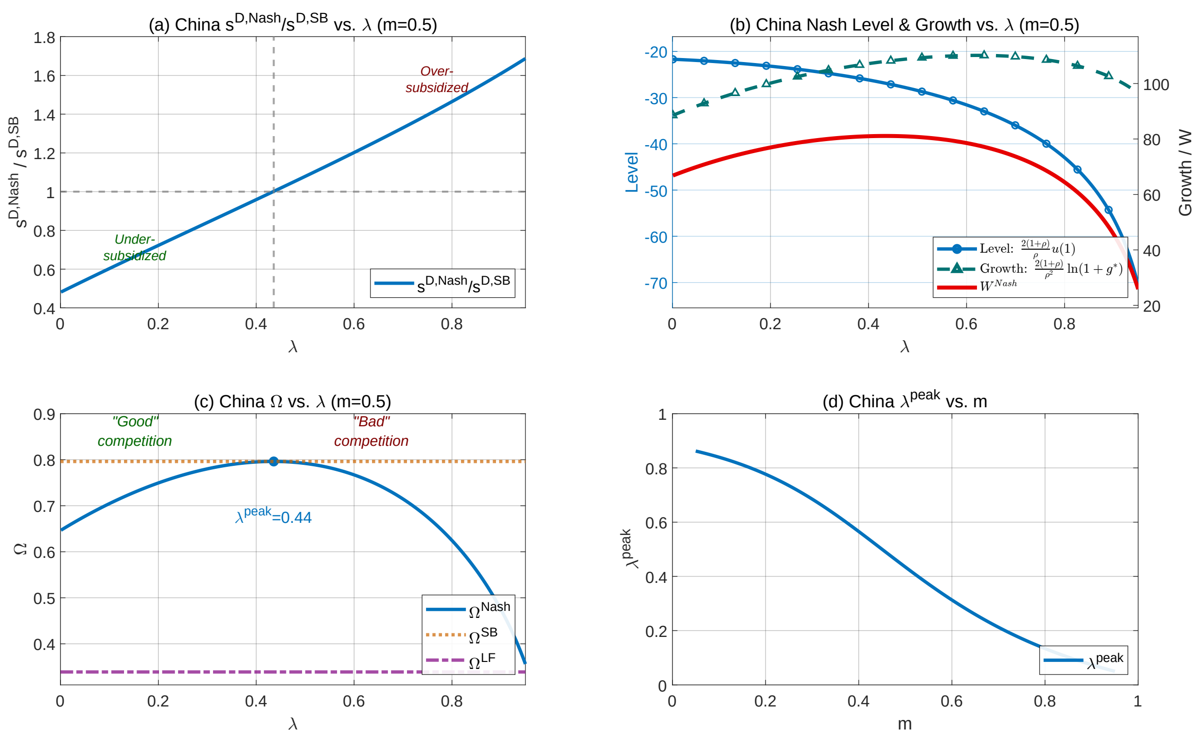

Figure 3: Tournament competition under the China calibration.

Notes: China calibration (city = region); Panels (a)–(c) evaluated at baseline $m = 0.50$. Panel (a): Nash equilibrium subsidy ratio $s^{D,Nash}/s^{D,SB}$ against λ; the horizontal dashed line marks Ratio $= 1$ and the vertical dashed line marks λ^{peak}. Panel (b): decomposition into the level component $\frac{2(1+\rho)}{\rho}u(t=1)$ (blue solid, left axis) and the growth component $\frac{2(1+\rho)}{\rho^2}\ln(1+g^*)$ (teal dashed, right axis), together with total welfare $\mathcal{W}^{Nash} =$ Level+Growth (red solid, right axis). Panel (c): consumption equivalent welfare $\Omega^{Nash}(\lambda)$ against $\lambda \in (0,1)$, with horizontal lines at Ω^{SB} and Ω^{LF} and the peak marked at λ^{peak}; $\lambda < \lambda^{peak}$ corresponds to "good competition" (under-subsidized) and $\lambda > \lambda^{peak}$ to "bad competition" (over-subsidized). Panel (d): optimal tournament intensity λ^{peak} against m.

5.3. Robustness

We assess robustness by perturbing each of twelve key parameters (ρ, σ, α, τ, ϕ, η, β, ξ, $\chi^{*,LF}$, $g^{*,LF}$, $\bar{H}$, and T) by $\pm 20\%$ from its baseline value.[7] For each perturbation, the scale

[7] The government tenure T is rounded to the nearest integer (minimum 1) after perturbation and applies only to the China calibration, which features the tournament mechanism. The EU calibration, which lacks

parameters (κ^D, κ^M) are jointly re-calibrated to match the laissez-faire targets ($g^{*,LF}$ and $\chi^{*,LF}$). To summarize the sensitivity of the main results to all perturbations simultaneously, each figure in Appendix E plots the baseline curve together with an envelope (shaded band) that covers the full range of perturbed outcomes at every grid point.

Two main results follow. First, the welfare ranking $\Omega^{SB} \geq \Omega^{LF}$ is preserved across all perturbations under both calibrations (Figures E.1–E.2), confirming that the gains from centralized coordination are robust. Second, the inverted-U shape of $\Omega^{Nash}(\lambda)$ (Figure E.3) and the monotonically declining pattern of $\lambda^{peak}(m)$ (Figure E.4) are both preserved across all perturbations under the China calibration, confirming that these results are robust. The optimal tournament intensity should be moderate and decline with digital adaptability.

6. Model Extensions

The baseline model adopts a representative manufacturing firm, Cobb-Douglas preferences, symmetric regions, and frictionless data flows. This section enriches the framework by relaxing these assumptions in turn, showing that the core mechanism extends naturally to richer environments.

6.1. Heterogeneous Manufacturing Firms

The baseline model treats all production lines within a representative firm as sharing a single digital adaptability m. This subsection replaces the representative firm with a unit continuum of manufacturing firms, each producing a differentiated variety. Firms are indexed by their difficulty of digital transformation $\tilde{m} \in [0, 1]$, drawn from a uniform distribution. A lower $\tilde{m}$ indicates stronger compatibility with frontier digital technology; firms with high $\tilde{m}$ remain weakly aligned with the frontier.

Each firm's baseline technology before R&D is the maximum of the digital spillover and its own lagged technology:

$$Z_i^{\tilde{m}-}(t) = \max\big\{(1-\tilde{m}) \cdot Z_i^D(t-1), Z_i^M(t-1)\big\}.$$

On the balanced growth path, the adoption threshold $\tilde{m}^*$ is determined by the indifference condition $(1-\tilde{m}^*)Z^D = Z^M$, yielding

$$\tilde{m}^* = 1 - \frac{1}{\chi^*} = \frac{\chi^* - 1}{\chi^*}.$$

a tournament, has 22 perturbations (11 parameters × 2 directions). For the EU calibration, the −20% perturbation of the technology ratio $\chi^{*,LF}$ would yield $1.20 \times 0.80 = 0.96 < 1$, violating the constraint $\chi^{*,LF} > 1$ (the digital sector's technological lead over manufacturing). This perturbation is therefore floored at $\chi^{*,LF} = 1.01$. All other parameter perturbations remain within their theoretical bounds.

Firms with $\tilde{m} \leq \tilde{m}^*$ adopt digital technology ($Z_i^{\tilde{m}-}(t) = (1-\tilde{m})Z_i^D(t-1)$), while those with $\tilde{m} > \tilde{m}^*$ retain traditional technology ($Z_i^{\tilde{m}-}(t) = Z_i^M(t-1)$). Crucially, $\tilde{m}^*$ is endogenous, and a higher technology ratio raises $\tilde{m}^*$, expanding the set of digital adopters.

Define the digital spillover share of aggregate baseline technology

$$\begin{aligned}\mathcal{M}(\chi^*) &\equiv \frac{\int_0^{\tilde{m}^*} Z_i^{\tilde{m}-}(t)d\tilde{m}}{\int_0^1 Z_i^{\tilde{m}-}(t)d\tilde{m}} = \frac{\int_0^{\tilde{m}^*}(1-\tilde{m})Z_i^D(t-1)d\tilde{m}}{\int_0^{\tilde{m}^*}(1-\tilde{m})Z_i^D(t-1)d\tilde{m} + \int_{\tilde{m}^*}^1 Z_i^M(t-1)d\tilde{m}} \\ &= \frac{\tilde{m}^* - (\tilde{m}^*)^2/2}{\tilde{m}^* - (\tilde{m}^*)^2/2 + (1-\tilde{m}^*)/\chi^*} = \frac{(\chi^*)^2 - 1}{(\chi^*)^2 + 1}.\end{aligned}$$

This ratio measures the fraction of aggregate baseline technology attributable to digital spillovers. In the baseline model, the corresponding share $m\chi^*/(m\chi^* + 1 - m)$ is governed by the exogenous parameter m, so the extent of digital penetration is fixed regardless of the economy's technological state. The heterogeneous-firm extension endogenizes this share through $\mathcal{M}(\chi^*)$, which is increasing in χ^*. As the digital sector's technological lead widens, the adoption threshold $\tilde{m}^*$ rises and the aggregate digital spillover share increases accordingly.

This extension enriches the microstructure of the baseline model and preserves, even strengthens, the main results. Specifically, because $\mathcal{M}(\chi^*)$ is increasing in χ^*, a rise in digital technology not only raises manufacturing productivity but also expands the set of digital adopters, generating additional data and further accelerating digital innovation. This endogenous amplification creates a stronger self-reinforcing loop than the baseline with fixed m. Moreover, the heterogeneous-firm model adds an extensive margin to the welfare analysis. A subsidy now affects not only the intensity of digital activity within each firm but also the share of firms that adopt digital technology. Moderate subsidies draw more firms into the digital adoption pool, while excessive subsidies induce adoption by firms with low adaptability, yielding diminishing returns. The inverted-U welfare pattern therefore becomes steeper, but its qualitative shape is unchanged.

6.2. CES Cross-Sector Preferences

The baseline model adopts a Cobb-Douglas period utility, $u_i(t) = \beta \ln C_i^D(t) + (1-\beta)\ln C_i^M(t)$, imposing a unit elasticity of substitution between digital services and manufactured goods. In the structural transformation literature, Ngai and Pissarides (2007) and Desmet and Rossi-Hansberg (2014) adopt CES preferences with an elasticity of substitution below unity (cross-sector complementarity), so that faster productivity growth in manufacturing shifts expenditure and labor toward the slower-growing service sector. Here, however, the digital sector exhibits faster productivity growth driven by data-augmented innovation, distinguishing it from the traditional low-growth service sector. This subsection replaces the

Cobb-Douglas aggregator with a CES form and examines how complementarity interacts with faster digital growth.

The CES period utility is rewritten as

$$u_i(t) = \frac{\varepsilon}{\varepsilon - 1} \ln \left[\beta^{1/\varepsilon} \big(C_i^D(t) \big)^{(\varepsilon-1)/\varepsilon} + (1-\beta)^{1/\varepsilon} \big(C_i^M(t) \big)^{(\varepsilon-1)/\varepsilon} \right],$$

where $\varepsilon \in (0,1)$ is the elasticity of substitution between digital services and manufactured goods. The digital expenditure share depends on relative prices:

$$\omega = \frac{\beta \big(P_i^D \big)^{1-\varepsilon}}{\beta \big(P_i^D \big)^{1-\varepsilon} + (1-\beta) \big(P_i^M \big)^{1-\varepsilon}}.$$

Because the two goods are complements, faster productivity growth in the frontier digital sector lowers its relative price and reduces the digital expenditure share ω.

This price dependence transmits to the labor market equilibrium. In the baseline, the digital labor share $\Theta_L^{LF} = \beta(1-\varphi)/(1-\varphi\beta)$ is independent of technology levels. Under CES, the analog expression replaces β with ω:

$$\Theta_L^{CES} = \frac{\omega(1-\varphi)}{1-\varphi\omega},$$

where ω depends on relative prices, which in turn depend on the technology ratio χ^*. Because $\omega < \beta$, the digital labor share Θ_L^{CES} falls below its Cobb-Douglas counterpart Θ_L^{LF}, and labor therefore reallocates toward manufacturing.

This extension preserves the main results while introducing a price-mediated labor reallocation channel absent from the baseline. The complementarity reallocates labor toward manufacturing, which lowers digital data generation but increases manufacturing data feedback. These two effects partially offset each other, so the net impact on the growth rate is quantitative rather than qualitative. Furthermore, CES preferences create an indirect dampening effect on digital subsidies. A digital subsidy attracts more labor and talent into the digital sector, accelerating innovation and lowering its relative price. Under complementarity, the falling relative price shifts expenditure and labor back toward manufacturing, partially counteracting the subsidy's direct effect. This dampening effect moderates both the benefits of well-calibrated subsidies and the costs of over-subsidization. The Cobb-Douglas specification is therefore adopted in the main analysis as the tractable special case that delivers closed-form solutions without loss of qualitative generality.

6.3. Asymmetric Regions

The baseline derives results under symmetric equilibrium, where labor and talent split equally across regions and both set identical subsidies. In practice, however, regions differ in their initial levels of digital and manufacturing technology. This subsection incorporates such asymmetry and shows that regional asymmetry generates persistent divergence and amplifies the welfare cost of decentralized competition.

Suppose region A starts with higher technology levels in both sectors, $Z_A^D > Z_B^D$ and $Z_A^M > Z_B^M$. The core mechanism operates through factor mobility. Labor and digital talent flow toward the higher-productivity region A, which offers higher wages and higher returns to digital talent. First, talent inflows directly accelerate region A's digital technology improvement. Second, labor inflows broaden production activity, generating richer data input and further raising the pace of innovation. Third, a higher digital technology level strengthens the cross-sector spillover, boosting manufacturing productivity and feeding back additional data from manufacturing consumption into the innovation process.

Region B experiences the reverse. Talent drain directly reduces its innovation rate. Less economic activity means less data generation and a smaller knowledge input into the innovation process. A lower digital technology level weakens the spillover to manufacturing, further depressing productivity. These opposing dynamics widen the productivity gap between the two regions, drawing even more factors from region B to region A. The inter-regional technology gap therefore widens monotonically over time.

Under decentralized Nash competition, the lagging region responds to its competitive disadvantage by offering larger digital subsidies ($s_B^D > s_A^D$). This "desperate subsidization" is socially inefficient and amplifies the welfare cost of fiscal competition. Region B's lower productivity means that the same subsidy rate attracts fewer factors than in region A, because the total compensation package, including wages and subsidy-augmented returns, remains lower. To compensate, region B must offer disproportionately higher subsidies. These require commensurately higher manufacturing taxes ($t_B > t_A$) through the government budget constraint. The higher taxes further depress manufacturing output and reduce the data generated from manufacturing activity, partially undoing the intended stimulus to the digital sector. The lagging region is thus trapped in a fiscal bind. It must subsidize to stay competitive, but the taxes needed to finance the subsidies erode the very manufacturing base that feeds digital innovation.

The welfare cost of decentralized competition is larger under asymmetry than under symmetry. First, the over-subsidization losses in both regions are amplified. The lagging region over-subsidizes more intensively. The advanced region also over-subsidizes, though less intensively, because it must match its rival's factor-attraction efforts. Second, asymmetry introduces an additional factor misallocation loss that is zero under symmetry. Uncoordinated subsidies distort the inter-regional distribution of labor and talent away from the social optimum, creating a deadweight loss from the inefficient spatial allocation of factors. The misallocation loss is convex in the degree of asymmetry, and larger regional gaps translate into greater gains from coordination.

6.4. Data Markets

The baseline model treats data as a costless byproduct of economic activity. Consumption and R&D automatically generate data that flows into the digital sector without friction. In practice, however, data trading involves costs that depend on ownership structure (Jones and Tonetti, 2020; Acemoglu et al., 2022; Acquisti et al., 2016). Following Jones and Tonetti (2020), let $\tilde{x}^D \in [0,1]$, $\tilde{x}^M \in [0,1]$, and $\tilde{x}^R \in [0,1]$ denote the fractions of digital consumption data, manufacturing consumption data, and R&D data that are actually used in the digital sector. The data generation equation becomes

$$\tilde{\mathcal{D}}_i(t-1) = \tilde{x}^D \cdot c_{ii}^D(t-1) + \frac{m\chi_i(t-1)}{m\chi_i(t-1) + (1-m)} \cdot \left[\tilde{x}^M \cdot c_{ii}^M(t-1) + \tilde{x}^R \cdot \psi_i^M(t-1)\right],$$

which nests the baseline when $\tilde{x}^D = \tilde{x}^M = \tilde{x}^R = 1$. Two cases of data ownership illustrate how different cost structures reduce these fractions and lower the equilibrium growth rate.

Case 1. Firm ownership. Each sector's firms own the data generated within that sector. Digital firms fully own digital consumption data, so $\tilde{x}^D = 1$. To obtain manufacturing data, digital firms must purchase them from manufacturing firms. Selling consumption data exposes the seller to creative destruction, because competitors can use purchasing patterns to develop substitute products. This cost $\zeta^M(\tilde{x}^M)$ is increasing and convex with $\zeta^M(0) = 0$. Selling R&D data poses a more direct threat, at cost $\zeta^R(\tilde{x}^R)$, also increasing and convex with $\zeta^R(0) = 0$. Manufacturing firms equate marginal revenue with marginal creative-destruction cost, yielding $\tilde{x}^{M*} < 1$ and $\tilde{x}^{R*} < 1$.

Case 2. Consumer and producer ownership. Suppose instead that consumers own all consumption data from both sectors, while manufacturing firms retain ownership of R&D data. Consumers who sell their data suffer a privacy cost denoted by $\gamma^D(\tilde{x}^D)$ for digital consumption data and $\gamma^M(\tilde{x}^M)$ for manufacturing consumption data. Both cost functions are increasing and convex with $\gamma^D(0) = \gamma^M(0) = 0$. The period utility function becomes

$$\tilde{u}_i(t) = \beta \ln C_i^D(t) + (1-\beta) \ln C_i^M(t) - \gamma^D(\tilde{x}^D) - \gamma^M(\tilde{x}^M).$$

Consumers choose $\tilde{x}^{D*}$ and $\tilde{x}^{M*}$ by equating marginal revenue with marginal privacy cost, yielding $\tilde{x}^{D*} < 1$ and $\tilde{x}^{M*} < 1$. Manufacturing firms face the same creative-destruction cost

$\zeta^R(\tilde{x}^R)$ as in Case 1, so $\tilde{x}^{R*} < 1$.

Under both ownership structures, the data trading fractions fall below unity. The effective data intensity satisfies $\tilde{\Phi}^* < \Phi^*$, and the balanced growth rate becomes

$$\tilde{g}^* = \kappa^D \cdot (\tilde{\Phi}^*)^{1-\xi} \cdot \left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)} \cdot \frac{\bar{H}}{2} - 1 < g^*.$$

This extension partially internalizes the data feedback externality, because data now carries a market price that reflects part of its value to digital innovation. However, private data ownership also leads to under-sharing relative to the social optimum (Jones and Tonetti, 2020). Privacy and creative-destruction costs further reduce data usage and weaken the data feedback channel. The digital technology spillover externality remains unaffected, since it operates through the nonrival diffusion of frontier technology. The balanced growth rate is therefore lower and the potential gains from coordinated intervention are greater. Beyond digital subsidies, policies that reduce data trading costs, such as strengthening data property rights, standardizing data formats, and designing privacy-preserving trading mechanisms, can improve welfare by narrowing the gap between available and utilized data.

7. Conclusion

This paper examines whether decentralized fiscal competition can correct the bidirectional externalities between digital services and manufacturing that private markets fail to internalize. We develop a dynamic two-region spatial general equilibrium model in which frontier digital technology spills over to manufacturing, while manufacturing activity generates data that feeds back into digital innovation. The resulting growth loop strengthens with the manufacturing sector's digital adaptability, yet laissez-faire under-allocates resources to the digital services sector and is Pareto-inefficient. A centralized coordinator using digital output subsidies financed by manufacturing taxes delivers efficiency gains over laissez-faire, establishing a Second-Best benchmark. Tournament competition among local governments produces a non-monotonic welfare response to tournament intensity. Moderate competition partially corrects the externalities and raises allocative efficiency, while excessive competition leads to over-subsidization. At the welfare peak, the decentralized outcome coincides with the centralized coordinator's optimum. Numerical calibrations to the Chinese and EU economies confirm these results and show that the optimal tournament intensity decreases as manufacturing becomes more digitally adaptable.

These findings carry implications for the design of intergovernmental fiscal relations in the digital economy. First, bidirectional externalities between the digital and manufacturing sectors imply that laissez-faire cannot deliver an efficient allocation of resources. Digital subsidies are justified as a corrective response to well-defined market failures. This provides a welfare-theoretic rationale for initiatives such as China's "Digital China" strategy, the EU's Digital Decade program, and the US CHIPS and Science Act. Second, the non-monotonic welfare response to tournament intensity implies that intergovernmental competition should balance externality correction against over-subsidization. Policymakers should preserve moderate competitive incentives while establishing safeguards against excessive fiscal rivalry. As manufacturing becomes more digitally adaptable, the scope for welfare-improving competition narrows, and tournament incentives should be recalibrated downward accordingly. Third, the asymmetric-region extension shows that uncoordinated competition between technologically unequal regions may generate persistent divergence. Early coordinated intervention can mitigate this divergence and improve aggregate welfare. More broadly, intergovernmental fiscal institutions should harness the corrective potential of decentralized competition while preventing competition from overshooting into welfare-reducing territory.

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Appendix A Descriptive Statistics

Table A.1 presents the summary statistics of all city- and firm-level variables used in the analysis. The distributional statistics suggest considerable heterogeneity in both city and firm digitalization across the sample.

Panel A reports city-level variables. The average value of local digitalization measured by $Digit\_local1$ is 2.29, with a standard deviation of 4.39, indicating substantial variation across cities. The corresponding measure for other cities, $Digit\_other1$, has a mean of 2.29 and a smaller standard deviation of 2.37. When considering only invention patents, the mean values are 1.20 for $Digit\_local2$ and $Digit\_other2$, suggesting similar average levels of technological innovation locally and in other cities. Their standard deviations are 2.68 and 1.31, respectively.

Panel B reports firm-level variables. The mean of $Digit\_firm1$ is 4.60 and that of $Digit\_firm2$ is 2.99. Their standard deviations are 17.30 and 11.78, respectively. At both the city and firm levels, variables measured under the broader definition, which includes invention patents and utility model patents, exhibit greater variation than those measured under the narrower definition, which includes invention patents only.

Table A.1: Descriptive Statistics

Variable	N	Mean	S.D.	Min	p25	p50	p75	Max
Panel A: City-level variables								
$Digit\_local1$	2,511	2.29	4.39	0.02	0.27	0.73	2.15	29.76
$Digit\_other1$	2,511	2.29	2.37	0.02	0.74	1.48	2.77	13.67
$Digit\_local2$	2,511	1.20	2.68	0.01	0.08	0.26	0.92	18.34
$Digit\_other2$	2,511	1.20	1.31	0.02	0.34	0.69	1.47	6.86
Economic development	2,511	10.34	0.56	9.06	9.94	10.30	10.70	11.69
Population density	2,511	5.72	0.91	3.15	5.14	5.81	6.40	7.88
Industrial structure	2,511	47.02	10.37	19.25	40.99	47.53	53.53	72.83
Panel B: Firm-level variables								
$Digit\_firm1$	17,436	4.60	17.30	0.00	0.00	0.00	0.00	136.29
$Digit\_firm2$	17,436	2.99	11.78	0.00	0.00	0.00	0.00	93.20
Size	17,436	22.01	1.18	19.08	21.15	21.88	22.71	25.95
Lev	17,436	0.41	0.20	0.05	0.25	0.40	0.56	0.88
Roa	17,436	0.04	0.06	−0.26	0.01	0.04	0.07	0.20
Tang	17,436	0.23	0.16	0.00	0.11	0.20	0.32	0.70
Cash	17,436	0.05	0.07	−0.16	0.01	0.05	0.09	0.24
Age	17,436	2.83	0.34	1.79	2.64	2.89	3.04	3.47
Top1	17,436	34.09	14.42	8.80	22.90	32.22	43.40	74.18

Notes: Panel A reports city-level variables for 280 cities over 2011–2019. Panel B reports firm-level variables for 2,516 A-share listed firms. Variables with suffix 1 include invention patents and utility model patents, while those with suffix 2 include invention patents only. All variable definitions follow Section 2.2.

Appendix B Laissez-Faire Equilibrium Derivations and Proof of Proposition 1

This appendix provides complete laissez-faire equilibrium derivations and proves Proposition 1.

B.1 Consumer Problem

Expenditure shares. The representative consumer in region i has period utility $u_i(t) = \beta \ln C_i^D(t) + (1-\beta)\ln C_i^M(t)$ and maximizes $\mathcal{V}_i = \sum_{t=1}^{\infty}(1+\rho)^{-(t-1)}u_i(t)$. The Cobb-Douglas structure yields constant expenditure shares $P_i^D(t)C_i^D(t) = \beta E_i(t)$ and $P_i^M(t)C_i^M(t) = (1-\beta)E_i(t)$, hence:

$$\frac{P_i^D(t)C_i^D(t)}{P_i^M(t)C_i^M(t)} = \frac{\beta}{1-\beta}. \tag{B1}$$

Bilateral demands and price indices. Standard CES expenditure minimization over the CES aggregator yields the bilateral demands and price indices.

Digital services. The bilateral demands are:

$$c_{ii}^D(t) = \alpha\left(\frac{p_i^D(t)}{P_i^D(t)}\right)^{-\sigma} C_i^D(t),$$

$$c_{ji}^D(t) = (1-\alpha)\left(\frac{p_j^D(t)}{P_i^D(t)}\right)^{-\sigma} C_i^D(t),$$

where the digital price index is:

$$P_i^D(t) = \left[\alpha(p_i^D(t))^{1-\sigma} + (1-\alpha)(p_j^D(t))^{1-\sigma}\right]^{1/(1-\sigma)}.$$

Manufacturing. With consumer-facing prices $\tilde{p}_{ii}^M(t) = p_i^M(t)$ and $\tilde{p}_{ji}^M(t) = \tau p_j^M(t)$, the analogous CES expenditure minimization yields the bilateral demands:

$$c_{ii}^M(t) = \alpha\left(\frac{p_i^M(t)}{P_i^M(t)}\right)^{-\sigma} C_i^M(t),$$

$$c_{ji}^M(t) = (1-\alpha)\left(\frac{\tau p_j^M(t)}{P_i^M(t)}\right)^{-\sigma} C_i^M(t),$$

where the manufacturing price index is:

$$P_i^M(t) = \left[\alpha(p_i^M(t))^{1-\sigma} + (1-\alpha)(\tau p_j^M(t))^{1-\sigma}\right]^{1/(1-\sigma)}.$$

B.2 Digital Services Sector

Digital firm problem. The digital firm has production function $Y_i^D(t) = Z_i^D(t)(L_i^D(t))^\eta$ and marginal cost:

$$MC_i^D(t) = \frac{w_i^L(t)}{\eta Z_i^D(t)(L_i^D(t))^{\eta-1}}.$$

The optimal pricing condition $p_i^D(t) = \frac{\sigma}{\sigma-1} \cdot MC_i^D(t)$ yields the labor first-order condition:

$$w_i^L(t)L_i^D(t) = \frac{\eta(\sigma-1)}{\sigma} p_i^D(t)Y_i^D(t).$$

Digital talent earns the residual:

$$w_i^H(t)H_i^D(t) = p_i^D(t)Y_i^D(t) - w_i^L(t)L_i^D(t) = \left[1 - \frac{\eta(\sigma-1)}{\sigma}\right] p_i^D(t)Y_i^D(t).$$

B.3 Manufacturing Sector

Manufacturing firm problem. The manufacturing firm has production function $Y_i^M(t) = Z_i^M(t)(L_i^M(t))^\eta$ and incurs R&D cost $p_i^M(t)\psi_i^M(t)$. Marginal cost is:

$$MC_i^M(t) = \frac{w_i^L(t)}{\eta Z_i^M(t)(L_i^M(t))^{\eta-1}}.$$

The optimal price $p_i^M(t) = \frac{\sigma}{\sigma-1} \cdot MC_i^M(t)$ yields the labor first-order condition:

$$w_i^L(t)L_i^M(t) = \frac{\eta(\sigma-1)}{\sigma} p_i^M(t)Y_i^M(t).$$

The R&D first-order condition becomes:

$$p_i^M(t)\psi_i^M(t) = \frac{\phi(\sigma-1)}{\sigma} p_i^M(t)Y_i^M(t) \equiv \varphi p_i^M(t)Y_i^M(t), \tag{B2}$$

where $\varphi \equiv \phi(\sigma-1)/\sigma$ denotes the manufacturing R&D expenditure share.

B.4 Sectoral Labor Allocation

Inter-regional allocation. Both types of labor are perfectly mobile across the two regions, so wages equalize in equilibrium: $w_A^L = w_B^L \equiv w^L$ and $w_A^H = w_B^H \equiv w^H$.

Regular labor. The digital firm's FOC in region i gives $w^L = \eta(\sigma-1)/\sigma \cdot p_i^D(t)Z_i^D(t)(L_i^D(t))^{\eta-1}$, and similarly the manufacturing FOC gives $w^L = \eta(\sigma-1)/\sigma \cdot p_i^M(t)Z_i^M(t)(L_i^M(t))^{\eta-1}$. In symmetric equilibrium, technology levels and prices are identical across regions ($Z_A^D = Z_B^D$, $Z_A^M = Z_B^M$, $p_A^D = p_B^D$, $p_A^M = p_B^M$), so the wage-equalization conditions require

$$L_A^D = L_B^D, \qquad L_A^M = L_B^M.$$

Combined with the aggregate labor market clearing $\sum_i (L_i^D + L_i^M) = \bar{L}$, each region employs exactly half the total regular labor:

$$L_A^D + L_A^M = L_B^D + L_B^M = \frac{\bar{L}}{2}.$$

Digital talent. Talent H_i^D earns the residual profit in the digital sector: $w^H H_i^D = [1 - \eta(\sigma - 1)/\sigma]p_i^D(t)Y_i^D(t)$. Under symmetry ($p_A^D Y_A^D = p_B^D Y_B^D$), equalization of w^H requires

$$H_A^D = H_B^D = \frac{\bar{H}}{2}.$$

Thus, perfect factor mobility and symmetric fundamentals ensure that each region receives $\bar{L}/2$ regular labor and $\bar{H}/2$ digital talent, and the remaining degree of freedom is how each region's regular labor is allocated across sectors.

Intra-regional labor allocation. From the digital labor FOC and manufacturing labor FOC above:

$$\frac{w^L(t)L_i^D(t)}{w^L(t)L_i^M(t)} = \frac{\eta(\sigma-1)/\sigma p_i^D(t)Y_i^D(t)}{\eta(\sigma-1)/\sigma p_i^M(t)Y_i^M(t)} = \frac{p_i^D(t)Y_i^D(t)}{p_i^M(t)Y_i^M(t)}.$$

The Cobb-Douglas expenditure share Eq. (B1) gives:

$$P_i^D(t)C_i^D(t) = \beta E_i(t), \qquad P_i^M(t)C_i^M(t) = (1-\beta)E_i(t).$$

In symmetric equilibrium ($p_i^D(t) = p_j^D(t)$, $c_{ij}^D(t) = c_{ji}^D(t)$), digital firm revenue equals local digital expenditure:

$$p_i^D(t)Y_i^D(t) = p_i^D(t)\big(c_{ii}^D(t) + c_{ij}^D(t)\big) = p_i^D(t)c_{ii}^D(t) + p_j^D(t)c_{ji}^D(t) = P_i^D(t)C_i^D(t) = \beta E_i(t).$$

Since $P_i^D(t) = p_i^D(t)$ in symmetric equilibrium,

$$C_i^D(t) = Y_i^D(t). \tag{B3}$$

For manufacturing, in symmetric equilibrium ($p_i^M(t) = p_j^M(t)$, $c_{ij}^M(t) = c_{ji}^M(t)$), the firm's export revenue equals consumers' import expenditure, so:

$$p_i^M(t)\underbrace{\big(Y_i^M(t) - \psi_i^M(t)\big)}_{=c_{ii}^M(t)+\tau c_{ij}^M(t)} = p_i^M(t)c_{ii}^M(t) + \tau p_j^M(t)c_{ji}^M(t) = P_i^M(t)C_i^M(t) = (1-\beta)E_i(t).$$

Using $\psi_i^M(t) = \varphi Y_i^M(t)$ from Eq. (B2), net output is $(1-\varphi)Y_i^M(t)$, so total manufacturing revenue is:

$$p_i^M(t)Y_i^M(t) = \frac{(1-\beta)E_i(t)}{1-\varphi}.$$

The trade balance also gives $P_i^M(t)C_i^M(t) = p_i^M(t)(1-\varphi)Y_i^M(t)$. In symmetric equilibrium, $P_i^M(t) = p_i^M(t)[\alpha + (1-\alpha)\tau^{1-\sigma}]^{1/(1-\sigma)}$, so

$$C_i^M(t) = \big[\alpha + (1-\alpha)\tau^{1-\sigma}\big]^{-1/(1-\sigma)}(1-\varphi)Y_i^M(t). \tag{B4}$$

Combining the two sectors:

$$\frac{p_i^D(t)Y_i^D(t)}{p_i^M(t)Y_i^M(t)} = \frac{\beta(1-\varphi)}{1-\beta}.$$

Substituting into the labor ratio above:

$$\frac{L_i^D(t)}{L_i^M(t)} = \frac{\beta(1-\varphi)}{1-\beta}.$$

Define the digital labor share $\Theta_L \equiv L_i^D(t)/\big(L_i^D(t)+L_i^M(t)\big)$. From the labor ratio above:

$$\Theta_L = \frac{\beta(1-\varphi)}{1-\varphi\beta}. \tag{B5}$$

B.5 Data Feedback

Data generation. The data input is $\mathcal{D}_i(t) = c_{ii}^D(t) + \frac{m\chi_i(t)}{m\chi_i(t)+(1-m)}[c_{ii}^M(t)+\psi_i^M(t)]$, where the manufacturing data weight equals the digital technology share in manufacturing's composite technology base Eq. (3). For digital services, from Eq. (B3) and the home consumption share $c_{ii}^D(t)/C_i^D(t) = \alpha$,

$$c_{ii}^D(t) = \alpha Y_i^D(t).$$

For manufacturing, from Eq. (B4) and the CES home expenditure share $\alpha/[\alpha+(1-\alpha)\tau^{1-\sigma}]$, home consumption is:

$$c_{ii}^M(t) = \frac{\alpha}{\alpha+(1-\alpha)\tau^{1-\sigma}}(1-\varphi)Y_i^M(t).$$

The home R&D investment, which also generates data, is $\psi_i^M(t) = \varphi Y_i^M(t)$ (R&D uses domestic manufacturing goods). Adding the two data sources:

$$c_{ii}^M(t)+\psi_i^M(t) = \frac{\alpha(1-\varphi)}{\alpha+(1-\alpha)\tau^{1-\sigma}}Y_i^M(t)+\varphi Y_i^M(t) = \left[\frac{\alpha(1-\varphi)}{\alpha+(1-\alpha)\tau^{1-\sigma}}+\varphi\right]Y_i^M(t).$$

Substituting into $\mathcal{D}_i(t) = c_{ii}^D(t) + \frac{m\chi_i(t)}{m\chi_i(t)+(1-m)}[c_{ii}^M(t)+\psi_i^M(t)]$:

$$\mathcal{D}_i(t) = \alpha Y_i^D(t) + \frac{m\chi_i(t)}{m\chi_i(t)+(1-m)} \cdot \left[\frac{\alpha(1-\varphi)}{\alpha+(1-\alpha)\tau^{1-\sigma}}+\varphi\right]Y_i^M(t).$$

Effective data intensity. On the BGP, $\chi_i(t) = \chi^*$ is constant. From the production functions, the output ratio satisfies $Y_i^M/Y_i^D = (1/\chi^*)\big[(1-\Theta_L)/\Theta_L\big]^\eta$. Substituting into the data expression above to eliminate Y_i^M, and noting that the data weight $\frac{m\chi^*}{m\chi^*+(1-m)}$ multiplied by $1/\chi^*$ simplifies to $\frac{m}{m\chi^*+(1-m)}$, the data input can be written as $\mathcal{D}_i(t) = \Phi^*(\bar{L}/2)^\eta Z_i^D(t)$, where the effective data intensity is

$$\Phi^* = \alpha\Theta_L^\eta + \frac{m}{m\chi^*+(1-m)}\left[\frac{\alpha(1-\varphi)}{\alpha+(1-\alpha)\tau^{1-\sigma}}+\varphi\right](1-\Theta_L)^\eta, \tag{B6}$$

with Θ_L given by Eq. (B5) and $\varphi \equiv \phi(\sigma-1)/\sigma$ by Eq. (B2) under laissez-faire.

B.6 Balanced Growth Path

On the balanced growth path (BGP), both sectors grow at the common rate g^* and labor allocations are constant: $L_i^D = \Theta_L \bar{L}/2$, $L_i^M = (1-\Theta_L)\bar{L}/2$, $H_i^D = \bar{H}/2$.

Equal sector growth rates. From $Y_i^D(t) = Z_i^D(t)(L_i^D)^\eta$ and $Y_i^M(t) = Z_i^M(t)(L_i^M)^\eta$ with constant labor, both sectors' output grows at the same rate as their respective technology levels. Since χ^* is constant on the BGP, both Z_i^D and Z_i^M grow at rate g^*, and hence $g_{Y^D} = g_{Y^M} = g^*$.

Balanced growth rate g^*. The digital technology evolution Eq. (1) is $Z_i^D(t) = \kappa^D(Z_i^D(t-1))^\xi \mathcal{D}_i(t-1)^{1-\xi} H_i^D(t)$. On the BGP, dividing both sides by $Z_i^D(t-1)$ and substituting $\mathcal{D}_i(t-1) = \Phi^*(\bar{L}/2)^\eta Z_i^D(t-1)$:

$$
\begin{aligned}
1 + g^* &= \kappa^D \big(Z_i^D(t-1)\big)^{\xi-1} \big[\Phi^*(\bar{L}/2)^\eta Z_i^D(t-1)\big]^{1-\xi} \frac{\bar{H}}{2} \\
&= \kappa^D \cdot (\Phi^*)^{1-\xi} \cdot \left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)} \cdot \frac{\bar{H}}{2}.
\end{aligned}
$$

Hence:

$$
g^* = \kappa^D \cdot (\Phi^*)^{1-\xi} \cdot \left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)} \cdot \frac{\bar{H}}{2} - 1, \tag{B7}
$$

where Φ^* is given by Eq. (B6).

B.7 Technology Ratio

Derivation of χ^*. The R&D FOC gives $\psi_i^M(t) = \varphi \cdot Y_i^M(t) = \varphi \cdot Z_i^M(t)[(1-\Theta_L)\bar{L}/2]^\eta$. Substituting into the manufacturing technology equation $Z_i^M(t) = \kappa^M(\psi_i^M(t))^\phi (Z_i^{M-}(t))^{1-\phi}$ with baseline technology Eq. (2) $Z_i^{M-}(t) = mZ_i^D(t-1) + (1-m)Z_i^M(t-1)$, and dividing by $(Z_i^M(t))^\phi$:

$$
\big(Z_i^M(t)\big)^{1-\phi} = \kappa^M \varphi^\phi \left(\frac{(1-\Theta_L)\bar{L}}{2}\right)^{\phi\eta} \cdot \big[mZ_i^D(t-1) + (1-m)Z_i^M(t-1)\big]^{1-\phi}.
$$

On the BGP, χ^* is constant, so the bracketed term equals $[m\chi^* + (1-m)]Z_i^M(t-1)$. Using $Z_i^M(t) = (1+g^*)Z_i^M(t-1)$, the Z_i^M terms cancel:

$$
(1+g^*)^{1-\phi} = \kappa^M \varphi^\phi \left(\frac{(1-\Theta_L)\bar{L}}{2}\right)^{\phi\eta} \cdot \big[m\chi^* + (1-m)\big]^{1-\phi}.
$$

Substituting $1 + g^*$ from Eq. (B7) and solving for χ^*:

$$
\chi^* = \frac{1}{m} \left\{ \frac{\kappa^D(\Phi^*)^{1-\xi} \left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)} \frac{\bar{H}}{2}}{(\kappa^M)^{1/(1-\phi)} \varphi^{\phi/(1-\phi)} \left(\frac{(1-\Theta_L)\bar{L}}{2}\right)^{\phi\eta/(1-\phi)}} - (1-m) \right\}.
$$

Since Φ^* depends on χ^* through the output ratio (see Eq. (B6)), this equation is a single equation in χ^* that determines the equilibrium.

Existence, uniqueness, and convergence of (g^*, χ^*). The BGP requires both sectors to grow at the common rate g^*, i.e., $g^D(\chi^*) = g^M(\chi^*)$, where

$$1 + g^D(\chi) \equiv \kappa^D \big[\Phi(\chi)\big]^{1-\xi} \left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)} \frac{\bar{H}}{2},$$

$$1 + g^M(\chi) \equiv (\kappa^M)^{\frac{1}{1-\phi}} \varphi^{\frac{\phi}{1-\phi}} \left(\frac{(1-\Theta_L)\bar{L}}{2}\right)^{\frac{\phi\eta}{1-\phi}} \big[m\chi + (1-m)\big].$$

This is equivalent to the χ^* equation derived above, with $\Phi(\chi)$ given by Eq. (B6). Since the denominator $m\chi + (1-m)$ in $\Phi(\chi)$ is increasing, Φ and hence g^D are strictly decreasing in χ. The manufacturing growth rate g^M is linear and strictly increasing in χ.

Existence and uniqueness. As $\chi \to +\infty$: $\Phi(\chi) \to \alpha\Theta_L^\eta$, so g^D converges to a finite lower bound, while $g^M \to +\infty$. At $\chi = 0$: both growth rates are finite, with $g^D(0) > g^M(0)$ under the maintained parameterization. Since $g^D - g^M$ is continuous, strictly decreasing, positive at 0, and diverges to $-\infty$, it has exactly one zero, yielding the unique $\chi^{**} > 0$ with $g^{**} = g^D(\chi^{**}) = g^M(\chi^{**}) > 0$ and $\Phi^{**} = \Phi(\chi^{**}) > \alpha\Theta_L^\eta$.

Convergence. Off the BGP, labor allocations remain pinned by static equilibrium conditions (they depend only on parameters), so the sectoral growth rates are functions of $\chi_i(t-1)$ alone. The technology ratio therefore evolves as

$$\chi_i(t) = \frac{1 + g^D\big(\chi_i(t-1)\big)}{1 + g^M\big(\chi_i(t-1)\big)} \cdot \chi_i(t-1) \equiv G\big(\chi_i(t-1)\big).$$

For $\chi < \chi^{**}$: $g^D > g^M$, so $G(\chi) > \chi$; for $\chi > \chi^{**}$: $g^D < g^M$, so $G(\chi) < \chi$. At χ^{**}, differentiation gives

$$G'(\chi^{**}) = 1 - \underbrace{\frac{m\chi^{**}}{m\chi^{**} + (1-m)}}_{\in (0,1)} \left[1 + (1-\xi)\left(1 - \frac{\alpha\Theta_L^\eta}{\Phi^{**}}\right)\right].$$

Since $\xi \in (0,1)$ and $\Phi^{**} > \alpha\Theta_L^\eta$, the bracketed expression lies in $(1, 2-\xi)$, which yields $G'(\chi^{**}) \in (\xi - 1, 1) \subset (-1, 1)$. Hence the fixed point is locally stable, and combined with the directional adjustment property and the uniqueness of χ^{**}, the BGP is globally asymptotically stable: $\chi_i(t) \to \chi^{**}$ and $g_i(t) \to g^{**}$ from any initial condition. □

B.8 Welfare

On the BGP, both $C_i^D(t)$ and $C_i^M(t)$ grow at rate g^*. The representative consumer's lifetime utility is $\mathcal{V}_i = \sum_{t=1}^{\infty}(1+\rho)^{-(t-1)} u_i(t)$, where $u_i(t) = \beta \ln C_i^D(t) + (1-\beta)\ln C_i^M(t)$. Since $u_i(t) =$

$u_i(t=1) + (t-1)\ln(1+g^*)$:

$$\mathcal{V}_i^{BGP} = \sum_{t=1}^{\infty} \frac{u_i(t=1) + (t-1)\ln(1+g^*)}{(1+\rho)^{t-1}} = \frac{1+\rho}{\rho} u_i(t=1) + \ln(1+g^*) \sum_{t=1}^{\infty} \frac{t-1}{(1+\rho)^{t-1}}$$
$$= \frac{1+\rho}{\rho} u_i(t=1) + \frac{1+\rho}{\rho^2} \ln(1+g^*),$$

where we use $\sum_{t=1}^{\infty}(1+\rho)^{-(t-1)} = (1+\rho)/\rho$ and $\sum_{t=1}^{\infty}(t-1)(1+\rho)^{-(t-1)} = (1+\rho)/\rho^2$.

Initial-period utility. From Eq. (B3), $C_i^D(t=1) = Y_i^D(t=1) = Z_i^D(t=1)(\Theta_L \bar{L}/2)^\eta$. From Eq. (B4), $C_i^M(t=1) = [\alpha + (1-\alpha)\tau^{1-\sigma}]^{-1/(1-\sigma)}(1-\varphi)Z_i^M(t=1)[(1-\Theta_L)\bar{L}/2]^\eta$. Using $Z_i^D(t=1) = \chi^* Z_i^M(t=1)$:

$$u_i(t=1) = \ln\left[Z_i^M(t=1)(\chi^*)^\beta \Theta_L^{\eta\beta}(1-\Theta_L)^{\eta(1-\beta)}(\bar{L}/2)^\eta \left[\alpha + (1-\alpha)\tau^{1-\sigma}\right]^{-\frac{1-\beta}{1-\sigma}}(1-\varphi)^{1-\beta}\right].$$

Aggregate welfare. In symmetric equilibrium, aggregate lifetime welfare is:

$$\mathcal{W} = \sum_{i \in \{A,B\}} \mathcal{V}_i^{BGP} = \frac{2(1+\rho)}{\rho} u(t=1) + \frac{2(1+\rho)}{\rho^2} \ln(1+g^*).$$

The growth effect is amplified by an additional factor of $1/\rho$ relative to the level effect, reflecting the compounding of growth gains over the infinite horizon.

B.9 Proof of Proposition 1

From Eq. (B2) and Eq. (B5), the laissez-faire allocations $\varphi^{LF} = \phi(\sigma-1)/\sigma$ and $\Theta_L^{LF} = \beta(1-\varphi^{LF})/(1-\varphi^{LF}\beta)$ are independent of m. By Eq. (B7),

$$g^{*,LF} = \kappa^D \cdot (\Phi^{*,LF})^{1-\xi} \cdot \left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)} \cdot \frac{\bar{H}}{2} - 1,$$

so $g^{*,LF}$ is strictly increasing in $\Phi^{*,LF}$. It suffices to show $d\Phi^{*,LF}/dm > 0$.

Step 1: Eliminating χ^*. Multiplying both sides of Eq. (B6) by $[m\chi^* + (1-m)]$:

$$\left[\Phi^* - \alpha(\Theta_L^{LF})^\eta\right]\left[m\chi^* + (1-m)\right] = m\left[\frac{\alpha(1-\varphi^{LF})}{\alpha + (1-\alpha)\tau^{1-\sigma}} + \varphi^{LF}\right](1-\Theta_L^{LF})^\eta.$$

From the BGP growth-rate equalization (Section B.7), $m\chi^* + (1-m)$ is proportional to $(\Phi^*)^{1-\xi}$:

$$m\chi^* + (1-m) = \frac{\kappa^D\,(\bar{L}/2)^{\eta(1-\xi)}\,\bar{H}/2}{(\kappa^M)^{1/(1-\phi)}\,(\varphi^{LF})^{\phi/(1-\phi)}\,[(1-\Theta_L^{LF})\bar{L}/2]^{\phi\eta/(1-\phi)}} \cdot (\Phi^*)^{1-\xi}.$$

Substituting the second equation into the first and dividing by the positive coefficient of $(\Phi^*)^{1-\xi}$

reduces the system to a single implicit equation in Φ^*:

$$\left[\Phi^* - \alpha(\Theta_L^{LF})^\eta\right](\Phi^*)^{1-\xi} = \frac{m\,(\kappa^M)^{1/(1-\phi)}\,(\varphi^{LF})^{\phi/(1-\phi)}\,(\bar{L}/2)^{\phi\eta/(1-\phi)}}{\kappa^D\,(\bar{L}/2)^{\eta(1-\xi)}\,\bar{H}/2} \times \left[\frac{\alpha(1-\varphi^{LF})}{\alpha+(1-\alpha)\tau^{1-\sigma}} + \varphi^{LF}\right](1-\Theta_L^{LF})^{\eta/(1-\phi)}. \quad \text{(B8)}$$

Step 2: Monotonicity. The LHS of Eq. (B8) is strictly increasing in Φ^* for $\Phi^* > \alpha(\Theta_L^{LF})^\eta$:

$$\frac{d}{d\Phi}\left[(\Phi - \alpha\Theta_L^\eta)\,\Phi^{1-\xi}\right] = \Phi^{1-\xi} + (1-\xi)(\Phi - \alpha\Theta_L^\eta)\,\Phi^{-\xi} > 0,$$

since $\xi \in (0,1)$ and both terms are positive. The RHS of Eq. (B8) is linear in m with a positive coefficient independent of Φ^*. By the implicit function theorem, $d\Phi^{*,LF}/dm > 0$ for all $m \in (0,1)$.

Step 3: Growth premium. As $m \to 0^+$, the RHS of Eq. (B8) vanishes, so $\Phi^{*,LF} \to \alpha\,(\Theta_L^{LF})^\eta$ and

$$\lim_{m\to 0^+} g^{*,LF}(m) = \kappa^D\,\alpha^{1-\xi}\,(\Theta_L^{LF})^{\eta(1-\xi)}\left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)}\frac{\bar{H}}{2} - 1.$$

For all $m \in (0,1)$, $\Phi^{*,LF}(m) > \lim_{m\to 0^+}\Phi^{*,LF}$, hence

$$\Delta g^{LF}(m) > 0, \qquad \frac{d\Delta g^{LF}}{dm} = \underbrace{(1-\xi)\kappa^D(\Phi^{*,LF})^{-\xi}\left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)}\frac{\bar{H}}{2}}_{>0}\cdot\frac{d\Phi^{*,LF}}{dm} > 0. \quad \square$$

Appendix C First-Best Optimality and Proof of Proposition 2

This appendix establishes the First-Best (FB) planner's optimality conditions, compares them with the laissez-faire (LF) equilibrium, and proves Proposition 2.

C.1 First-Best Optimality and Laissez-Faire Comparison

FB planner's problem. The First-Best planner directly chooses (Θ_L, φ) to maximize

$$\mathcal{W}^{FB}(\Theta_L,\varphi) = \frac{2(1+\rho)}{\rho}u(t=1;\Theta_L,\varphi) + \frac{2(1+\rho)}{\rho^2}\ln\big(1+g^*(\Theta_L,\varphi)\big), \quad \text{(C1)}$$

subject to the bidirectional-spillover equilibrium conditions Eq. (5)–Eq. (7) with $(\Theta_L^{LF},\varphi^{LF})$ replaced by (Θ_L,φ). The initial-period utility is

$$u(t=1;\Theta_L,\varphi) = \ln\Big[Z_i^M(t=1)\,(\chi^*)^\beta\,(\Theta_L)^{\eta\beta}\,(1-\Theta_L)^{\eta(1-\beta)}\,(\bar{L}/2)^\eta \times \left[\alpha+(1-\alpha)\tau^{1-\sigma}\right]^{-\frac{1-\beta}{1-\sigma}}(1-\varphi)^{1-\beta}\Big]. \quad \text{(C2)}$$

FB first-order conditions. Differentiating Eq. (C1) with respect to Θ_L and decomposing into the level effect (from initial-period utility) and the growth effect (from the balanced growth rate):

$$\frac{\partial \mathcal{W}^{FB}}{\partial \Theta_L} = \frac{2(1+\rho)}{\rho} \left\{ \underbrace{\left\{ \frac{\eta\beta}{\Theta_L} - \frac{\eta(1-\beta)}{1-\Theta_L} + \frac{\beta[m\chi^* + (1-m)]}{m\chi^*} \cdot \left[\frac{\eta\phi}{(1-\Theta_L)(1-\phi)} + \frac{1-\xi}{\Phi^*}\frac{\partial \Phi^*}{\partial \Theta_L} \right] \right\}}_{\text{level effect}} + \underbrace{\frac{1-\xi}{\rho\Phi^*}\frac{\partial \Phi^*}{\partial \Theta_L}}_{\text{growth effect}} \right\} = 0, \tag{C3}$$

and in φ:

$$\frac{\partial \mathcal{W}^{FB}}{\partial \varphi} = \frac{2(1+\rho)}{\rho} \left\{ \underbrace{-\frac{1-\beta}{1-\varphi} + \frac{\beta[m\chi^* + (1-m)]}{m\chi^*} \cdot \left[\frac{1-\xi}{\Phi^*}\frac{\partial \Phi^*}{\partial \varphi} - \frac{\phi}{(1-\phi)\varphi} \right]}_{\text{level effect}} + \underbrace{\frac{1-\xi}{\rho\Phi^*}\frac{\partial \Phi^*}{\partial \varphi}}_{\text{growth effect}} \right\} = 0. \tag{C4}$$

In both equations, the *level effect* captures the welfare impact through initial-period utility $u(t = 1; \Theta_L, \varphi)$, and the *growth effect* captures the welfare gain from raising g^* through Φ^*.

Laissez-faire benchmark. The LF labor FOC equates the private marginal returns across sectors,

$$\frac{\beta}{\Theta_L^{LF}} = \frac{1-\beta}{(1-\Theta_L^{LF})(1-\varphi^{LF})}, \tag{C5}$$

which reproduces Eq. (4); and the LF R&D FOC equates the private marginal product of R&D to its marginal cost,

$$\varphi^{LF} = \frac{\phi(\sigma-1)}{\sigma}. \tag{C6}$$

Comparing Eq. (C5)–Eq. (C6) with Eq. (C3)–Eq. (C4): substituting the LF condition Eq. (C5) into Eq. (C3) yields $\frac{\eta\beta}{\Theta_L} - \frac{\eta(1-\beta)}{1-\Theta_L} = \frac{\eta(1-\beta)\varphi}{(1-\Theta_L)(1-\varphi)} > 0$, so the first two terms of the level effect are strictly positive at LF. The product term multiplied by the χ^*-elasticity term and the entire growth effect are social returns not internalized by private agents. In Eq. (C4), only the consumption cost $-(1-\beta)/(1-\varphi)$ is reflected in the private calculus Eq. (C6); the χ^*-elasticity term and the growth effect are not internalized.

C.2 Proof of Proposition 2: Externalities Invalidate LF Efficiency

Reformulating the equilibrium system. Multiplying Eq. (6) by $[m\chi^* + (1-m)]$ and using Eq. (7) to substitute the denominator, the equilibrium system collapses into a single equation for Φ^* alone (for given Θ_L, φ):

$$F(\Phi, m; \Theta_L, \varphi) \equiv \Phi^{2-\xi} - \alpha\Theta_L^{\eta}\Phi^{1-\xi} - \frac{mR(\varphi)(1-\Theta_L)^{\eta/(1-\phi)}}{\Lambda_0(\varphi)} = 0, \tag{C7}$$

where $R(\varphi) \equiv \frac{\alpha(1-\varphi)}{\alpha+(1-\alpha)\tau^{1-\sigma}} + \varphi$ is the manufacturing data feedback rate in Eq. (6), and $\Lambda_0(\varphi)$ is the parameter ratio such that Eq. (7) reads $m\chi^* + (1-m) = \Lambda_0(\varphi)(1-\Theta_L)^{-\phi\eta/(1-\phi)}\Phi^{*1-\xi}$.

The LHS of Eq. (C7) is a strictly increasing function of Φ for $\Phi > \alpha\Theta_L^\eta$, so a unique interior solution $\Phi^*(m;\Theta_L,\varphi) > 0$ exists; as $m \to 0^+$, $\Phi^*(m;\Theta_L,\varphi) \to \alpha\Theta_L^\eta$.

Lemma 1. *The implicit solution $\Phi^*(m;\Theta_L,\varphi)$ of Eq. (C7) satisfies:*

(i) $\frac{\partial\Phi^*}{\partial\Theta_L} > 0$ *whenever* $\Theta_L \leq \beta$*;*

(ii) $\frac{\partial\Phi^*}{\partial\varphi} > 0$.

Proof. By the implicit function theorem, $\dfrac{\partial\Phi^*}{\partial y} = -\dfrac{\partial F/\partial y}{\partial F/\partial\Phi}$, where

$$\frac{\partial F}{\partial\Phi} = \Phi^{-\xi}\big[(2-\xi)\Phi - (1-\xi)\alpha\Theta_L^\eta\big] > 0 \quad \text{for all } \Phi^* \geq \alpha\Theta_L^\eta.$$

Since $\frac{\partial F}{\partial\Phi} > 0$ throughout, the sign of each comparative static is opposite to the sign of $\frac{\partial F}{\partial y}$.

(i) Reallocating labor toward the digital sector raises Φ^* through two channels, that is, a direct productivity channel (higher Θ_L increases digital output) and an indirect feedback channel (through the equilibrium spillover). Differentiating Eq. (C7) with respect to Θ_L and substituting gives

$$\frac{\partial F}{\partial\Theta_L} = \eta\Phi^{1-\xi}\left[\frac{\Phi^* - \alpha\Theta_L^\eta}{(1-\phi)(1-\Theta_L)} - \alpha\Theta_L^{\eta-1}\right].$$

The sign is negative—and hence $\frac{\partial\Phi^*}{\partial\Theta_L} > 0$—if and only if

$$\Phi^* - \alpha\Theta_L^\eta < \alpha\Theta_L^{\eta-1}(1-\phi)(1-\Theta_L).$$

To verify this inequality for all $m \in (0,1)$, we derive an explicit upper bound on the LHS using Eq. (C7). Since $F(\Phi^*, m; \Theta_L, \varphi) = 0$, we have

$$\Phi^* - \alpha\Theta_L^\eta = \frac{m\,R(\varphi)\,(1-\Theta_L)^{\eta/(1-\phi)}}{\Lambda_0(\varphi)\,(\Phi^*)^{1-\xi}}.$$

Because $\Phi^* \geq \alpha\Theta_L^\eta$ and $m < 1$, the LHS is bounded above by

$$\Phi^* - \alpha\Theta_L^\eta < \frac{R(\varphi)\,(1-\Theta_L)^{\eta/(1-\phi)}}{\Lambda_0(\varphi)\,(\alpha\Theta_L^\eta)^{1-\xi}}.$$

Meanwhile, the RHS of the target inequality is $\alpha\Theta_L^{\eta-1}(1-\phi)(1-\Theta_L)$. It therefore suffices to show

$$\frac{R(\varphi)\,(1-\Theta_L)^{\eta/(1-\phi)}}{\Lambda_0(\varphi)\,(\alpha\Theta_L^\eta)^{1-\xi}} \leq \alpha\Theta_L^{\eta-1}(1-\phi)(1-\Theta_L),$$

which rearranges to $R(\varphi)/[\Lambda_0(\varphi)\,\alpha^{2-\xi}\,(1-\phi)] \leq \Theta_L^{\eta(2-\xi)-1}\,(1-\Theta_L)^{1-\eta/(1-\phi)}$. The left-hand side is a finite constant depending only on parameters (not on m or Θ_L), while the right-hand side is a strictly positive, continuous function of Θ_L on $(0, \beta]$ that is bounded away from zero. Under the maintained parameterization (in particular $\eta(2-\xi)-1 > -1$ and $\eta+\phi < 1$, which ensures $1-\eta/(1-\phi) > 0$), the inequality holds for all $\Theta_L \in (0, \beta]$ and all $m \in (0, 1)$.[8]

(ii) A higher R&D share φ affects Φ^* through the manufacturing data feedback rate $R(\varphi)$ and the technology parameter $\Lambda_0(\varphi)$. Both effects work in the same direction:

$$\frac{dR}{d\varphi} = \frac{(1-\alpha)\tau^{1-\sigma}}{\alpha + (1-\alpha)\tau^{1-\sigma}} > 0, \qquad \frac{\partial \ln \Lambda_0}{\partial \varphi} = -\frac{\phi}{(1-\phi)\varphi} < 0.$$

Combining these, $\frac{\partial F}{\partial \varphi} = -\frac{m(1-\Theta_L)^{\eta/(1-\phi)}}{\Lambda_0}\big[\frac{dR}{d\varphi} - R \cdot \frac{\partial \ln \Lambda_0}{\partial \varphi}\big] < 0$, hence $\frac{\partial \Phi^*}{\partial \varphi} > 0$. □

Step 1: $\Theta_L^{LF} < \beta$. We first show that the laissez-faire digital labor share lies strictly below the digital output elasticity β. From Eq. (4):

$$\Theta_L^{LF} = \frac{\beta(1-\varphi^{LF})}{1-\varphi^{LF}\beta} < \beta \quad \Longleftrightarrow \quad \varphi^{LF}\beta < \varphi^{LF},$$

which holds since $\beta < 1$. This bound is used in Step 2 to ensure Lemma 1(i) applies.

Step 2: $\partial \mathcal{W}^{FB}/\partial \Theta_L > 0$ at $(\Theta_L^{LF}, \varphi^{LF})$. Evaluated at the LF allocation, the two terms in Eq. (C3) behave as follows.

(a) Level effect: substituting the LF condition Eq. (C5), i.e. $\beta/\Theta_L = (1-\beta)/[(1-\Theta_L)(1-\varphi)]$, into the first two terms yields $\frac{\eta\beta}{\Theta_L} - \frac{\eta(1-\beta)}{1-\Theta_L} = \frac{\eta(1-\beta)\varphi}{(1-\Theta_L)(1-\varphi)} > 0$. The χ^*-elasticity term $\big[\frac{\eta\phi}{(1-\Theta_L)(1-\phi)} + \frac{1-\xi}{\Phi^*}\frac{\partial \Phi^*}{\partial \Theta_L}\big]$ is strictly positive, with the second element positive by Lemma 1(i). Hence the level effect is strictly positive.

(b) Growth effect: $\frac{\partial \Phi^*}{\partial \Theta_L} > 0$ by Lemma 1(i), so this term is strictly positive.

Since both the level effect and the growth effect are strictly positive, we conclude

$$\left.\frac{\partial \mathcal{W}^{FB}}{\partial \Theta_L}\right|_{(\Theta_L^{LF}, \varphi^{LF})} > 0. \tag{C8}$$

Step 3: $\Theta_L^{FB} > \Theta_L^{LF}$. The first two terms of the level effect in Eq. (C3) satisfy $\frac{\eta\beta}{\Theta_L} - \frac{\eta(1-\beta)}{1-\Theta_L} = \frac{\eta(\beta-\Theta_L)}{\Theta_L(1-\Theta_L)} \geq 0$ for any $\Theta_L \leq \beta$. The χ^*-elasticity term $\big[\frac{\eta\phi}{(1-\Theta_L)(1-\phi)} + \frac{1-\xi}{\Phi^*}\frac{\partial \Phi^*}{\partial \Theta_L}\big]$ is strictly positive. Together with the strictly positive growth effect, $\partial \mathcal{W}^{FB}/\partial \Theta_L > 0$ on the entire region $(0, \beta] \times (0, 1)$. In particular, evaluating at $(\Theta_L^{LF}, \varphi^{FB})$ shows that increasing Θ_L from Θ_L^{LF} strictly raises welfare, which contradicts the optimality of any $\Theta_L^{FB} \leq \Theta_L^{LF}$.

Step 4: Ambiguity of φ^{FB} vs. φ^{LF}. Unlike the unambiguous result for Θ_L, the welfare effect of raising the R&D share φ involves offsetting forces. In Eq. (C4), the consumption cost $-(1-\beta)/(1-$

[8] An equivalent verification: the LHS of the original inequality is $\Phi^* - \alpha\Theta_L^{\eta}$, which from the implicit equation is proportional to m, while the RHS $\alpha\Theta_L^{\eta-1}(1-\phi)(1-\Theta_L)$ is independent of m. Since the LHS vanishes at $m = 0$ and grows at most linearly in m (with a bounded coefficient that depends on the finite constant R/Λ_0), and the RHS is a strictly positive constant, the inequality is preserved for all $m \in (0, 1)$ under the maintained parameterization.

$\varphi) < 0$ is unambiguously negative. The χ^*-elasticity term combines the data-enrichment channel $(1-\xi)\partial\Phi^*/(\Phi^*\partial\varphi) > 0$ (by Lemma 1(ii)) with the technology-catching-up channel $-\phi/[(1-\phi)\varphi] < 0$; its sign is thus ambiguous. The growth effect is strictly positive by the same lemma. The overall sign of $\partial\mathcal{W}^{FB}/\partial\varphi|_{LF}$ depends on the relative magnitude of these opposing forces, so $\varphi^{FB} \gtrless \varphi^{LF}$ is parameter-dependent.

Step 5: $\mathcal{W}^{FB} > \mathcal{W}^{LF}$. Finally, the FB allocation strictly dominates the LF outcome. By revealed preference, the FB optimum must yield welfare at least as high as welfare evaluated at the LF allocation: $\mathcal{W}^{FB} \geq \mathcal{W}^{FB}(\Theta_L^{LF}, \varphi^{LF}) = \mathcal{W}^{LF}$. The inequality is strict because Eq. (C8) shows that a marginal increase in Θ_L from Θ_L^{LF} strictly raises welfare, so the LF allocation cannot be optimal. □

Appendix D Government Intervention, Second-Best Allocation, Nash Equilibrium, and Proof of Proposition 3

This appendix derives the equilibrium under government intervention, characterizes the Second-Best (SB) allocation and the Nash equilibrium, and proves Proposition 3.

D.1 Equilibrium Under Government Intervention

D.1.1 Consumer Problem

Expenditure shares. The representative consumer in region i has period utility $u_i(t) = \beta \ln C_i^D(t) + (1-\beta)\ln C_i^M(t)$. Given total expenditure $E_i(t)$, the Cobb-Douglas structure yields:

$$P_i^D(t)C_i^D(t) = \beta E_i(t), \qquad P_i^M(t)C_i^M(t) = (1-\beta)E_i(t).$$

The expenditure ratio is:

$$\frac{P_i^D(t)C_i^D(t)}{P_i^M(t)C_i^M(t)} = \frac{\beta}{1-\beta}.$$

Consumer-facing prices and bilateral demands. Digital services are traded at $\tilde{p}_{ki}^D(t) = p_k^D(t)$ and manufacturing goods at $\tilde{p}_{ii}^M(t) = p_i^M(t)$, $\tilde{p}_{ji}^M(t) = \tau p_j^M(t)$. Consequently, the functional forms of bilateral demands, expenditure shares, markups ($\sigma/(\sigma-1)$ for both sectors), and price indices ($P_i^D(t)$, $P_i^M(t)$) remain the same expressions as in laissez-faire; their equilibrium values, however, differ because the manufacturing tax alters producer behavior through the FOCs derived below.

D.1.2 Sectoral FOCs Under Government Intervention

Digital sector. The digital firm has production function $Y_i^D(t) = Z_i^D(t)(L_i^D(t))^\eta$ and receives a per-unit subsidy s_i^D, so effective revenue is $(1+s_i^D)p_i^D(t)Y_i^D(t)$. The subsidized digital labor FOC is:

$$w_i^L(t)L_i^D(t) = \frac{\eta(\sigma-1)}{\sigma}(1+s_i^D)p_i^D(t)Y_i^D(t).$$

The digital talent earns the residual:

$$w_i^H(t)H_i^D(t) = \left[1 - \frac{\eta(\sigma-1)}{\sigma}\right](1+s_i^D)p_i^D(t)Y_i^D(t).$$

The talent clearing condition $w_A^H = w_B^H$ with $H_A^D + H_B^D = \bar{H}$ determines H_i^D endogenously under subsidies (s_i^D, s_j^D), reducing to $H_i^D = \bar{H}/2$ in symmetric equilibrium.

Manufacturing sector. The manufacturing firm has production function $Y_i^M(t) = Z_i^M(t)(L_i^M(t))^\eta$ and incurs R&D cost $p_i^M(t)\psi_i^M(t)$. Under the manufacturing tax, the firm retains only fraction $(1-t_i)$ of gross revenue $p_i^M(t)Y_i^M(t)$, so after-tax revenue is $(1-t_i)p_i^M(t)Y_i^M(t)$. The profit maximization problem yields FOCs where the tax wedge enters both the labor and R&D conditions:

$$w_i^L(t)L_i^M(t) = \frac{\eta(\sigma-1)}{\sigma}(1-t_i)p_i^M(t)Y_i^M(t),$$

$$p_i^M(t)\psi_i^M(t) = \frac{\phi(\sigma-1)}{\sigma}(1-t_i)p_i^M(t)Y_i^M(t) \equiv \varphi p_i^M(t)Y_i^M(t),$$

where the manufacturing tax reduces the R&D share to:

$$\varphi \equiv \frac{\psi_i^M(t)}{Y_i^M(t)} = \frac{\phi(\sigma-1)(1-t_i)}{\sigma} < \frac{\phi(\sigma-1)}{\sigma} \equiv \varphi^{LF},$$

so the manufacturing tax compresses the R&D share below the laissez-faire level $\phi(\sigma-1)/\sigma$.

Net manufacturing consumption. From the R&D FOC above: $\psi_i^M(t) = \varphi Y_i^M(t)$. Net manufacturing consumption (output minus R&D input) satisfies:

$$P_i^M(t)C_i^M(t) = p_i^M(t)\big[Y_i^M(t) - \psi_i^M(t)\big] = p_i^M(t)(1-\varphi)Y_i^M(t).$$

D.1.3 Sectoral Labor Allocation

Ratio of labor FOCs. From the digital labor FOC and manufacturing labor FOC above:

$$\frac{w_i^L(t)L_i^D(t)}{w_i^L(t)L_i^M(t)} = \frac{[\eta(\sigma-1)/\sigma](1+s_i^D)p_i^D(t)Y_i^D(t)}{[\eta(\sigma-1)/\sigma](1-t_i)p_i^M(t)Y_i^M(t)} = \frac{(1+s_i^D)}{(1-t_i)} \cdot \frac{p_i^D(t)Y_i^D(t)}{p_i^M(t)Y_i^M(t)}.$$

Applying the expenditure ratio. Consumer expenditure shares give $P_i^D(t)C_i^D(t)/[P_i^M(t)C_i^M(t)] = \beta/(1-\beta)$. Since $P_i^D(t)C_i^D(t) = p_i^D(t)Y_i^D(t)$ and $P_i^M(t)C_i^M(t) = p_i^M(t)(1-\varphi)Y_i^M(t)$:

$$\frac{p_i^D(t)Y_i^D(t)}{p_i^M(t)Y_i^M(t)} = \frac{\beta(1-\varphi)}{1-\beta}.$$

Substituting into the labor ratio above:

$$\frac{L_i^D(t)}{L_i^M(t)} = \frac{(1+s_i^D)\beta(1-\varphi)}{(1-t_i)(1-\beta)}.$$

Digital labor share. Recall $\Theta_L \equiv L_i^D(t)/\big(L_i^D(t)+L_i^M(t)\big)$. From the labor ratio above:

$$\Theta_L = \frac{(1+s_i^D)\beta(1-\varphi)}{(1+s_i^D)\beta(1-\varphi)+(1-t_i)(1-\beta)}.$$

General equilibrium conditions. **Government budget constraint.** Tax revenue $t_i p_i^M(t)Y_i^M(t)$ finances the digital subsidy $s_i^D p_i^D(t)Y_i^D(t)$:

$$t_i p_i^M(t)Y_i^M(t) = s_i^D p_i^D(t)Y_i^D(t).$$

From the expenditure ratio derived above, $p_i^D(t)Y_i^D(t)/(p_i^M(t)Y_i^M(t)) = \beta(1-\varphi)/(1-\beta)$. Since $\varphi = \phi(\sigma-1)(1-t_i)/\sigma$ depends on t_i itself, substituting into the budget constraint yields an implicit equation in t_i:

$$t_i = s_i^D \cdot \frac{\beta\big[1-\phi(\sigma-1)(1-t_i)/\sigma\big]}{1-\beta}.$$

Expanding the right-hand side and collecting terms in t_i gives the closed-form solution:

$$t_i = \frac{s_i^D\beta[1-\phi(\sigma-1)/\sigma]}{(1-\beta)-s_i^D\beta\phi(\sigma-1)/\sigma}.$$

Substituting back, the equilibrium policy-adjusted R&D share is:

$$\varphi = \frac{\phi(\sigma-1)(1-t_i)}{\sigma} = \frac{\phi(\sigma-1)}{\sigma}\cdot\frac{(1-\beta)-s_i^D\beta}{(1-\beta)-s_i^D\beta\phi(\sigma-1)/\sigma}, \tag{D1}$$

and substituting with $1-t_i = [(1-\beta)-s_i^D\beta]/[(1-\beta)-s_i^D\beta\phi(\sigma-1)/\sigma]$ and $1-\varphi = [1-\phi(\sigma-1)/\sigma](1-\beta)/[(1-\beta)-s_i^D\beta\phi(\sigma-1)/\sigma]$, cancelling the common factor yields:

$$\Theta_L = \frac{(1+s_i^D)\beta(1-\varphi)}{(1+s_i^D)\beta(1-\varphi)+(1-t_i)(1-\beta)} = \frac{(1+s_i^D)\beta[1-\phi(\sigma-1)/\sigma]}{1-\phi(\sigma-1)\beta(1+s_i^D)/\sigma}. \tag{D2}$$

D.1.4 Data Feedback and Growth

Data generation. The data input is $\mathcal{D}_i(t) = c_{ii}^D(t) + \frac{m\chi(t)}{m\chi(t)+(1-m)}[c_{ii}^M(t)+\psi_i^M(t)]$. In symmetric equilibrium, digital home consumption is $c_{ii}^D(t) = \alpha Y_i^D(t)$, manufacturing home consumption is $c_{ii}^M = \frac{\alpha(1-\varphi)}{\alpha+(1-\alpha)\tau^{1-\sigma}}Y_i^M$, and home R&D is $\psi_i^M = \varphi Y_i^M$. Adding:

$$c_{ii}^M(t)+\psi_i^M(t) = \left[\frac{\alpha(1-\varphi)}{\alpha+(1-\alpha)\tau^{1-\sigma}}+\varphi\right]Y_i^M(t).$$

Substituting:

$$\mathcal{D}_i(t) = \alpha Y_i^D(t) + \frac{m\chi(t)}{m\chi(t)+(1-m)}\cdot\left[\frac{\alpha(1-\varphi)}{\alpha+(1-\alpha)\tau^{1-\sigma}}+\varphi\right]Y_i^M(t).$$

Effective data intensity. Define the effective data intensity Φ^* (constant on the BGP). Using

$Y_i^M/Y_i^D = (1/\chi^*) \cdot [(1-\Theta_L)/\Theta_L]^\eta$:

$$\Phi^* = \alpha(\Theta_L)^\eta + \frac{m}{m\chi^* + (1-m)}\left[\frac{\alpha(1-\varphi)}{\alpha + (1-\alpha)\tau^{1-\sigma}} + \varphi\right](1-\Theta_L)^\eta. \tag{D3}$$

Compared to the laissez-faire expression, both Θ_L and φ are policy-adjusted: Θ_L is given by Eq. (D2) and $\varphi < \varphi^{LF}$ by Eq. (D1). Θ_L enters Φ^* directly through $(\Theta_L)^\eta$ and $(1-\Theta_L)^\eta$, φ enters directly through the bracket, and both enter indirectly through χ^*.

Growth rate. From the digital technology evolution:

$$g^* = \kappa^D \cdot (\Phi^*)^{1-\xi} \cdot \left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)} \cdot \frac{\bar{H}}{2} - 1. \tag{D4}$$

Technology ratio. The manufacturing tax reduces the R&D share to $\varphi = \phi(\sigma-1)(1-t_i)/\sigma$. With this modified R&D share:

$$\chi^* = \frac{1}{m}\left\{\frac{\kappa^D(\Phi^*)^{1-\xi}\left(\frac{\bar{L}}{2}\right)^{\eta(1-\xi)}\frac{\bar{H}}{2}}{(\kappa^M)^{1/(1-\phi)}(\varphi)^{\phi/(1-\phi)}\left(\frac{(1-\Theta_L)\bar{L}}{2}\right)^{\phi\eta/(1-\phi)}} - (1-m)\right\}. \tag{D5}$$

Equations Eq. (D3), Eq. (D4), and Eq. (D5) jointly determine (χ^*, g^*, Φ^*), with the policy-adjusted labor share Θ_L given by Eq. (D2) and the policy-adjusted R&D share φ given by Eq. (D1).

D.1.5 Welfare

Initial-period utility. Under the manufacturing tax, the R&D share becomes $\varphi = \phi(\sigma-1)(1-t_i)/\sigma$, so net manufacturing consumption satisfies $C_i^M(t=1) = [\alpha + (1-\alpha)\tau^{1-\sigma}]^{-1/(1-\sigma)}(1-\varphi)Z_i^M(t=1)[(1-\Theta_L)\bar{L}/2]^\eta$. Digital consumption remains $C_i^D(t=1) = Y_i^D(t=1) = Z_i^D(t=1)(\Theta_L\bar{L}/2)^\eta$. Using $Z_i^D(t=1) = \chi^* Z_i^M(t=1)$:

$$u_i(t=1) = \ln\Big[Z_i^M(t=1)\,(\chi^*)^\beta\,(\Theta_L)^{\eta\beta}\,(1-\Theta_L)^{\eta(1-\beta)}\,(\bar{L}/2)^\eta \times \left[\alpha + (1-\alpha)\tau^{1-\sigma}\right]^{-\frac{1-\beta}{1-\sigma}}(1-\varphi)^{1-\beta}\Big]. \tag{D6}$$

Representative consumer's welfare. Region i's representative consumer's lifetime welfare on the BGP is:

$$\mathcal{V}_i = \frac{1+\rho}{\rho}u_i(t=1) + \frac{1+\rho}{\rho^2}\ln(1+g^*),$$

where $u_i(t=1)$ is given by Eq. (D6) and g^* by Eq. (D4).

D.2 Second-Best Optimal Subsidy

SB planner's problem. The coordinated planner maximizes aggregate welfare $\mathcal{W} = \frac{2(1+\rho)}{\rho}u(t=1) + \frac{2(1+\rho)}{\rho^2}\ln(1+g^*)$ by choosing a common subsidy s^D, taking the general equilibrium as given. The budget constraint determines t residually.

First-order condition. Differentiating $\mathcal{W}$ with respect to s^D and setting equal to zero:

$$\frac{d\mathcal{W}}{ds^D} = \frac{2(1+\rho)}{\rho}\frac{du(t=1)}{ds^D} + \frac{2(1+\rho)}{\rho^2} \cdot \frac{1}{1+g^*} \cdot \frac{dg^*}{ds^D} = 0. \tag{D7}$$

Level effect. Differentiating Eq. (D6):

$$\frac{du(t=1)}{ds^D} = \beta\frac{d\ln\chi^*}{ds^D} + \eta\frac{d\Theta_L}{ds^D}\left(\frac{\beta}{\Theta_L} - \frac{1-\beta}{1-\Theta_L}\right) + (1-\beta)\frac{d\ln(1-\varphi)}{ds^D}.$$

Growth effect. The balanced growth rate $g^* = \kappa^D(\Phi^*)^{1-\xi}(\bar{L}/2)^{\eta(1-\xi)}\bar{H}/2 - 1$ depends on s^D only through the effective data intensity Φ^*, so

$$\frac{dg^*}{ds^D} = (1-\xi)\frac{1+g^*}{\Phi^*}\frac{d\Phi^*}{ds^D}.$$

The policy s^D enters Φ^* through $\Theta_L(s^D)$ and $\varphi(s^D)$ only (with χ^* pinned down residually by Eq. (D5)):

$$\frac{d\Phi^*}{ds^D} = \underbrace{\frac{d\Phi^*}{d\Theta_L}\frac{d\Theta_L}{ds^D}}_{\text{sectoral reallocation}(+)} + \underbrace{\frac{d\Phi^*}{d\varphi}\frac{d\varphi}{ds^D}}_{\text{R\&D compression}(?)},$$

where $d/d\Theta_L$ and $d/d\varphi$ denote *total* derivatives that incorporate the induced response of χ^* via Eq. (D5).

Signs of the two channels. Differentiating Eq. (D2) and Eq. (D1) with respect to s^D:

$$\frac{d\Theta_L}{ds^D} = \frac{\beta[1-\phi(\sigma-1)/\sigma]}{[1-\phi(\sigma-1)\beta(1+s^D)/\sigma]^2} > 0,$$

$$\frac{d\varphi}{ds^D} = -\frac{\phi(\sigma-1)\beta[1-\phi(\sigma-1)/\sigma](1-\beta)}{\sigma[(1-\beta)-s^D\beta\phi(\sigma-1)/\sigma]^2} < 0.$$

By Lemma 1(i)–(ii), $d\Phi^*/d\Theta_L > 0$ on $\Theta_L \leq \beta$ and $d\Phi^*/d\varphi > 0$ when $\tau > 1$ (where $d/d\Theta_L$ and $d/d\varphi$ are total derivatives along Eq. (D5)). Combining: the sectoral reallocation channel $\frac{d\Phi^*}{d\Theta_L}\frac{d\Theta_L}{ds^D} > 0$, and the R&D compression channel $\frac{d\Phi^*}{d\varphi}\frac{d\varphi}{ds^D} < 0$.

Existence and uniqueness of $s^{D,SB} > 0$. Define the budget-binding subsidy $\bar{s}^D \equiv (1-\beta)/\beta$, at which $t_i \to 1^-$ and $\varphi \to 0^+$.

At $s^D = 0$: $\Theta_L = \Theta_L^{LF} < \beta$ (Proposition 2), so $d\Theta_L/ds^D > 0$ and, by Lemma 1(i), $d\Phi^*/ds^D > 0$, hence $dg^*/ds^D > 0$. For the level effect, from Eq. (D6):

$$\left.\frac{du(t=1)}{ds^D}\right|_{s^D=0} = \beta\frac{d\ln\chi^*}{ds^D} + \eta\frac{d\Theta_L}{ds^D}\underbrace{\left(\frac{\beta}{\Theta_L^{LF}} - \frac{1-\beta}{1-\Theta_L^{LF}}\right)}_{>0 \text{ since } \Theta_L^{LF}<\beta} + (1-\beta)\underbrace{\frac{-d\varphi/ds^D}{1-\phi(\sigma-1)/\sigma}}_{>0}.$$

All three terms are positive, so $d\mathcal{W}/ds^D|_{s^D=0} > 0$. As $s^D \to \bar{s}^{D-}$, $\Theta_L \to 1^-$ so the term $\eta(1-\beta)\ln(1-\Theta_L) \to -\infty$ dominates, driving $d\mathcal{W}/ds^D < 0$. By continuity and the intermediate value theorem, there exists $s^{D,SB} \in (0, \bar{s}^D)$ satisfying Eq. (D7).

D.3 Nash Equilibrium Subsidy and Proof of Proposition 3

D.3.1 Nash Best-Response Problem

Region i takes region j's subsidy s_j^D as given and chooses s_i^D to maximize its tournament objective. Given s_j^D, labor market clearing determines the equilibrium allocation $L_i(s_i^D, s_j^D)$ endogenously, as labor migrates until marginal products are equalized across regions,

$$w_i\big(s_i^D, L_i\big) = w_j\big(s_j^D, \bar{L} - L_i\big),$$

which implicitly defines $L_i = L_i(s_i^D, s_j^D)$ with $\partial L_i / \partial s_i^D > 0$ (a higher local subsidy attracts labor) and $\partial L_i / \partial s_j^D < 0$. Similarly, digital talent migrates until talent wages are equalized:

$$w_i^H\big(s_i^D, H_i^D\big) = w_j^H\big(s_j^D, \bar{H} - H_i^D\big),$$

which implicitly defines $H_i^D = H_i^D(s_i^D, s_j^D)$ with $\partial H_i^D / \partial s_i^D > 0$ (a higher subsidy raises digital revenue and attracts talent) and $\partial H_i^D / \partial s_j^D < 0$.

Region i's growth rate therefore depends on both subsidies:

$$g_i^*\big(s_i^D, s_j^D\big) \equiv g^*\big(s_i^D, L_i(s_i^D, s_j^D), H_i^D(s_i^D, s_j^D)\big).$$

Region i's government, with tenure T periods, solves:

$$\max_{s_i^D} (1-\lambda)\big[\Gamma_1(T) u_i(t=1) + \Gamma_2(T)\ln(1+g_i^*)\big] + \lambda\Big[g_i^*\big(s_i^D, s_j^D\big) - g_j^*\big(s_i^D, s_j^D\big)\Big],$$

where $\Gamma_1(T)$ and $\Gamma_2(T)$ are defined in Eq. (16) and $\lambda \in (0,1)$. To state the first-order condition, we distinguish two channels through which s_i^D affects region i's growth rate. The *intra-regional policy channel*, denoted $\partial g_i^* / \partial s_i^D |_{L_i, H_i^D}$, operates through the within-region allocation objects $\Theta_{L,i}$, φ_i, and χ_i^*, holding the inter-regional factor allocation (L_i, H_i^D) fixed. The *inter-regional factor channel* captures the additional effect of cross-regional factor inflows attracted by the subsidy ($\partial L_i / \partial s_i^D > 0$, $\partial H_i^D / \partial s_i^D > 0$). The total effect on g_i^* decomposes as:

$$\frac{dg_i^*}{ds_i^D} = \underbrace{\left.\frac{\partial g_i^*}{\partial s_i^D}\right|_{L_i, H_i^D}}_{\text{intra-regional policy}} + \underbrace{\frac{\partial g_i^*}{\partial L_i}\cdot\frac{\partial L_i}{\partial s_i^D} + \frac{\partial g_i^*}{\partial H_i^D}\cdot\frac{\partial H_i^D}{\partial s_i^D}}_{\text{inter-regional factor}},$$

where $\partial g_i^* / \partial H_i^D = (1+g_i^*)/H_i^D > 0$ and $\partial g_i^* / \partial L_i = \eta(1-\xi)(1+g_i^*)/L_i > 0$. The rival region's growth responds only through the inter-regional factor channel (with reversed signs):

$$\frac{\partial g_j^*}{\partial s_i^D} = \frac{\partial g_j^*}{\partial L_j}\cdot\frac{\partial L_j}{\partial s_i^D} + \frac{\partial g_j^*}{\partial H_j^D}\cdot\frac{\partial H_j^D}{\partial s_i^D} < 0.$$

We assume that each government evaluates local welfare through the intra-regional policy channel only, taking the inter-regional factor allocation as given. The tournament term, by contrast, reflects the full growth differential including both the intra-regional policy effect and the inter-regional factor effect, because tournament competition inherently involves rivalry over mobile factors (L_i and H_i^D) across regions.

The first-order condition is:

$$(1-\lambda)\left[\Gamma_1(T)\frac{\partial u_i(t=1)}{\partial s_i^D}\bigg|_{L_i,H_i^D}+\frac{\Gamma_2(T)}{1+g_i^*}\frac{\partial g_i^*}{\partial s_i^D}\bigg|_{L_i,H_i^D}\right]+\lambda\left[\frac{dg_i^*}{ds_i^D}-\frac{\partial g_j^*}{\partial s_i^D}\right]=0. \tag{D8}$$

By symmetry, region j solves the analogous problem.

D.3.2 Factor Allocation and Grabbing Elasticities

We now derive closed-form expressions for $L_i(s_i^D, s_j^D)$ and $H_i^D(s_i^D, s_j^D)$ and their partial derivatives, which are needed for the tournament term.

Step 1: Digital labor share is a function of s_i^D alone. From Eq. (D2), the within-region digital labor share in region i is

$$\Theta_{L,i}=\frac{(1+s_i^D)\beta[1-\phi(\sigma-1)/\sigma]}{1-\phi(\sigma-1)\beta(1+s_i^D)/\sigma}, \tag{D9}$$

which depends only on s_i^D.

Step 2: Labor allocation across regions. Each region's digital sector employs $L_i^D=\Theta_{L,i}L_i$ labor with production $Y_i^D=Z_i^D(\Theta_{L,i}L_i)^\eta$. The digital labor FOC gives:

$$w_i^L=\frac{\eta(\sigma-1)}{\sigma}(1+s_i^D)p_i^D Z_i^D\Theta_{L,i}^{\eta-1}L_i^{\eta-1}.$$

Under symmetric technology ($Z_i^D=Z_j^D$) and free trade of digital services ($\tau^D=1$), variety prices satisfy $p_i^D=p_j^D$. The wage equalization condition $w_i^L=w_j^L$ then becomes:

$$(1+s_i^D)\Theta_{L,i}^{\eta-1}L_i^{\eta-1}=(1+s_j^D)\Theta_{L,j}^{\eta-1}(\bar{L}-L_i)^{\eta-1}. \tag{D10}$$

Define $\varpi_i\equiv(1+s_i^D)\Theta_{L,i}^{\eta-1}$. Since $\eta\in(0,1)$, Eq. (D10) solves uniquely for

$$L_i=\frac{\varpi_i^{1/(1-\eta)}}{\varpi_i^{1/(1-\eta)}+\varpi_j^{1/(1-\eta)}}\bar{L}. \tag{D11}$$

Step 3: Talent allocation across regions. Talent earns the residual in the digital sector: $w_i^H H_i^D=[1-\eta(\sigma-1)/\sigma](1+s_i^D)p_i^D Y_i^D$. The wage equalization $w_i^H=w_j^H$ requires:

$$\frac{(1+s_i^D)(\Theta_{L,i}L_i)^\eta}{H_i^D}=\frac{(1+s_j^D)(\Theta_{L,j}(\bar{L}-L_i))^\eta}{\bar{H}-H_i^D}.$$

Define $\nu_i \equiv (1+s_i^D)\Theta_{L,i}^{\eta}$. Substituting Eq. (D11):

$$\frac{H_i^D}{\bar{H}-H_i^D} = \frac{\nu_i}{\nu_j}\left(\frac{L_i}{\bar{L}-L_i}\right)^{\eta} = \frac{\nu_i}{\nu_j}\left(\frac{\varpi_i}{\varpi_j}\right)^{\eta/(1-\eta)}.$$

Since $\nu_i/\nu_j = [(1+s_i^D)\Theta_{L,i}^{\eta}]/[(1+s_j^D)\Theta_{L,j}^{\eta}]$ and $(\varpi_i/\varpi_j)^{\eta/(1-\eta)} = [(1+s_i^D)\Theta_{L,i}^{\eta-1}]^{\eta/(1-\eta)}/[(1+s_j^D)\Theta_{L,j}^{\eta-1}]^{\eta/(1-\eta)}$, combining the exponents of $(1+s_i^D)$: $1+\eta/(1-\eta) = 1/(1-\eta)$; and combining the exponents of $\Theta_{L,i}$: $\eta + \eta(\eta-1)/(1-\eta) = 0$. That is, $\Theta_{L,i}$ cancels entirely, yielding:

$$H_i^D = \frac{(1+s_i^D)^{1/(1-\eta)}}{(1+s_i^D)^{1/(1-\eta)} + (1+s_j^D)^{1/(1-\eta)}}\bar{H}. \tag{D12}$$

The digital labor share $\Theta_{L,i}$ drops out because labor migration Eq. (D11) exactly offsets the sectoral reallocation.

Step 4: Talent-grabbing elasticity at the symmetric point. Differentiating Eq. (D12) by the quotient rule:

$$\frac{\partial H_i^D}{\partial s_i^D} = \frac{\frac{1}{1-\eta}(1+s_i^D)^{\frac{\eta}{1-\eta}}(1+s_j^D)^{\frac{1}{1-\eta}}}{\left[(1+s_i^D)^{\frac{1}{1-\eta}} + (1+s_j^D)^{\frac{1}{1-\eta}}\right]^2}\bar{H}.$$

At $s_i^D = s_j^D = s^D$, both terms in the denominator equal $(1+s^D)^{1/(1-\eta)}$, so

$$\left.\frac{\partial H_i^D}{\partial s_i^D}\right|_{\text{sym}} = \frac{\bar{H}}{4(1-\eta)(1+s^D)}.$$

Step 5: Labor-grabbing elasticity at the symmetric point. Since Eq. (D11) has the same contest-function structure as Eq. (D12), by the same quotient-rule argument as Step 4, differentiating with respect to s_i^D and evaluating at $\varpi_i = \varpi_j$ gives:

$$\left.\frac{\partial L_i}{\partial s_i^D}\right|_{\text{sym}} = \frac{\bar{L}}{4(1-\eta)}\cdot\frac{1}{\varpi_i}\frac{\partial \varpi_i}{\partial s_i^D}.$$

From Eq. (D9), $\frac{1}{\Theta_{L,i}}\frac{\partial \Theta_{L,i}}{\partial s_i^D} = 1/[(1+s_i^D)(1-\phi(\sigma-1)\beta(1+s_i^D)/\sigma)]$, so

$$\frac{1}{\varpi_i}\frac{\partial \varpi_i}{\partial s_i^D} = \frac{1}{1+s_i^D} + \frac{\eta-1}{\Theta_{L,i}}\frac{\partial \Theta_{L,i}}{\partial s_i^D} = \frac{1}{1+s^D}\cdot\frac{\eta-\phi(\sigma-1)\beta(1+s^D)/\sigma}{1-\phi(\sigma-1)\beta(1+s^D)/\sigma}.$$

Substituting:

$$\left.\frac{\partial L_i}{\partial s_i^D}\right|_{\text{sym}} = \frac{\bar{L}}{4(1+s^D)}\left[\frac{1}{1-\eta} - \frac{1}{1-\phi(\sigma-1)\beta(1+s^D)/\sigma}\right].$$

Step 6: Tournament growth differential. By symmetry of factor outflows ($\partial L_j/\partial s_i^D =$

$-\partial L_i/\partial s_i^D$ and $\partial H_j^D/\partial s_i^D = -\partial H_i^D/\partial s_i^D$), the tournament term is:

$$\left.\frac{\partial g_i^*}{\partial s_i^D}\right|_{\text{sym}} - \left.\frac{\partial g_j^*}{\partial s_i^D}\right|_{\text{sym}} = \left.\frac{\partial g^*}{\partial s^D}\right|_{L,H} + 2\frac{\partial g^*}{\partial L_i}\frac{\partial L_i}{\partial s_i^D} + 2\frac{\partial g^*}{\partial H_i^D}\frac{\partial H_i^D}{\partial s_i^D}. \tag{D13}$$

Using $\partial g^*/\partial H_i^D = (1+g^*)/(\bar{H}/2)$ and $\partial g^*/\partial L_i = \eta(1-\xi)(1+g^*)/(\bar{L}/2)$, the talent-grabbing contribution is:

$$2 \cdot \frac{2(1+g^*)}{\bar{H}} \cdot \frac{\bar{H}}{4(1-\eta)(1+s^D)} = \frac{1+g^*}{(1-\eta)(1+s^D)}, \tag{D14}$$

and the labor-grabbing contribution is:

$$\frac{\eta(1-\xi)(1+g^*)}{1+s^D}\left[\frac{1}{1-\eta} - \frac{1}{1-\phi(\sigma-1)\beta(1+s^D)/\sigma}\right]. \tag{D15}$$

D.3.3 Symmetric Nash Equilibrium

The symmetric Nash equilibrium $s^{D*,Nash}$ satisfies both FOCs at $s_i^D = s_j^D = s^{D*,Nash}$. At the symmetric point $u_A(t=1) = u_B(t=1)$, $g_A^* = g_B^*$, and $L_A = L_B = \bar{L}/2$. Evaluating Eq. (D8) at the symmetric point:

$$(1-\lambda)\left[\Gamma_1(T)\left.\frac{\partial u_i(t=1)}{\partial s_i^D}\right|_{\substack{\text{sym}\\L,H}} + \frac{\Gamma_2(T)}{1+g^*}\left.\frac{\partial g_i^*}{\partial s_i^D}\right|_{\substack{\text{sym}\\L,H}}\right] + \lambda\left[\left.\frac{\partial g^*}{\partial s^D}\right|_{L,H} + 2\frac{\partial g^*}{\partial L_i}\frac{\partial L_i}{\partial s_i^D} + 2\frac{\partial g^*}{\partial H_i^D}\frac{\partial H_i^D}{\partial s_i^D}\right]_{\text{sym}} = 0,$$

where the tournament bracket is Eq. (D13). Compared with the SB planner, the regional government's finite tenure compresses the growth weight: $\Gamma_2(T)/\Gamma_1(T) < 1/\rho$, strictly for every finite T.

D.3.4 Proof of Proposition 3

Part (i): $s^{D*,Nash}(\lambda)$ is strictly increasing in λ. Denote the welfare bracket (intra-regional policy channel) by $\mathcal{B}^W(s^D) \equiv \Gamma_1(T)\frac{\partial u_i(t=1)}{\partial s_i^D}\big|_{L,H} + \frac{\Gamma_2(T)}{1+g^*}\frac{\partial g_i^*}{\partial s_i^D}\big|_{L,H}$ and the tournament bracket (full growth differential) by $\mathcal{B}^T(s^D) \equiv \frac{\partial g^*}{\partial s^D}\big|_{L,H} + 2\frac{\partial g^*}{\partial L_i}\frac{\partial L_i}{\partial s_i^D} + 2\frac{\partial g^*}{\partial H_i^D}\frac{\partial H_i^D}{\partial s_i^D}$, both evaluated at the symmetric point. The FOC is $F(s^D, \lambda) \equiv (1-\lambda)\mathcal{B}^W + \lambda\mathcal{B}^T = 0$. By the implicit function theorem:

$$\frac{ds^{D*,Nash}}{d\lambda} = -\frac{\partial F/\partial \lambda}{\partial F/\partial s^D}.$$

The denominator $\partial F/\partial s^D < 0$ under standard second-order conditions. The numerator:

$$\frac{\partial F}{\partial \lambda} = -\mathcal{B}^W + \mathcal{B}^T.$$

At the Nash equilibrium, $F = 0$ implies $(1-\lambda)\mathcal{B}^W = -\lambda\mathcal{B}^T$. By Eq. (D13), $\mathcal{B}^T$ consists of the intra-regional policy effect plus the two factor-grabbing terms Eq. (D14)–Eq. (D15). The talent-grabbing term $(1+g^*)/[(1-\eta)(1+s^D)]$ is strictly positive, and the labor-grabbing term is positive whenever $\phi(\sigma-1)\beta(1+s^D)/\sigma<\eta$, i.e., away from the budget limit. When this condition holds, $\mathcal{B}^T>0$, so $\mathcal{B}^W<0$ at Nash and $\partial F/\partial\lambda=-\mathcal{B}^W+\mathcal{B}^T>0$. Therefore:

$$\frac{ds^{D*,Nash}}{d\lambda}=-\frac{(+)}{(-)}>0.$$

Hence $s^{D*,Nash}(\lambda)$ is **strictly increasing** in λ.

Parts (ii)–(iii): Inverted-U welfare. We prove these in three steps.

Step 1: Single-crossing of $\mathcal{W}(s^D)$ at $s^{D,SB}$. Existence of a positive maximizer $s^{D,SB}\in(0,\bar{s}^D)$ with $\bar{s}^D\equiv(1-\beta)/\beta$ (the budget-binding subsidy, $t_i\to 1^-$) follows from Section D.2. Change variables from s^D to Θ_L using Eq. (D9): since $d\Theta_L/ds^D>0$ on $[0,\bar{s}^D)$, the mapping is smooth and strictly increasing, with Θ_L ranging over $[\Theta_L^{LF},1)$ as s^D varies over $[0,\bar{s}^D)$. Substituting Eq. (C2) with $\varphi=\varphi(\Theta_L)$ (strictly decreasing via Eq. (13)), and collecting terms that do not depend on Θ_L into $\bar{\mathcal{W}}\equiv\frac{2(1+\rho)}{\rho}\left[\ln Z_i^M(t{=}1)+\eta\ln\frac{\bar{L}}{2}-\frac{1-\beta}{1-\sigma}\ln\big(\alpha+(1-\alpha)\tau^{1-\sigma}\big)\right]$:

$$\begin{aligned}\mathcal{W}(s^D(\Theta_L))=\bar{\mathcal{W}}+\frac{2(1+\rho)}{\rho}\big[&\eta\beta\ln\Theta_L+\eta(1-\beta)\ln(1-\Theta_L)+\beta\ln\chi^*(\Theta_L)\\&+(1-\beta)\ln(1-\varphi(\Theta_L))\big]+\frac{2(1+\rho)}{\rho^2}\ln(1+g^*(\Theta_L)).\end{aligned}$$

The labor-allocation term is strictly concave with maximum at $\Theta_L=\beta$; $\ln(1-\varphi)$ is strictly increasing; and χ^*,g^* are strictly increasing on $(\Theta_L^{LF},\beta]$ by Lemma 1. Boundary behavior $d\mathcal{W}/ds^D|_{s^D=0}>0$ (Section D.2) and $\mathcal{W}\to-\infty$ as $\Theta_L\to 1^-$, combined with the standard second-order condition, give single-crossing: $d\mathcal{W}/ds^D>0$ on $[0,s^{D,SB})$ and <0 on $(s^{D,SB},\bar{s}^D)$.

Step 2: $s^{D*,Nash}(\lambda=0)<s^{D,SB}<s^{D*,Nash}(\lambda\to 1^-)$. At $\lambda=0$, the Nash FOC reduces to $\mathcal{B}^W(s^D)=0$. Under symmetry ($L_i=\bar{L}/2$, $H_i^D=\bar{H}/2$), $\mathcal{B}^W$ and $d\mathcal{W}/ds^D$ share identical marginal objects and differ only in time weights: the SB planner uses $((1+\rho)/\rho,(1+\rho)/\rho^2)$, the official uses $(\Gamma_1(T),\Gamma_2(T))$ with $\Gamma_2(T)/\Gamma_1(T)<1/\rho$ strictly for every finite T. Since $\partial g^*/\partial s^D|_{L,H}>0$ on a neighborhood of LF, under-weighting growth lowers the zero of $\mathcal{B}^W$: $s^{D*,Nash}(\lambda=0)<s^{D,SB}$; positivity follows from $\mathcal{B}^W(s^D=0)>0$.

At $\lambda=1$, the FOC reduces to $\mathcal{B}^T(s^D)=0$ with

$$\mathcal{B}^T(s^D)=\underbrace{\left.\frac{\partial g^*}{\partial s^D}\right|_{L,H}}_{\text{intra-regional policy}}+\underbrace{\frac{1+g^*}{(1-\eta)(1+s^D)}}_{\text{talent grabbing}}+\underbrace{\frac{\eta(1-\xi)(1+g^*)}{1+s^D}\left[\frac{1}{1-\eta}-\frac{1}{1-\phi(\sigma-1)\beta(1+s^D)/\sigma}\right]}_{\text{labor grabbing}}.$$

The talent-grabbing term is strictly positive on $[0,\bar{s}^D)$, and the labor-grabbing term is strictly positive whenever $\phi(\sigma-1)\beta(1+s^D)/\sigma<\eta$. Since $1+s^D<1/\beta$ on $[0,\bar{s}^D)$, a sufficient condition is $\phi(\sigma-1)/\sigma<\eta$, which guarantees $\mathcal{B}^T(s^{D,SB})>0$. As $s^D\to\bar{s}^{D-}$, $t_i\to 1^-$ and $\varphi\to 0^+$ via Eq. (13), so the direct-policy term drives $\mathcal{B}^T\to-\infty$. By continuity and the intermediate value theorem, $\mathcal{B}^T$ has a unique zero $\bar{s}^{D,\text{tour}}\in(s^{D,SB},\bar{s}^D)$, hence $s^{D*,Nash}(\lambda\to 1^-)=\bar{s}^{D,\text{tour}}>s^{D,SB}$.

Step 3: Existence of λ^{peak} and conclusion. By part (i) and Step 2, $s^{D*,Nash}(\cdot)$ is continuous and strictly increasing on $(0,1)$ with $s^{D*,Nash}(\lambda=0) < s^{D,SB} < s^{D*,Nash}(\lambda \to 1^-)$, so the intermediate value theorem yields a unique $\lambda^{peak} \in (0,1)$ with $s^{D*,Nash}(\lambda^{peak}) = s^{D,SB}$. Since aggregate welfare depends on λ only through the equilibrium subsidy, $\mathcal{W}^{Nash}(\lambda^{peak}) = \mathcal{W}(s^{D,SB}) = \mathcal{W}^{SB}$, giving (iii). For (ii), the chain rule

$$\frac{d\mathcal{W}^{Nash}}{d\lambda} = \frac{d\mathcal{W}}{ds^D}\bigg|_{s^{D*,Nash}(\lambda)} \cdot \frac{ds^{D*,Nash}}{d\lambda},$$

combined with Step 1 and part (i), gives $d\mathcal{W}^{Nash}/d\lambda > 0$ on $(0, \lambda^{peak})$ and < 0 on $(\lambda^{peak}, 1)$. □

Appendix E Robustness of Numerical Results

This appendix reports robustness checks for Section 5. Each of twelve key parameters (ρ, σ, α, τ, ϕ, η, β, ξ, $\chi^{*,LF}$, $g^{*,LF}$, $\bar{H}$, and T) is perturbed by $\pm 20\%$ from its baseline value. The government tenure T applies only to the China calibration (24 perturbations from 12 parameters); the EU calibration, which lacks a tournament mechanism, uses 22 perturbations from 11 parameters. For each perturbation, the scale parameters (κ^D, κ^M) are jointly re-calibrated to match the laissez-faire targets ($g^{*,LF}$ and $\chi^{*,LF}$). To summarize the sensitivity of the main results to all perturbations simultaneously, each figure plots the baseline curve together with an envelope (shaded band) that covers the full range of perturbed outcomes at every grid point.

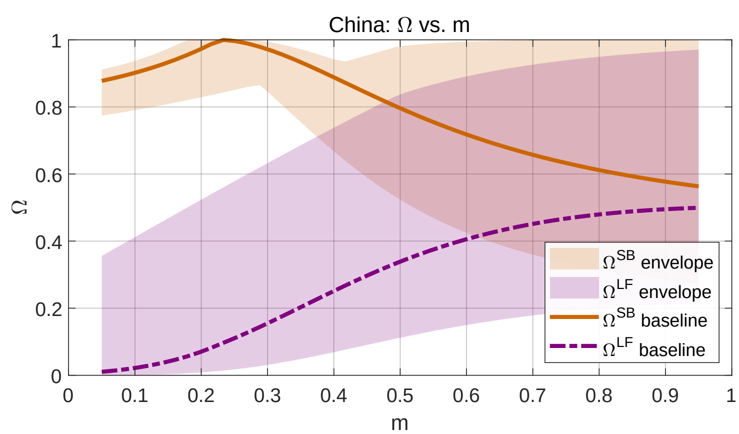

Figure E.1: Robustness of $\Omega^{SB}(m)$ and $\Omega^{LF}(m)$ under the China calibration.
Note: Solid lines: baseline. Shaded bands: envelope covering all 24 perturbations ($\pm 20\%$ of twelve parameters). Orange: Ω^{SB}. Purple: Ω^{LF}.

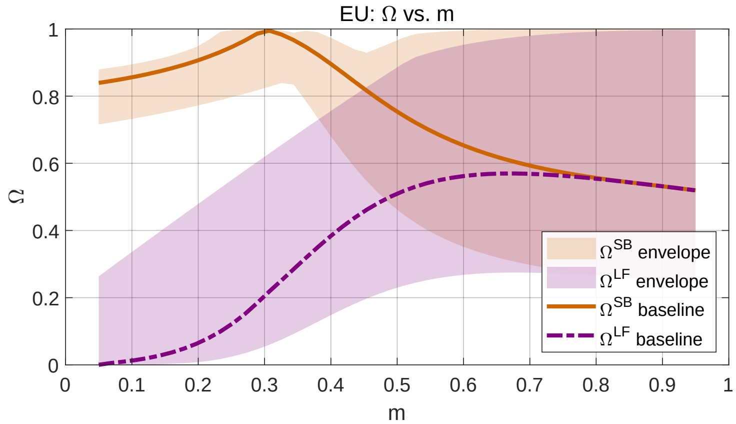

Figure E.2: Robustness of $\Omega^{SB}(m)$ and $\Omega^{LF}(m)$ under the EU calibration.
Note: Solid lines: baseline. Shaded bands: envelope covering all 22 perturbations (±20% of eleven parameters). Orange: Ω^{SB}. Purple: Ω^{LF}.

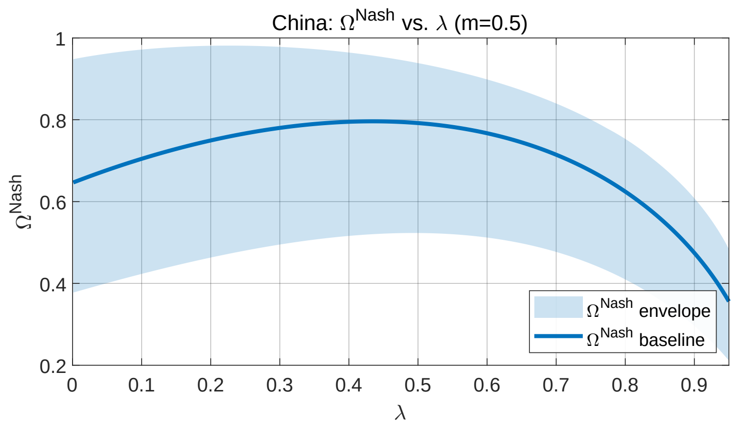

Figure E.3: Robustness of the inverted-U relationship $\Omega^{Nash}(\lambda)$ under the China calibration ($m = 0.50$).
Note: Solid line: baseline. Shaded band: envelope covering all 24 perturbations (±20% of twelve parameters).

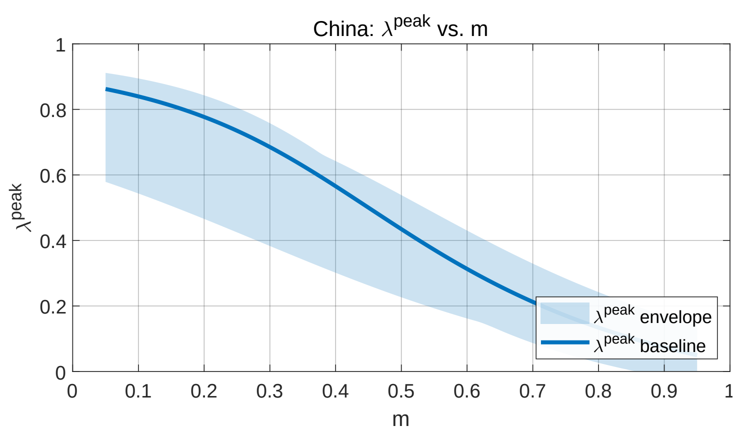

Figure E.4: Robustness of the optimal tournament intensity $\lambda^{peak}(m)$ under the China calibration.
Note: Solid line: baseline. Shaded band: envelope covering all 24 perturbations (±20% of twelve parameters).