2020 秋季本科时间序列

第 9 次作业答案

12 月 30 日

1. (a) $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + \varepsilon_t$, 其中 $\Phi_1 = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$,$\Phi_2 = \begin{pmatrix} d & e \\ 0 & f \end{pmatrix}$

   由$A(\mathscr{L}) = I - \Phi_1\mathscr{L} - \Phi_2\mathscr{L}^2$ 得 $A(\mathscr{L})X_t = \varepsilon_t$

   关于 $A(\mathscr{L})$ 的特征多项式为

$$\begin{aligned}\det[A(\mathscr{L})] &= \left|I - \Phi_1\mathscr{L} - \Phi_2\mathscr{L}^2\right| \\ &= \left|I - \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} Z - \begin{pmatrix} d & e \\ 0 & f \end{pmatrix} Z^2\right| \\ &= (1 - aZ - dZ^2)(1 - cZ - fZ^2) = 0\end{aligned}$$

   因为 X_t 序列平稳, 所以 $\det[A(\mathscr{L})] = 0$ 的零点都在单位圆外, 而方程中不含 b 和 e, 所以 X_t 序列的平稳性与 b 和 e 的取值无关

   (b) 将 (a) 中的方程展开可得

$$X_{1,t} = aX_{1,t-1} + dX_{1,t-2} + bX_{2,t-1} + cX_{2,t-2} + \varepsilon_{1,t} \tag{1}$$

$$X_{2,t} = cX_{2,t-1} + fX_{2,t-2} + \varepsilon_{2,t} \tag{2}$$

   对 (2) 式而言,$X_{2,t}$ 只受其自身滞后项 $X_{2,t-1}$ 和 $X_{2,t-2}$ 的影响, 所以 $X_{1,t}$ 不是 $X_{2,t}$ 的格兰杰原因

   对 (1) 式而言, 若 b=e=0 成立, 则 $X_{1,t}$ 只受其自身滞后项的影响，此时 $X_{2,t}$ 不是 $X_{1,t}$ 的格兰杰原因; 反之,$X_{2,t}$ 是 $X_{1,t}$ 的格兰杰原因

   (c) 令 $Y_t = \Psi Y_{t-1} + e_t$, 其中 $Y_t = \begin{pmatrix} X_t \\ X_{t-1} \end{pmatrix}$, $\Psi = \begin{pmatrix} \Phi_1 & \Phi_2 \\ I_{2\times2} & O_{2\times2} \end{pmatrix}$, $e_t = \begin{pmatrix} \varepsilon_t \\ 0 \end{pmatrix}$, 带入具体数值得

$$\Psi = \begin{pmatrix} 1.4 & 1 & -0.45 & -1 \\ 0 & 1.5 & 0 & -0.56 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

对 Φ 求取其特征值, 即 $\det[\lambda I-\Psi]=0$ 得根分别为 0.9,0.5,0.8,0.7

将其带入求得对应的特征向量矩阵 P 和其逆矩阵 P^{-1}

所以 $\Psi=P\Lambda P^{-1}$, 其中 $P=\begin{pmatrix}0.6690 & -0.4472 & -0.6178 & 0.5684\\ 0 & 0 & -0.0927 & 0.0758\\ 0.7433 & -0.8944 & -0.7722 & 0.8120\\ 0 & 0 & -0.1158 & 0.1083\end{pmatrix}$

$\Lambda=\begin{pmatrix}0.9 & & & \\ & 0.5 & & \\ & & 0.8 & \\ & & & 0.7\end{pmatrix}$

$P^{-1}=\begin{pmatrix}3.3634 & -16.8170 & -1.6817 & 6.7268\\ 2.7951 & -23.2924 & -2.5156 & 20.4973\\ 0 & -86.3301 & 0 & 60.4311\\ 0 & -92.3594 & 0 & 73.8875\end{pmatrix}$

由递推式展开再合并可得 $Y_t=\sum_{j=0}^{\infty}\Psi^j e_{t-j}=\sum_{j=0}^{\infty}P\Lambda^j P^{-1}e_{t-j}$

记 $\Psi^j=P\Lambda^j P^{-1}=\begin{pmatrix}M_{1j} & M_{2j}\\ M_{3j} & M_{4j}\end{pmatrix}$

可知 $B_j=M_{1j}$, 即 Ψ^j 的左上 2×2 矩阵, 由计算可得

$B_j=\begin{pmatrix}2.25\times 0.9^j-1.25\times 0.5^j & 10.42\times 0.5^j-52.5\times 0.7^j+53.33\times 0.8^j-11.25\times 0.9^j\\ 0 & 8\times 0.8^j-7\times 0.7^j\end{pmatrix}$

2. (a) 在 $p=1,2,4$ 之下, 计算出 $j=4,8,12,36$ 期的预测方差分解, 结果如图 1 所示, 每行对应不同的 p 值

代码如下:

```
library(readxl)
library(dplyr)
library(vars)
data <- read_xlsx('C:/Users/孔彤阳/Desktop/时间序列2020/hw8
/VAR_M2/CMTS_Q.xlsx')
CMTS <- data %>%
 dplyr::select(q_dates_data, logrealGDP_va, CPI, logM2) %>%
 filter(q_dates_data >= 1999.75 & q_dates_data<=2018.75)
diff_CMTS <- tibble(
 diff_logGDP = diff(CMTS$logrealGDP_va, lag = 1),
 diff_logCPI = diff(log(CMTS$CPI), lag = 1),
 diff_logM2 = diff(CMTS$logM2, lag = 1)
)
```

```
VAR1 <- VAR(diff_CMTS, p=1, type='const')
VAR2 <- VAR(diff_CMTS, p=2, type='const')
VAR3 <- VAR(diff_CMTS, p=4, type='const')

fe1 = fevd(VAR1, n.ahead=36)
fe1$diff_logGDP[c(4,8,12,36),]
fe1$diff_logCPI[c(4,8,12,36),]
fe1$diff_logM2[c(4,8,12,36),]

fe2 = fevd(VAR2, n.ahead=36)
fe2$diff_logGDP[c(4,8,12,36),]
fe2$diff_logCPI[c(4,8,12,36),]
fe2$diff_logM2[c(4,8,12,36),]

fe3 = fevd(VAR3, n.ahead=36)
fe3$diff_logGDP[c(4,8,12,36),]
fe3$diff_logCPI[c(4,8,12,36),]
fe3$diff_logM2[c(4,8,12,36),]
```

(b) 在 $p = 1$ 之下,3 个变量相互间的 Granger 因果关系检验, 报告 F 检验的统计量及其显著性水平, 结果如图 2

代码如下:

```
(Causal_test1 <- causality(VAR1, cause=c("diff_logGDP")))
(Causal_test2 <- causality(VAR1, cause=c("diff_logCPI")))
(Causal_test3 <- causality(VAR1, cause=c("diff_logM2")))

Granger <- tribble(
  ~"Cause", ~"F-Test", ~"P.value",
  #----------/---------------/
  "diff_logGDP", Causal_test1$Granger$statistic, Causal_
      test1$Granger$p.value,
  "diff_logCPI", Causal_test2$Granger$statistic, Causal_
      test2$Granger$p.value,
  "diff_logM2", Causal_test3$Granger$statistic, Causal_
      test3$Granger$p.value
)
```

```
knitr::kable(Granger, digits =4,
             caption = "Granger Test")
```

(c) 脉冲响应结果差别并不大, 方差分解可以看出, 在 GDP 波动的分解中,CPI 和 M2 的贡献变大了, 在其余两者的分解中,GDP 的贡献变小了, 具体结果如图 3 和图 4

代码如下:

```
GDP_E <- read_xlsx('C:/Users/孔彤阳/Desktop/时间序列2020/
   hw9/预期GDP增速.xlsx')
DATA <-  tibble(
  GDP = diff_CMTS$diff_logGDP - GDP_E$expected_q_GDP,
  CPI = diff_CMTS$diff_logCPI,
  M2 = diff_CMTS$diff_logM2
)
VAR4 <- VAR(DATA, p=1, type = "const")
#脉冲响应
plot(irf(VAR4))
#方差分解
fe4 = fevd(VAR4, n.ahead=36)
fe4$GDP[c(4,8,12,36),]
fe4$CPI[c(4,8,12,36),]
fe4$M2[c(4,8,12,36),]
```

```
##      diff_logGDP diff_logCPI diff_logM2
## [1,]   0.6966802  0.03958238  0.2637374
## [2,]   0.6516418  0.06967168  0.2786865
## [3,]   0.6514138  0.07003666  0.2785495
## [4,]   0.6512663  0.07010554  0.2786282
```

```
##      diff_logGDP diff_logCPI diff_logM2
## [1,]   0.2144632   0.7582138 0.02732300
## [2,]   0.2064273   0.7258190 0.06775365
## [3,]   0.2068488   0.7245875 0.06856367
## [4,]   0.2068548   0.7244953 0.06864995
```

```
##      diff_logGDP diff_logCPI diff_logM2
## [1,]  0.01622004   0.1353898  0.8483901
## [2,]  0.03533176   0.1580053  0.8066629
## [3,]  0.03581524   0.1578787  0.8063061
## [4,]  0.03585232   0.1579484  0.8061993
```

```
##      diff_logGDP diff_logCPI diff_logM2
## [1,]   0.6807123   0.1006613  0.2186263
## [2,]   0.6051858   0.1638907  0.2309236
## [3,]   0.6009030   0.1698097  0.2292872
## [4,]   0.6008687   0.1698373  0.2292940
```

```
##      diff_logGDP diff_logCPI diff_logM2
## [1,]   0.2397124   0.7250141 0.03527356
## [2,]   0.2478530   0.6748055 0.07734160
## [3,]   0.2434551   0.6726894 0.08385552
## [4,]   0.2433534   0.6726466 0.08400000
```

```
##      diff_logGDP diff_logCPI diff_logM2
## [1,] 0.004969006   0.1747840  0.8202470
## [2,] 0.010388375   0.2174447  0.7721669
## [3,] 0.013202817   0.2207025  0.7660947
## [4,] 0.013555490   0.2206678  0.7657767
```

```
##      diff_logGDP diff_logCPI diff_logM2
## [1,]   0.7177501   0.2032456 0.07900436
## [2,]   0.5541935   0.3777869 0.06801958
## [3,]   0.5377394   0.3857498 0.07651077
## [4,]   0.4963325   0.4098574 0.09381006
```

```
##      diff_logGDP diff_logCPI diff_logM2
## [1,]   0.2186504   0.7596281 0.02172146
## [2,]   0.1990555   0.7784705 0.02247402
## [3,]   0.1926317   0.7852183 0.02215008
## [4,]   0.1930411   0.7799428 0.02701603
```

```
##      diff_logGDP diff_logCPI diff_logM2
## [1,]  0.03411370   0.4361505  0.5297358
## [2,]  0.05342265   0.4189592  0.5276181
## [3,]  0.08462486   0.3990639  0.5163112
## [4,]  0.11053580   0.4562777  0.4331865
```

图 1: $p = 1, 2, 4$ 时,$j = 4, 8, 12, 36$ 期的预测方差分解

```
## $Granger
##
##  Granger causality H0: diff_logGDP do not Granger-cause
##  diff_logCPI diff_logM2
##
## data:  VAR object VAR1
## F-Test = 6.2767, df1 = 2, df2 = 213, p-value = 0.002246
##
##
## $Instant
##
##  H0: No instantaneous causality between: diff_logGDP and
##  diff_logCPI diff_logM2
##
## data:  VAR object VAR1
## Chi-squared = 2.2916, df = 2, p-value = 0.318
```

```
## $Granger
##
##  Granger causality H0: diff_logCPI do not Granger-cause
##  diff_logGDP diff_logM2
##
## data:  VAR object VAR1
## F-Test = 4.2358, df1 = 2, df2 = 213, p-value = 0.01571
##
##
## $Instant
##
##  H0: No instantaneous causality between: diff_logCPI and
##  diff_logGDP diff_logM2
##
## data:  VAR object VAR1
## Chi-squared = 1.9282, df = 2, p-value = 0.3813
```

```
## $Granger
##
##  Granger causality H0: diff_logM2 do not Granger-cause diff_logGDP
##  diff_logCPI
##
## data:  VAR object VAR1
## F-Test = 10.815, df1 = 2, df2 = 213, p-value = 3.36e-05
##
##
## $Instant
##
##  H0: No instantaneous causality between: diff_logM2 and
##  diff_logGDP diff_logCPI
##
## data:  VAR object VAR1
## Chi-squared = 0.59059, df = 2, p-value = 0.7443
```

Granger Test

Cause	F-Test	P.value
diff_logGDP	6.2767	0.0022
diff_logCPI	4.2358	0.0157
diff_logM2	10.8155	0.0000

图 2: Granger 因果关系与 F 检验

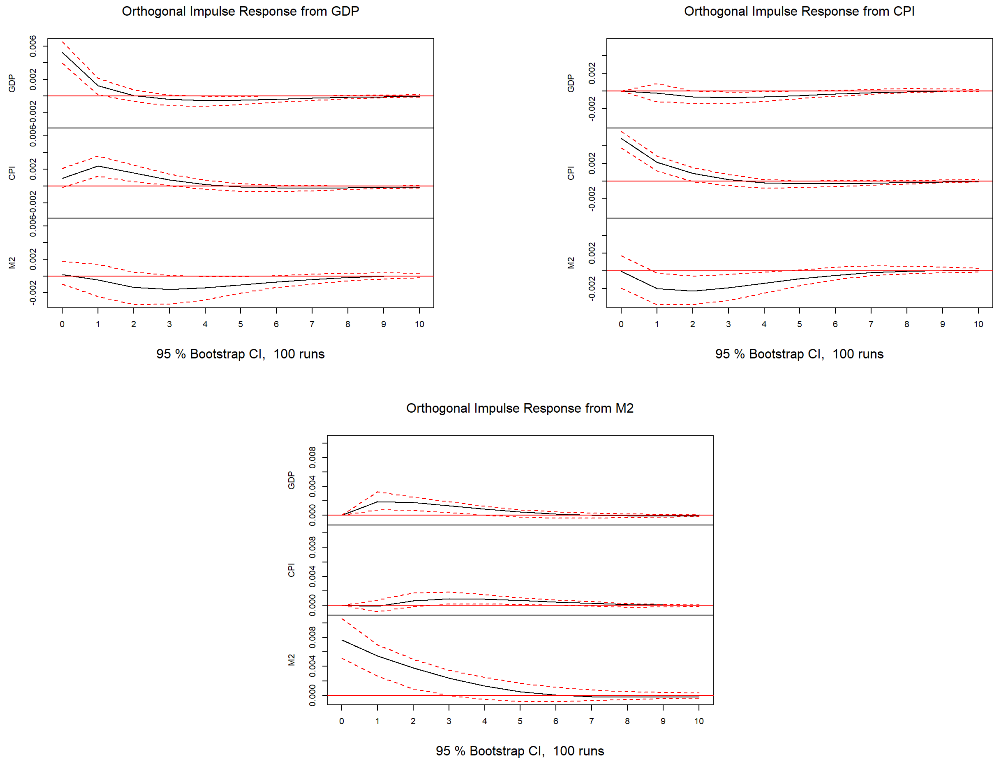

图 3: 脉冲响应

```
##            GDP        CPI        M2
## [1,] 0.7545049 0.02594834 0.2195467
## [2,] 0.7250773 0.04400243 0.2309203
## [3,] 0.7248236 0.04413971 0.2310367
## [4,] 0.7247539 0.04417274 0.2310734
```

```
##            GDP       CPI         M2
## [1,] 0.2476323 0.7220601 0.03030760
## [2,] 0.2389434 0.6940205 0.06703615
## [3,] 0.2396961 0.6928843 0.06741952
## [4,] 0.2396872 0.6928286 0.06748419
```

```
##             GDP       CPI        M2
## [1,] 0.03659603 0.1012372 0.8621668
## [2,] 0.06202387 0.1172800 0.8206961
## [3,] 0.06220283 0.1172294 0.8205678
## [4,] 0.06224798 0.1172640 0.8204880
```

图 4: 方差分解