2021 秋季本科时间序列

第 7 次作业答案

12 月 06 日

1. 对于 AR(2, 1)

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

$$(1 - \phi_1 \mathscr{L} - \phi_2 \mathscr{L}^2) X_t = (1 + \theta_1 \mathscr{L}) \varepsilon_t$$

$$A(\mathscr{L}) X_t = B(\mathscr{L}) \varepsilon_t$$

$$X_t = A(\mathscr{L})^{-1} B(\mathscr{L}) \varepsilon_t$$

则 $C(z) = A(z)^{-1}B(z) = \frac{1+\theta_1 z}{1-\phi_1 z - \phi_2 z^2}$

$$f_x(\omega) = G_(\omega) \cdot 1 = \sqrt{C(e^{\mathrm{i}\omega})C(e^{-\mathrm{i}\omega})} = \sqrt{\frac{B(e^{\mathrm{i}\omega})B(e^{-\mathrm{i}\omega})}{A(e^{\mathrm{i}\omega})A(e^{-\mathrm{i}\omega})}}$$

$$\begin{aligned}
S_x(\omega) &= f_x^2(\omega) S_\varepsilon(\omega) \\
&= \frac{1}{2\pi} f_x^2(\omega) \\
&= \frac{1}{2\pi} \frac{B(e^{\mathrm{i}\omega})B(e^{-\mathrm{i}\omega})}{A(e^{\mathrm{i}\omega})A(e^{-\mathrm{i}\omega})} \\
&= \frac{1}{2\pi} \frac{(1+\theta_1 e^{\mathrm{i}\omega})(1+\theta_1 e^{-\mathrm{i}\omega})}{(1-\phi_1 e^{\mathrm{i}\omega} - \phi_2 e^{2\mathrm{i}\omega})(1-\phi_1 e^{-\mathrm{i}\omega} - \phi_2 e^{-2\mathrm{i}\omega})} \\
&= \frac{1}{2\pi} \frac{(1+\theta_1^2) + 2\theta_1 \cos\omega}{1 + \phi^2 + \phi_2^2 - 2\phi_1(1-\phi_2)\cos\omega - 2\phi_2 \cos 2\omega}
\end{aligned}$$

带入 $\phi_1 = 1.3, \phi_2 = -0.4, \theta_1 = 0.6$ 可得

$$S_x(\omega) = \frac{1}{2\pi} \frac{1.36 + 1.2\cos\omega}{2.85 - 3.64\cos\omega + 0.8\cos 2\omega}$$

对于 AR(2)

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t$$

$$(1 - \phi_1 \mathscr{L} - \phi_2 \mathscr{L}^2) X_t = \varepsilon_t$$

$$A(\mathscr{L}) X_t = \varepsilon_t$$

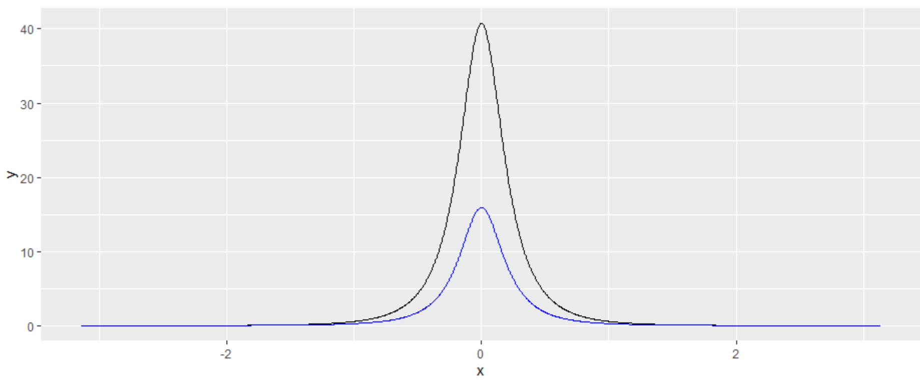
$$f_x(\omega)=\sqrt{\frac{1}{A(e^{i\omega})A(e^{-i\omega})}}=\frac{1}{1+\phi^2+\phi_2^2-2\phi_1(1-\phi_2)\cos\omega-2\phi_2\cos 2\omega}$$

带入 $\phi_1=1.3,\phi_2=-0.4$, 可得

$$S_x(\omega)=\frac{1}{2\pi}\frac{1}{2.85-3.64\cos\omega+0.8\cos 2\omega}$$

代码如下：

```
library (tidyverse)
x<-seq(-pi,pi,0.005)
y<-(0.6*cos(x)+0.68)/(pi*(2.85-3.64*cos(x)+0.8*cos(2*x)))
z<-1/(2*pi*(2.85-3.64*cos(x)+0.8*cos(2*x)))
num<-tibble(x,y,z)
ggplot(data=num)+
geom_line(mapping = aes(x=x,y=y))+
geom_line(mapping=aes(x=x,y=z),color="blue")
```



2. $X_t=\phi_1X_{t-1}+\phi_2X_{t-2}+\varepsilon_t$, 其中 $\phi_1=1.3,\phi_2=-0.4$

$$\begin{bmatrix}X_{t+1}\\X_t\end{bmatrix}=\begin{bmatrix}\phi_1&\phi_2\\0&0\end{bmatrix}\begin{bmatrix}X_t\\X_{t-1}\end{bmatrix}+\begin{bmatrix}\varepsilon_{t+1}\\0\end{bmatrix}$$

对任意的 $s\geq 2$

$$\begin{bmatrix}\hat{X}_{t+s|t}\\\hat{X}_{t+s-1|t}\end{bmatrix}=\begin{bmatrix}\phi_1&\phi_2\\1&0\end{bmatrix}\begin{bmatrix}\hat{X}_{t+s-1|t}\\\hat{X}_{t+s-2|t}\end{bmatrix}=\boldsymbol{A}\begin{bmatrix}\hat{X}_{t+s-1|t}\\\hat{X}_{t+s-2|t}\end{bmatrix}=\boldsymbol{A}^s\begin{bmatrix}\hat{X}_{t|t}\\\hat{X}_{t-1|t}\end{bmatrix}=\boldsymbol{A}^s\begin{bmatrix}X_t\\X_{t-1}\end{bmatrix}$$

则

$$det(\boldsymbol{A}-\lambda\boldsymbol{I})=det\begin{bmatrix}\phi_1-\lambda&\phi_2\\1&-\lambda\end{bmatrix}=\lambda^2-\phi_1\lambda-\phi_2=\lambda^2-1.3\lambda+0.4=0$$

解得 $\lambda_1=0.8,\lambda_2=0.5$

$$(\boldsymbol{A}-\lambda_1\boldsymbol{I})\boldsymbol{C}_1=\begin{bmatrix}\phi_1-\lambda_1&\phi_2\\1&-\lambda_1\end{bmatrix}\boldsymbol{C}_1=0\quad \boldsymbol{C}_1=\begin{bmatrix}\lambda_1\\1\end{bmatrix}$$

$$(\boldsymbol{A}-\lambda_2\boldsymbol{I})\boldsymbol{C}_2=\begin{bmatrix}\phi_1-\lambda_2 & \phi_2\\ 1 & -\lambda_2\end{bmatrix}\boldsymbol{C}_2=0 \quad \boldsymbol{C}_2=\begin{bmatrix}\lambda_2\\ 1\end{bmatrix}$$

因此

$$\boldsymbol{C}=\begin{bmatrix}\boldsymbol{C}_1 & \boldsymbol{C}_2\end{bmatrix}=\begin{bmatrix}\lambda_1 & \lambda_2\\ 1 & 1\end{bmatrix}$$

$$\boldsymbol{C}^{-1}=\frac{\boldsymbol{C}^*}{det\boldsymbol{C}}=\frac{1}{\lambda_1-\lambda_2}\begin{bmatrix}1 & \lambda_2\\ -1 & \lambda_1\end{bmatrix}$$

$$\boldsymbol{A}^s=\boldsymbol{C}\begin{bmatrix}\lambda_1^s & 0\\ 0 & \lambda_2^s\end{bmatrix}\boldsymbol{C}^{-1}$$

$$\begin{bmatrix}\hat{X}_{t+s|t}\\ \hat{X}_{t+s-1|t}\end{bmatrix}=\boldsymbol{A}^s\begin{bmatrix}X_t\\ X_{t-1}\end{bmatrix}=\frac{1}{\lambda_1-\lambda_2}\begin{bmatrix}\lambda_1^{s+1}-\lambda_2^{s+1} & -\lambda_1^{s+1}\lambda_2+\lambda_1\lambda_2^{s+1}\\ \lambda_1^s-\lambda_2^s & -\lambda_1^s\lambda_2+\lambda_1\lambda_2^s\end{bmatrix}\begin{bmatrix}X_t\\ X_{t-1}\end{bmatrix}$$

因此 $\hat{X}_{t+s|t}=\frac{1}{\lambda_1-\lambda_2}[(\lambda_1^{s+1}-\lambda_2^{s+1})X_t+(-\lambda_1^s\lambda_2+\lambda_1\lambda_2^s)X_{t-1}]$，其中 $\lambda_1=0.8,\lambda_2=0.5$

又 $|\lambda_1|<1,|\lambda_2|<1$

所以 $\lim_{s\to\infty}\lambda_1^s=0,\lim_{s\to\infty}\lambda_2^s=0$

$$\lim_{s\to\infty}\boldsymbol{A}^s=\lim_{s\to\infty}\boldsymbol{C}\begin{bmatrix}\lambda_1^s & 0\\ 0 & \lambda_2^s\end{bmatrix}\boldsymbol{C}^{-1}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\lim_{s\to\infty}\begin{bmatrix}\hat{X}_{t+s|t}\\ \hat{X}_{t+s-1|t}\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

即 $\lim_{s\to\infty}\hat{X}_{t+s|t}=0$

3. 已知：$\begin{bmatrix}X_{t+1}\\ X_t\end{bmatrix}=\boldsymbol{A}\begin{bmatrix}X_t\\ X_{t-1}\end{bmatrix}+\begin{bmatrix}\varepsilon_{t+1}\\ 0\end{bmatrix}$

则对任意的 $s\geq 2$

$$\begin{bmatrix}X_{t+s}\\ X_{t+s-1}\end{bmatrix}=\boldsymbol{A}^s\begin{bmatrix}X_t\\ X_{t-1}\end{bmatrix}+\begin{bmatrix}\varepsilon_{t+s}\\ 0\end{bmatrix}+\boldsymbol{A}\begin{bmatrix}\varepsilon_{t+s-1}\\ 0\end{bmatrix}+\cdots+\boldsymbol{A}^{s-1}\begin{bmatrix}\varepsilon_{t+1}\\ 0\end{bmatrix}$$

根据上题可知

$$\begin{bmatrix}\hat{X}_{t+s|t}\\ \hat{X}_{t+s-1|t}\end{bmatrix}=\boldsymbol{A}^s\begin{bmatrix}X_t\\ X_{t-1}\end{bmatrix}$$

因此可得

$$\begin{bmatrix}X_{t+s}-\hat{X}_{t+s|t}\\ X_{t+s-1}-\hat{X}_{t+s-1|t}\end{bmatrix}=\begin{bmatrix}\varepsilon_{t+s}\\ 0\end{bmatrix}+\boldsymbol{A}\begin{bmatrix}\varepsilon_{t+s-1}\\ 0\end{bmatrix}+\cdots+\boldsymbol{A}^{s-1}\begin{bmatrix}\varepsilon_{t+1}\\ 0\end{bmatrix}$$

带入 $\boldsymbol{A}$ 表达式可得：

$$X_{t+s}-\hat{X}_{t+s|t}=\varepsilon_{t+s}+\frac{1}{\lambda_1-\lambda_2}(\lambda_1^{1+1}-\lambda_2^{1+1})\varepsilon_{t+1}+\cdots+\frac{1}{\lambda_1-\lambda_2}(\lambda_1^{s-1+1}-\lambda_2^{s-1+1})\varepsilon_{t+1}$$

由于各期 ε 不相关

$$\begin{aligned}
\mathbb{E}[X_{t+s}-\hat{X}_{t+s|t}]^2&=\sigma_\varepsilon^2\big[1+\frac{1}{(\lambda_1-\lambda_2)^2}(\lambda_1^{1+1}-\lambda_2^{1+1})+\cdots+\frac{1}{(\lambda_1-\lambda_2)^2}(\lambda_1^{s-1+1}-\lambda_2^{s-1+1})\big]\\
&=\frac{\sigma_\varepsilon^2}{(\lambda_1-\lambda_2)^2}\big[(\lambda_1^{0+1}-\lambda_2^{0+1})^2+(\lambda_1^{1+1}-\lambda_2^{1+1})^2+\cdots+(\lambda_1^{s-1+1}-\lambda_2^{s-1+1})^2\big]\\
&=\frac{\sigma_\varepsilon^2}{(\lambda_1-\lambda_2)^2}\big[(\lambda_1^{0+1})^2+\cdots+(\lambda_1^{s})^2+(\lambda_2^{0+1})^2+\cdots+(\lambda_2^{s})^2-2\big((\lambda_1\lambda_2)^1+\cdots+(\lambda_1\lambda_2)^s\big)\big]
\end{aligned}$$

$$\lim_{s\to\infty}\mathbb{E}[X_{t+s}-\hat{X}_{t+s|t}]^2=\frac{\sigma_\varepsilon^2}{(\lambda_1-\lambda_2)^2}\big[\frac{\lambda_1^2}{1-\lambda_1^2}+\frac{\lambda_2^2}{1-\lambda_2^2}-\frac{2\lambda_1\lambda_2}{1-\lambda_1\lambda_2}\big]$$

计算 $\boldsymbol{A}$ 特征根时得 $\lambda^2-\phi_1\lambda-\phi_2=0$

所以 $\lambda_1+\lambda_2=\phi_1$， $\lambda_1\lambda_2=-\phi_2$， $\lambda_1^2+\lambda_2^2=\phi_1^2+2\phi_2$， $(\lambda_1-\lambda_2)^2=\phi_1^2+4\phi_2$

$$\begin{aligned}
\lim_{s\to\infty}\mathbb{E}[X_{t+s}-\hat{X}_{t+s|t}]^2&=\frac{\sigma_\varepsilon^2}{(\lambda_1-\lambda_2)^2}\big[\frac{\lambda_1^2}{1-\lambda_1^2}+\frac{\lambda_2^2}{1-\lambda_2^2}-\frac{2\lambda_1\lambda_2}{1-\lambda_1\lambda_2}\big]\\
&=\frac{\sigma_\varepsilon^2}{\phi_1^2+4\phi_2}\big[\frac{\lambda_1^2(1-\lambda_2^2)(1+\phi_2)+\lambda_2^2(1-\lambda_1^2)(1+\phi_2)+2\phi_2(1-\lambda_1^2)(1-\lambda_2^2)}{(1-\lambda_1^2)(1-\lambda_2^2)(1+\phi_2)}\big]\\
&=\frac{\sigma_\varepsilon^2}{\phi_1^2+4\phi_2}\frac{(\lambda_1^2-\phi_2^2)(1+\phi_2)+(\lambda_2^2-\phi_2^2)(1+\phi_2)+2\phi_2(1+\phi_2^2-\phi_1^2-2\phi_2)}{(1+\phi_2^2-\phi_1^2-2\phi_2)(1+\phi_2)}\\
&=\frac{\sigma_\varepsilon^2}{\phi_1^2+4\phi_2}\frac{\phi_1^2+4\phi_2-\phi_2(\phi_1^2+4\phi_2)}{(1+\phi_2^2-\phi_1^2-2\phi_2)(1+\phi_2)}\\
&=\sigma_\varepsilon^2\frac{1-\phi_2}{(1+\phi_2^2-\phi_1^2-2\phi_2)(1+\phi_2)}\\
&=\sigma_\varepsilon^2\frac{1-\phi_2}{(\phi_2+\phi_1-1)(\phi_2-\phi_1-1)(1+\phi_2)}\\
&=\mathrm{var}(X_t)
\end{aligned}$$

代入数据得

$$\lim_{s\to\infty}\mathbb{E}[X_{t+s}-\hat{X}_{t+s|t}]^2=\frac{700}{81}\sigma_\varepsilon^2$$

4. (a)

$$X_t=\varepsilon_t+\theta\varepsilon_{t-1}$$

$$X_{t+s}=\varepsilon_{t+s}+\theta\varepsilon_{t+s-1}$$

当 $s=1$ 时，$X_{t+1}=\varepsilon_{t+1}+\theta\varepsilon_t \quad \hat{X}_{t+1|t}=\theta\varepsilon_t$

当 $s=2$ 时，$X_{t+2}=\varepsilon_{t+2}+\theta\varepsilon_{t+1} \quad \hat{X}_{t+2|t}=0$

当 $s>2$ 时，不包含 t 及之前信息，有 $\hat{X}_{t+s|t}=0$

综上可得

$$\hat{X}_{t+s|t}=\begin{cases}\theta\varepsilon_t & s=1\\ 0 & s\geq 2\end{cases}$$

(b)

$$X_t = \varepsilon_t + \theta\varepsilon_{t-1}$$

两端同乘 X_t 取期望：$\gamma(0) = (1+\theta^2)\sigma_\varepsilon^2$

两端同乘 X_{t-1} 取期望：$\gamma(1) = \theta\sigma_\varepsilon^2$

当 $k \geq 2$ 时，$\gamma(k) = 0$

故

$$\gamma(k) = \begin{cases} (1+\theta^2)\sigma_\varepsilon^2 & k = 0 \\ \theta\sigma_\varepsilon^2 & k = 1 \\ 0 & k \geq 2 \end{cases}$$

当 $\sigma_\varepsilon^2 = 1$ 时，有

$$\gamma(k) = \begin{cases} 1+\theta^2 & k = 0 \\ \theta & k = 1 \\ 0 & k \geq 2 \end{cases}$$

则 θ 的样本矩估计量 $\hat{\theta} = \frac{1}{T}\sum[X_{t-1}X_t] \xrightarrow{a.s.} \gamma(1) = \theta$

故得证 $\hat{\theta}$ 具有一致性

(c) 由上问可知

$$\begin{cases} \gamma(0) = (1+\theta^2)\sigma_\varepsilon^2 \\ \gamma(1) = \theta\sigma_\varepsilon^2 \end{cases}$$

且 $|\theta| < 1$ 时序列平稳

解得

$$\begin{cases} \theta = \dfrac{\gamma(0) - \sqrt{\gamma^2(0) - 4\gamma^2(1)}}{2\gamma(1)} \\ \sigma_\varepsilon^2 = \dfrac{2\gamma^2(1)}{\gamma(0) - \sqrt{\gamma^2(0) - 4\gamma^2(1)}} \end{cases}$$

估计得到

$$\begin{cases} \hat{\theta} = \dfrac{\frac{1}{T}\sum X_t^2 - \sqrt{(\frac{1}{T}\sum X_t^2)^2 - 4(\frac{1}{T}\sum X_{t-1}X_t)^2}}{2\frac{1}{T}\sum X_{t-1}X_t} = \dfrac{\sum X_t^2 - \sqrt{(\sum X_t^2)^2 - 4(\sum X_{t-1}X_t)^2}}{2\sum X_{t-1}X_t} \\ \hat{\sigma}_\varepsilon^2 = \dfrac{2\frac{1}{T}(\sum X_{t-1}X_t)^2}{\sum X_t^2 - \sqrt{(\sum X_t^2)^2 - 4(\sum X_{t-1}X_t)^2}} \end{cases}$$

(d)

$$\mathbf{\Sigma} = \sigma_\varepsilon^2 \begin{bmatrix} (1+\theta^2) & \theta & 0 & \cdots & 0 \\ \theta & (1+\theta^2) & \theta & \cdots & \cdots \\ 0 & \theta & (1+\theta^2) & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & (1+\theta^2) \end{bmatrix}$$

$\boldsymbol{\Sigma}$ 为三对角矩阵，$det\boldsymbol{\Sigma} = \frac{\theta^{2(T+1)}-1}{\theta^2-1}\sigma_\varepsilon^2$

$$\log L(\theta, \sigma_\varepsilon^2|\boldsymbol{X}) = -\frac{T}{2}\log 2\pi - \frac{1}{2}\log det\boldsymbol{\Sigma} - \frac{1}{2}\boldsymbol{X}^\intercal\boldsymbol{\Sigma}^{-1}\boldsymbol{X}$$

5. (a) 根据课件中定义可知：X, Y 服从二元正态分布

$$Z = (Y - \mathbb{E}Y) - \frac{\text{cov}(X,Y)}{\text{var}(X)}(X - \mathbb{E}X), \quad \mathbb{E}Z = 0$$

$$\begin{aligned}
\text{cov}(Z,X) &= \mathbb{E}ZX - \mathbb{E}Z\mathbb{E}X \\
&= \mathbb{E}ZX \\
&= \mathbb{E}[(Y-\mathbb{E}Y)X - \frac{\text{cov}(X,Y)}{\text{var}(X)}(X-\mathbb{E}X)X] \\
&= \mathbb{E}(YX - \mathbb{E}YX) - \frac{\text{cov}(X,Y)}{\text{var}(X)}[\mathbb{E}(X^2) - (\mathbb{E}X)^2)] \\
&= \text{cov}(X,Y) - \frac{\text{cov}(X,Y)}{\text{var}(X)}\text{var}(X) \\
&= 0
\end{aligned}$$

故得证 X 与 Z 独立

(b)

$$\begin{aligned}
\mathbb{E}(Y - g(x))^2 &= \mathbb{E}[Y - (Y-Z) + (Y-Z) - g(X)]^2 \\
&= \mathbb{E}(Z + (Y - Z - g(X)))^2 \\
&= \mathbb{E}Z^2 + \mathbb{E}(Y - Z - g(X))^2 + 2\mathbb{E}[Z(Y - Z - g(X))] \\
&= \mathbb{E}Z^2 + \mathbb{E}(Y - Z - g(X))^2 + 2\mathbb{E}[Z(Y-Z)] - 2\mathbb{E}[Zg(X)]
\end{aligned}$$

由上问可知 X 与 Z 独立，且 $\mathbb{E}Z = 0$

因此 $\mathbb{E}[Zg(X)] = \mathbb{E}Z\mathbb{E}g(X) = 0$

$$\mathbb{E}[Z(Y-Z)] = \mathbb{E}[\mathbb{E}[Z(Y-Z)|X]] = (Y-Z)\mathbb{E}[Z|X] = (Y-Z)\mathbb{E}Z = 0$$

$$\mathbb{E}(Y - g(x))^2 = \mathbb{E}Z^2 + \mathbb{E}(Y - Z - g(X))^2 \geq \mathbb{E}Z^2 = \mathbb{E}[Y - (Y-Z)]^2$$

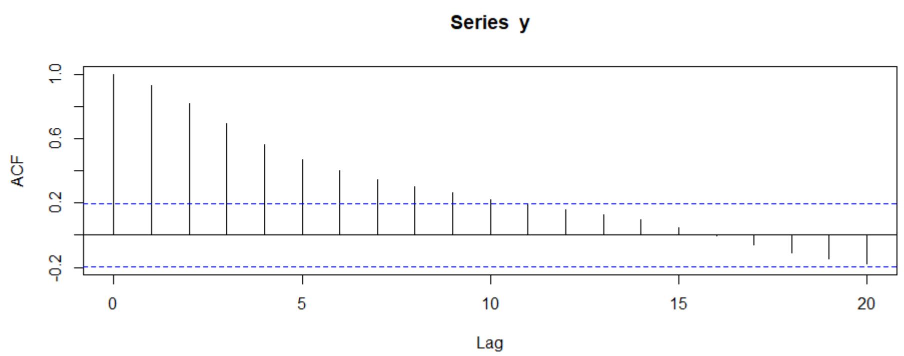
故得证 $Y - Z = \mathbb{E}(Y|X)$ 是 X 对 Y 的最小均方预测误差函数

(c) 已知 Z 与 X 独立、$Y-\mathbb{E}(Y|X)$ 与 X 独立且 $\mathbb{E}(Y|X)=Y-Z$

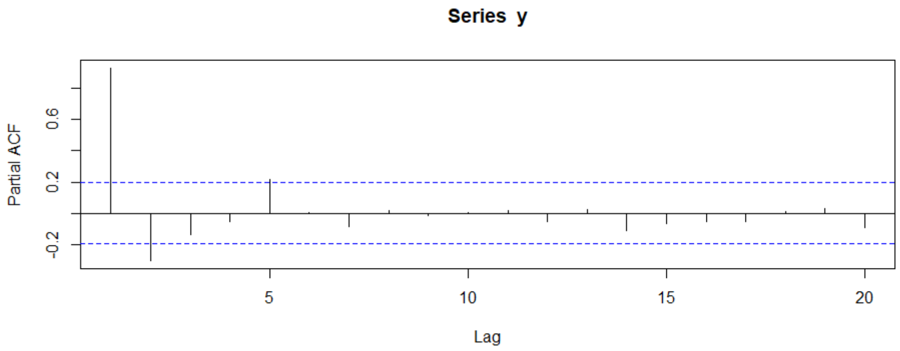
$$\begin{aligned}
\operatorname{var}(Y|X) &= \mathbb{E}\left[(Y-\mathbb{E}(Y|X))^2|X\right] \\
&= \mathbb{E}\left[Y-\mathbb{E}(Y|X)\right]^2 \\
&= \mathbb{E}\left[Y-\mathbb{E}Y+\mathbb{E}Y-\mathbb{E}(Y|X)\right]^2 \\
&= \mathbb{E}[Y-\mathbb{E}Y]^2+\mathbb{E}\left[\mathbb{E}Y-\mathbb{E}(Y|X)\right]^2+2\mathbb{E}\left[(Y-\mathbb{E}Y)(\mathbb{E}Y-\mathbb{E}(Y|X))\right] \\
&= \operatorname{var}(Y)+\mathbb{E}\left[\frac{\operatorname{cov}(X,Y)}{\operatorname{var}(X)}(X-\mathbb{E}X)\right]^2-2\mathbb{E}\left[(Y-\mathbb{E}Y)\frac{\operatorname{cov}(X,Y)}{\operatorname{var}(X)}(X-\mathbb{E}X)\right] \\
&= \operatorname{var}Y+\frac{\operatorname{cov}^2(X,Y)}{\operatorname{var}(X)}-2\frac{\operatorname{cov}^2(X,Y)}{\operatorname{var}(X)} \\
&= \operatorname{var}Y-\frac{\operatorname{cov}^2(X,Y)}{\operatorname{var}(X)}
\end{aligned}$$

6.

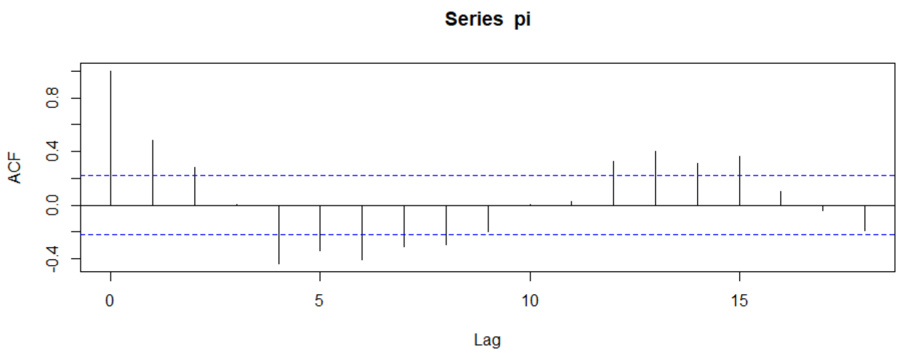
```
data<- read.csv("C:/Users/DELL/Desktop/TS07.csv")
data$rGDP<-data$NominalGDP/data$GDPDeflator
library(forecast)
#(a)
#构造GDP季节同比增速序列{y}
y<-vector()
for (i in 1:100)
{
y[i]=(data$rGDP[i+4]/data$rGDP[i]-1)*100
}
#进行建模估计
acf(y)#由图知, max.q=10
```



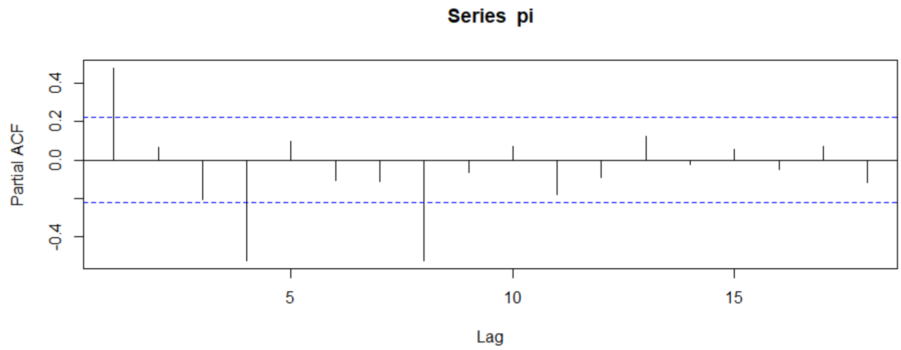

```
pacf(y)#由图知, max.p=2
```



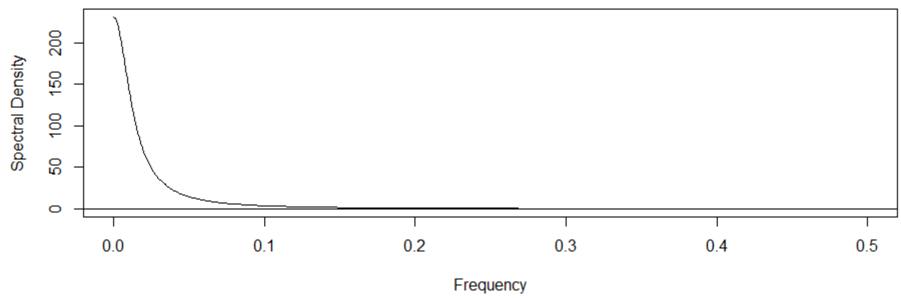

```
#最优ARMA滞后阶数， 并汇报估计结果
y_best<-auto.arima(y,d=0,D=0,seasonal=F,trace=TRUE,method="ML")
y_best#最优： ARIMA(1,0,2)
#(b)
#构造CPI季度同比增速序列{π}， 考虑97后的数据
pi<-vector()
for (i in 21:100)
{
pi[i-20]=(data$CPI[i+4]/data$CPI[i]-1)*100
}
pi<-diff(pi,1)
#进行建模估计
acf(pi)#由图知， max.q=14
```



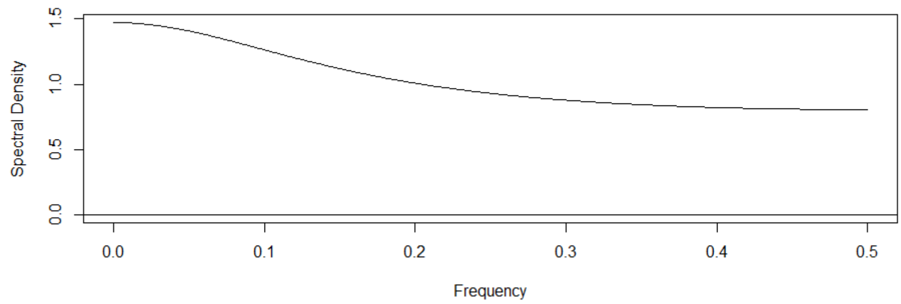

```
pacf(pi)#由图知， max.p=8
```



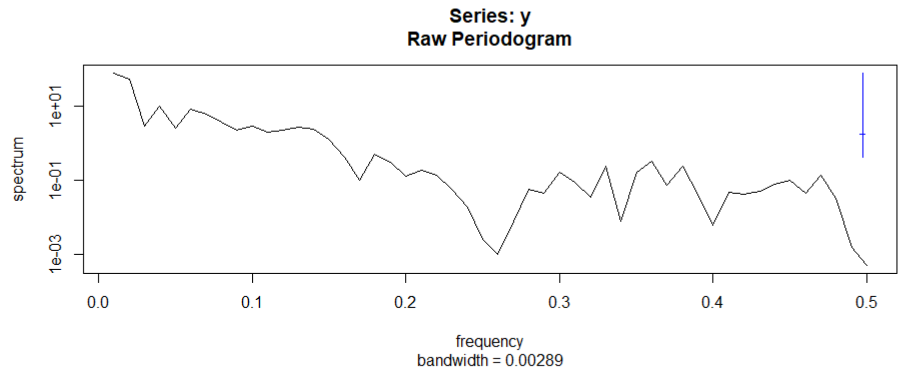

```
pi_best=auto.arima(pi,d=0,D=0,seasonal=F,method="ML")
pi_best#ARIMA(2,0,2)
#(c)
library(TSA)
ARMAspec(list(ar=y_best$coef[[1]],ma=y_best$coef[[2]]))
```



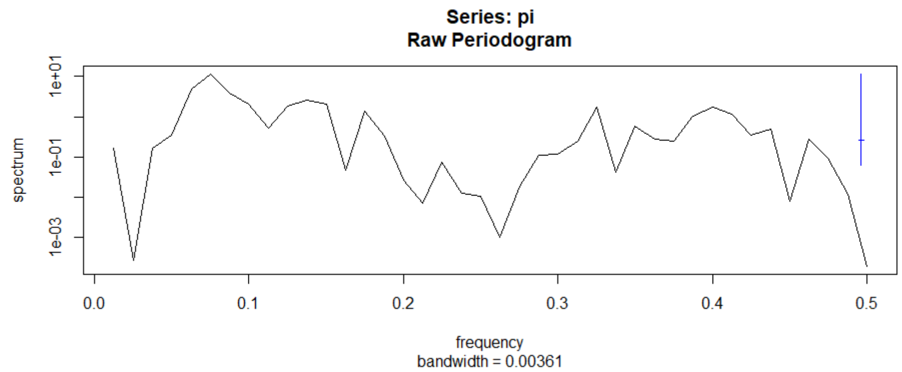

```
ARMAspec(list(ar=pi_best$coef[[1]],ma=pi_best$coef[[2]]))
```




```
spec.pgram(y)
```




```
spec.pgram(pi)
```




```
#理论谱密度和样本谱密度差别较大，
#估计得到的ARMA不能很好反映出序列的波动
#(d)
y_OLS<-auto.arima(y,max.p=4,max.q=0,d=0,D=0,seasonal=F,method="CSS")
y_OLS
y_ML<-arima(y,c(2,0,0),method = "ML")
y_ML

pi_OLS<-auto.arima(pi,max.p=2,max.q=0,d=0,D=0,seasonal=F,method="CSS")
pi_OLS
pi_ML<-arima(pi,c(1,0,0),method = "ML")
pi_ML
#OLS和极大似然法结果在数值上类似，但OLS的估计项数更多。
#e
#样本谱密度图像中y和pi均有明显的高频波动特征，
#因此需要加入MA项加以刻画。
```