From Business Cycle Accounting to Money and Banking

Shengxing Zhang

Introduction

- Most modern dynamic macro models have at their core a prototypical real business cycle model
- Different frictions, adjustment costs, and shocks represent deviations from the basic RBC framework such as adjustment costs, and shocks.
- Chari, Kehoe, and McGrattan (2007, Econometrica) develop a methodology which they call "business cycle accounting."
- They start with a basic one sector real business cycle model, and introduce four exogenous stochastic variables which they call wedges.
- These wedges are purely meant as reduced form accounting devices – the wedges could emerge because of exogenous shocks, or because of some friction or adjustment cost which means that the basic RBC model is mis-specified.
- We will use it as a framework to think how microfounded models affect aggregate dynamics.

The standard RBC model

- A representative household who consumes, supplies labor and owns the capital stock.
- Preference of the household: $\sum_{t=0}^{\infty} \beta^t \cdot u(C_t, N_t)$, $u(C_t, N_t) = U(C_t) - v(N_t)$, $0 < \beta < 1$.
- The household owns a technology to convert consumption goods into investment goods one-to-one. With I_t units of investment at period t,

$$K_{t+1} = (1-\delta)K_t + I_t.$$

  where δ is the depreciation rate on capital.
- A representative firm with Cobb-Douglas production function. Output at period t,

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

  Technology shock: $\ln A_{t+1} = \rho \ln A_t + \varepsilon_t$, $|\rho| < 1$, $\varepsilon_t \sim N(0, \sigma^2)$.
- The household is the equity holder of the firm.
- Two competitive markets. Labor market with wage rate w_t. Capital market with rental rate R_t.

The household's problem

- The budget constraint of the household is

$$C_t + K_{t+1} - (1-\delta)K_t \leq w_t N_t + R_t K_t + \Pi_t$$

- Given K_0, the household's optimization problem is

$$\max_{C_t, N_t, K_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - v(N_t) \right]$$

subject to the budget constraints for all t. The corresponding Lagrangian is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - v(N_t) \right.$$
$$\left. + \lambda_t \left(w_t N_t + R_t K_t + \Pi_t - C_t - K_{t+1} + (1-\delta)K_t \right) \right]$$

The household's problem

- The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \rightarrow U'(C_t) = \lambda_t$$
$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \rightarrow v'(N_t) = \lambda_t w_t$$
$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \rightarrow \lambda_t = \beta E_t \lambda_{t+1}(R_{t+1} + (1-\delta))$$

They can be combined to yield

$$v'(N_t) = U'(C_t) w_t$$
$$U'(C_t) = \beta E_t U'(C_{t+1})(R_{t+1} + (1-\delta))$$

The firm's problem

The firm chooses capital and labor demand given the wage rate w_t, the rental rate, R_t.

$$\max_{N_t, K_t} \mathbb{E}_0 \sum_{t=0}^{\infty} M_t \left(A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - R_t K_t \right)$$

where $M_t = \beta^t C_0 / C_t$ is the stochastic discount factor for the equity holder of the firm. The first order conditions are

$$MPK_t = A_t \alpha \left(\frac{N_t}{K_t} \right)^{1-\alpha} = R_t$$

$$MPL_t = A_t (1-\alpha) \left(\frac{K_t}{N_t} \right)^{\alpha} = w_t$$

Equilibrium analysis of the decentralized model

We can combine first order conditions from the firm and household problems to yield equilibrium conditions for C_t, N_t, K_{t+1}, Y_t, I_t, w_t, R_t, and A_t

$$
\begin{aligned}
U'(C_t) &= \beta E_t\left[U'(C_{t+1})\left(MPK_{t+1} + 1 - \delta\right)\right] \\
v'(N_t) &= U'(C_t)MPL_t \\
K_{t+1} &= I_t + (1-\delta)K_t \\
Y_t &= A_t K_t^{\alpha} N_t^{1-\alpha} \\
I_t &= Y_t - C_t \\
w_t &= MPL_t \\
R_t &= MPK_t \\
\ln A_t &= \rho \ln A_{t-1} + \varepsilon_t
\end{aligned}
$$

Introducing wedges

We now introduce wedges to household's budget constraint

$$C_t + (1+\tau_t^I)I_t \leq (1-\tau_t^N)w_t N_t + R_t K_t + \Pi_t - T_t$$

where $I_t = K_{t+1} - (1-\delta)K_t$, T_t is a lumpsum tax

- τ_t^I is like a tax on investment. It alters the relative price between consumption and investment. $1+\tau_t^I$: *the investment wedge*
- τ_t^N is a tax on labor income. $1-\tau_t^N$: *the labor wedge*
- The Lagrangian for the household is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[U(C_t) - v(N_t) + \lambda_t \Big((1-\tau_t^N) w_t N_t + R_t K_t + \Pi_t - C_t - (1+\tau_t^I)\left(K_{t+1} - (1-\delta)K_t\right) \Big) \Big]$$

The household's problem

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0 \rightarrow U'(C_t) = \lambda_t$$
$$\frac{\partial \mathcal{L}}{\partial N_t} = 0 \rightarrow v'(N_t) = \lambda_t(1-\tau_t^N)w_t$$
$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \rightarrow (1+\tau_t^I)\lambda_t = \beta E_t \lambda_{t+1}(R_{t+1} + (1-\delta)(1+\tau_{t+1}^I))$$

They can be combined to yield

$$v'(N_t) = \left(1-\tau_t^N\right) U'(C_t) w_t$$
$$\left(1+\tau_t^I\right) U'(C_t) = \beta E_t U'(C_{t+1}) \left[R_{t+1} + (1-\delta)(1+\tau_{t+1}^I)\right]$$

- A_t: *the efficiency wedge*
- G_t : *the government consumption wedge*
  The lump tax is chosen to balance the government's budget: $T_t = G_t - \tau_t^I I_t - \tau_t^N w_t N_t$.

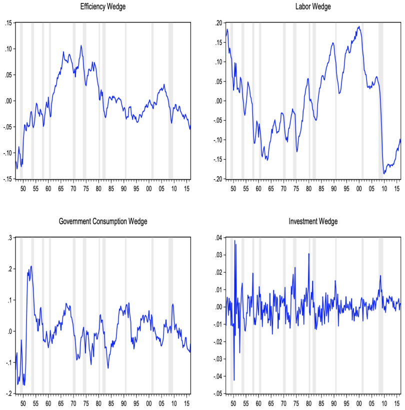
Equilibrium analysis of the decentralized model

Not counting the four exogenous wedges, the equilibrium conditions for C_t, N_t, K_{t+1}, Y_t, I_t, w_t, R_t are summarized by

$$
\begin{aligned}
(1+\tau_t^I)U'(C_t) &= \beta E_t\left[U'(C_{t+1})\left(MPK_{t+1} + (1-\delta)(1+\tau_{t+1}^I)\right)\right] \\
v'(N_t) &= \left(1-\tau_t^N\right)U'(C_t)MPL_t \\
K_{t+1} &= I_t + (1-\delta)K_t \\
Y_t &= A_t K_t^\alpha N_t^{1-\alpha} \\
I_t &= Y_t - C_t - G_t \\
w_t &= MPL_t \\
R_t &= MPK_t \\
\ln A_t &= \rho \ln A_{t-1} + \varepsilon_t
\end{aligned}
$$

Measuring the wedges in the data

Collect data on output, the capital stock, consumption, investment, and labor hours. And assume $\beta = 0.99$, $\delta = 0.02$, $\alpha = 1/3$, and $\chi = 1$.



Interpreting the wedges

- The labor and investment wedges do not have a clear interpretation as exogenous shocks. These could reflect exogenous shocks, or they could represent some mis-specification of the model along some dimension. Since the publication of their paper, there has been a substantial amount of research aimed at explaining the labor wedge.
- An alternative explanation of the labor wedge is simply a shock to the disutility from labor, $\nu_t \theta N_t^{\chi} = \frac{1}{C_t} w_t$. ν_t is isomorphic to a time-varying tax on labor income. Precisely because the model-implied labor wedge moves around so much in the data, papers which seek to formally estimate the stochastic properties of different structural shocks very often find that labor supply shocks are very important drivers of the business cycle.
- The more provocative claim in CKM's paper is that the lack of importance of the investment wedge means that research focusing on financial shocks and frictions is not likely to be a fruitful avenue for future research. Many feel that this claim is too strong, and it is not difficult to write down a model with a type of financial constraint that manifests directly as a labor wedge

The goal of this section is to establish equivalence results between a given detailed economy and the prototype one-sector growth model with wedges

Consider several detailed economies

- Money: cash-in-advance constraint
- Bank collateral constraints

Monetary model – Cash-in-advance constraint

- A representative household chooses consumption, labour supply, and money holding to maximize

$$\sum_{t=1}^{\infty} \beta^t \left[U(C_t) - v(N_t)\right]$$

subject to

$$W_t N_t + M_t \geq P_t C_t + M_{t+1}$$

and a cash-in-advance (CIA) constraint

$$M_t \geq P_t C_t$$

- Production function: $f(N_t) = A_t N_t$ so $MPL_t = A_t$

$$\left.\begin{array}{rl} \beta U'(C_{t+1}) \frac{W_t}{P_{t+1}} & = v'(N_t) \\ W_t & = A_t P_t \end{array}\right\} \Rightarrow {\color{red}(1 - \tau_t^N)} U'(c_t) A_t = v'(n_t), \text{ where } \tau_t^N = 1 - \frac{\frac{\beta u'(c_{t+1})}{u'(c_t)}}{\frac{P_{t+1}}{P_t}}$$

Bank collateral constraints

- Consider an economy populated by a household of workers and bankers. Each type is of total measure 1.
- The worker supply labor and return their wages to the household.
- Each banker manage a bank that transfers nonnegative dividends to the household.
- The family share consumption risks. So the family as a whole as preference

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

Given the initial asset holdings of bankers and workers, b_{H0}, and d_0. The household chooses C_t, N_t, D_{t+1} subject to the budget constraint

$$C_t + E_t q_{t+1} d_{t+1} + (1-\sigma)\sigma^{-1}\bar{n} \leq w_t N_t + d_t + X_t$$

where $d_{t+1} \geq \bar{d}$ denotes deposits, q_{t+1} is the price of the deposit, $\bar{d} < 0$ is large. X_t is the dividend paid by banks, $\bar{n}$ is initial equity given to each newly formed bank, $\frac{1-\sigma}{\sigma}$ is the measure of newly formed banks.

Household's problem

The Lagrangian for the household is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[U(C_t) - v(N_t) + \lambda_t \big(w_t N_t + d_t + X_t \\ - C_t - (1-\sigma)\sigma^{-1}\bar{n} - E_t q_{t+1} d_{t+1}\big)\Big]$$

From the first order conditions, we have (because $\bar{d}$ is negative enough, households

$$v'(N_t) = U'(C_t) w_t$$
$$q_{t+1} U'(C_t) = \beta \pi(s_{t+1}) U'(C_{t+1})$$

Banks' problem

At the beginning of each period, an idiosyncratic random shock is realized at each existing bank. With probability σ , the bank will continue in operation until the next period. With probability $1-\sigma$, the bank ceases to exist, and pays out all of its accumulated net worth to the households. Having banks die is a simple way to ensure that they do not build up enough equity.
Bankers' budget constraint (denote $\widetilde{R} = R + 1 - \delta$):

$$x_t + k_{t+1} - d_{t+1} \leq \tilde{R}_t(s^t)k_t - d_t \equiv n_t$$

subject to a collateral constraint for each s_{t+1},

$$d_{t+1} \leq \gamma \widetilde{R}_{t+1} k_{t+1}$$

and $x_t \geq 0$.

Banks' problem

The bank chooses $\{k_{t+1}, d_{t+1}, x_t\}$ to maximize

$$E_0 \sum_{t=0}^{\infty} q_t(\sigma x_t + (1-\sigma)n_t)$$

subject to the budget constraint, collateral constraint and the constraint that dividend payment cannot be negative, where M_t is the stochastic discount factor of the household.

$$\begin{aligned}\mathcal{L} = E_0 \sum_{t=0}^{\infty} \{ & q_t(\sigma x_t + (1-\sigma)n_t) \\ & + \lambda_t[\tilde{R}_{t+1}k_t - d_t - x_t - k_{t+1} + m_{t+1}d_{t+1}] \\ & + \mu_{t+1}\left(\gamma \tilde{R}_{t+1}k_{t+1} - d_{t+1}\right) + \eta_{xt}x_t \}\end{aligned}$$

where $n_t = \tilde{R}_t k_t - d_t$, $m_{t+1} = \frac{q_{t+1}}{q_t} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$

Banks' problem

The first order conditions are

$$\begin{aligned} k_{t+1} :& \lambda_t = [q_{t+1}(1-\sigma) + \lambda_{t+1} + \gamma\mu_{t+1}]\,[R_{t+1} + (1-\delta)] \\ d_{t+1} :& \beta\frac{u'(C_{t+1})}{u'(C_t)}\lambda_t = q_{t+1}(1-\sigma) + \lambda_{t+1} + \mu_{t+1} \\ x_t :& \lambda_t = q_t\sigma + \eta_{xt} \end{aligned}$$

$$1 = \beta\frac{u'(C_{t+1})}{u'(C_t)}\,[R_{t+1} + (1-\delta)]\left[1 - \frac{(1-\gamma)\mu_{t+1}}{q_{t+1}(1-\sigma) + \lambda_{t+1} + \mu_{t+1}}\right]$$

Credit frictions impose a wedge on the *gross* return of capital:

$$1 = E_t\beta\frac{u'(C_{t+1})}{u'(C_t)}\,(R_{t+1} + (1-\delta))\,(1-\tau^K_{t+1})$$

In the benchmark BCA, the wedge on investment is on the net return, kind of:

$$1 = E_t\beta\frac{u'(C_{t+1})}{u'(C_t)}\left(\frac{1}{1+\tau^I_t}R_{t+1} + (1-\delta)\frac{1+\tau^I_{t+1}}{1+\tau^I_t}\right)$$