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Making sovereign debt safe with a financial stability fund[☆]

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ABSTRACT

We develop an optimal design of a Financial Stability Fund that coexists with the international debt market. The sovereign can borrow defaultable bonds on the private international market, while having with the Fund a long-term contingent contract subject to limited enforcement constraints. The Fund contract does not have *ex ante* conditionality, but requires an accurate country-specific risk-assessment (DSA), accounting for the Fund contract. The Fund periodically announces the level of liabilities the country can sustain to achieve the constrained efficient allocation. The Fund is only required *minimal absorption* of the sovereign debt, but it must provide insurance (Arrow-securities) to the country. Furthermore, with the Fund *all sovereign debt is safe independently of the seniority structure*; however, for the Fund, seniority may require a greater *minimal absorption* than a *pari passu* regime. We calibrate our model to the Italian economy and show it would have had a more efficient path of debt accumulation with the Fund.

1. Introduction

In the last few years, the public debt-to-GDP ratio has reached historic levels in the European Union (EU).[1] This is the result of three consecutive crises — the global financial crisis of 2007–2009, the European sovereign debt crisis of 2010–2012 and the

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[1] According to AMECO, General Government Gross Debt in 2022: Euro area 94%, Italy 145%, Portugal 115%, Spain 114% and Greece 171%.

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COVID-19 crisis. In response to these crises, important institutional and policy changes took place, making the Euro area and the EU more resilient but, for the time being, more indebted.[2] As a result of these changes, at the end of 2021, Euro area institutions were playing a leading role in their sovereign debt market, holding more than 30% of the sovereign debt of all Euro area countries.[3] Nevertheless, the question of how to efficiently stabilize the sovereign debt – for example, with complementary official lending programmes – remains open.

To address this question, we design a Financial Stability Fund (Fund) as a *constrained efficient mechanism*, in line with Ábrahám et al. (2022).[4] While the latter assumes that the Fund absorbs all the sovereign debt of a country and focuses on the borrower's perspective, we emphasize the lender's side of the contract and derive the optimal relationship between the private competitive lenders and the Fund. More precisely, we assume that sovereign countries can raise debt in the private international market and in the Fund.[5] While private international lenders solely offer credit (i.e. long-term non-contingent defaultable bonds), the Fund provides both credit and insurance (i.e. Arrow securities) in the form of long-term state-contingent securities. The Fund's intervention is constrained to prevent default and, therefore, it also takes into account the country's indebtedness (i.e. commitments) with private lenders, which brings the issue of whether the Fund possesses seniority. In line with the official lending practice, we consider two regimes: *pari passu* (i.e. no seniority) and *seniority* of Fund's liabilities over private liabilities. The Fund is also constrained to satisfy a strict *debt sustainability analysis* (DSA), which requires that the expected present value of the sovereign's future surpluses (net savings) can *always* cover the country's debt liabilities with the Fund and the private lenders. This constraint has three simultaneous consequences: *i*) it prevents permanent transfers from the Fund to the country which, in the context of a union (as share holders of the Fund), means that there are 'no undesired transfers' across countries, nor debt mutualization, *ii*) it prevents excessive lending when debt is safe (i.e. it is a *non-excessive-lending* constraint), and *iii*) it provides more recursive structure to the Fund contract since a no point in time the Fund would have a negative overhang if it had to restart.[6]

The Fund does not impose *ex ante* conditions, provided there can be feasible contracts with the country, given its existing sovereign debt.[7] This requires upfront a detailed risk-assessment of the country and a calibration of the economy which allow the Fund to compute the optimal borrowing policy the sovereign should adopt. This policy defines the *total* debt holdings and insurance necessary to reach the *constrained efficient allocation*. Then, in any given period, the Fund plays a dual role with respect to the country with a long-term contract: first, it announces the total liabilities that the country can sustain for next period, provided they maintain the contract with the Fund; second, after the country has contracted some, or all, of its debt liabilities with private lenders, the Fund implements its contract, for the period, with its insurance and, if needed, its additional lending. Our characterization of the Fund is a Nash *Recursive Competitive Equilibrium* (RCE). The Fund does not play the role of a Ramsey planner, since it lacks the authority to fully control the market transactions between the private lenders and the sovereign borrower. In particular, it takes the decisions of the private lenders as given and *vice versa*.

The characterization of this RCE implementation is remarkable. First, the Fund stabilizes the entire indebtedness of the sovereign. In other words, *the entire sovereign debt becomes safe*, without default risk. Second, as we assume that there is sufficient private demand for safe assets, there is only need for a *minimal intervention policy* (MIP) of the Fund in the sovereign debt market. Such intervention consists of an insurance component with an additional guarantee on long-term debt holdings by private lenders when the DSA binds. Third, all sovereign debt is safe independently of the seniority structure. However, seniority of the Fund may require a greater debt absorption by the Fund than a *pari passu* regime. Fourth, the Fund – as capacity announcer and provider of insurance and, when needed, debt – implements a unique *constrained efficient* allocation which features no default, therefore, no debt-dilution, and no excess lending. In sum, the literature on sovereign debt has mostly focused on the borrower's default decision, we contribute by characterizing the lenders' optimal policy and its impact on the sovereign debt market.

The first three elements contain novel aspects that deserve explanation.[8] First, *the entire sovereign debt* is split between the competitive private lenders and the Fund. The Fund contract makes the privately held debt safe, while the Fund's debt holdings become a safe asset in its balance sheet allowing the Fund to issue safe debt (say, eurobonds) to finance its absorption, therefore *the entire sovereign debt becomes safe assets*.

Second, the depreciation of the value of the debt can take different forms: when debt is nominal, with inflation; when debt is real and defaultable, with default and dilution, and when the debt is real and perceived safe, with excessive lending. As we have already mentioned, the DSA constraint is, in fact, a *non-excessive-lending* constraint, when it is binding results in a negative spread, a price signal that lenders should not purchase new debt, but also that, if they can, they should sell their holdings of long-term debt

[2] In particular, starting the European Banking Union, founding the European Stability Mechanism, implementing asset purchasing programmes by the ECB, some including purchases of Euro area sovereign debt, and the COVID-19 Next Generation EU (NGEU) programme of the EU making, *de facto*, the European Commission the world's largest official lender, with unprecedented emissions of EU debt.

[3] Particularly, the sovereign debt holdings by Euro area institutions represents for Cyprus, Italy, Portugal and Spain more than 40% of their GDP and for Greece more than 120%.

[4] The main difference with respect to Ábrahám et al. (2022) is threefold. First, we do not consider an exclusive contract between the Fund and the contracting countries. Second, we use growth shocks to better analyze the interest rate-growth differential (i.e. $r - g$). Third, we abstract from moral hazard as we focus on the lending side of the contract.

[5] The adjective 'private' is used to distinguish lenders on the international market relative to the Fund.

[6] We keep this *strict* feature through our analysis, however, it can easily be extended to allow for 'desired transfers' in particular states – say, a pandemic – not properly accounted in the risk-sharing component of the Fund contract.

[7] There may be very high levels of debt that may require restructuring to make the Fund contract feasible or the country may prefer to implement some *ex ante* reforms to improve its risk-profile; that is, the Fund can, and should, have a *menu of Fund contracts* depending on different risk profiles.

[8] We postpone the explanation of the 'fourth element' to the discussion of the literature.

in exchange for riskless assets with a better return. Expectations of these *sudden stop* turbulences can harm the value of long-term bonds. That is, the Fund's MIP can be seen as a prudential policy: if the DSA binds, the Fund is willing to absorb "whatever it takes" of the existing stock of long-term debt, while keeping its commitment to provide insurance, in order to repel the turmoil.

Third, seniority is usually rationalized based on the fact that official lenders are, ultimately, backed by public resources and therefore should have priority in default proceedings. As a result, seniority introduces a partial default risk (default to private lenders but not the Fund), which increases with the fraction of privately held sovereign debt. The Fund contract is designed to make debt safe independently of the seniority structure. Thus, from the perspective of the borrowing country and the private lenders, as long as debt is safe, the seniority structure is irrelevant. However, to avoid partial default, the Fund must be able to commit to absorb enough private debt as to make the sovereign debt country indifferent between partial default and the repayment of private lenders. This commitment – the MIP with seniority – may be substantially larger than the MIP with *pari passu*; therefore, seniority may be a burden for the Fund.

As we said, our analysis enables a comparison with existing lending institutions such as the European Stability Mechanism (ESM) and the International Monetary Fund (IMF). We show that the Fund without seniority might need to absorb less debt in our environment, while the ESM and the IMF usually require seniority in their lending programmes.[9] Moreover, while it is true that official lending institutions conduct DSAs as a necessary condition to guarantee credits, it is not the case that their resulting debt contracts provide insurance against future DSAs, as the Fund does. In other words, international lending institutions base their lending policy on one of several scenarios — e.g. the 'most likely,' the 'politically preferred,' or the 'worst case' scenario. In contrast, the Fund contract risk-shares among these different scenarios or paths. That is, it provides additional transfers in the worst scenario in exchange for higher payments in the best scenario.[10]

We conduct a quantitative analysis in which we calibrate the outside option of the Fund – an incomplete market economy with defaults – to Italy for the period 1992Q1–2019Q4. Unlike Greece, Portugal and Spain, Italy did not participate to any official lending support during the European sovereign debt crisis. It therefore offers the possibility to conduct counterfactual analyses.

The main results of our quantitative inquiry are twofold. First, with the Fund, the Italian debt would have been free of default risk. This is due to the Fund state-contingent credit line being designed to support a countercyclical fiscal policy with respect to exogenous shocks, but also contingent to the states that endogenous enforcement constraints become binding: reassessing the value of primary surpluses to avoid default, and risk-sharing across states when the DSA would be binding in some state. Importantly, we show that the sovereign benefits from a greater debt absorption capacity compared to the standard incomplete market economy with defaults. Particularly, receiving state-contingent transfers from the Fund, the sovereign can accumulate debt in states in which defaults would usually happen. Quantitatively, we find in the steady-growth economy with the Fund substantial welfare gains.

Second, we argue that by accessing the Fund, Italy would have had a more stable evolution of its indebtedness. Using the decomposition of Cochrane (2020, 2022), we show that, in the last two decades, Italy largely increased its public indebtedness despite large primary surpluses. This is due to a strongly positive interest rate-growth differential ($r - g$) dominating the debt accumulation process. The positive differential is a combination of a relatively low, and unstable, growth of the Italian economy with an important risk premium on the Italian sovereign debt. We show that, by accessing the Fund, the Italian government would have reduced these perverse effects and therefore would have ended up with a lower indebtedness. The model predicts that the Italian indebtedness by the end of 2019 would have been around 80% of GDP rather than 135% if Italy could have joined the Fund in 2000.

Our work is related to the sovereign debt literature pioneered by Eaton and Gersovitz (1981) and subsequently extended by Aguiar and Gopinath (2006) and Arellano (2008).[11] As in Ábrahám et al. (2022), our benchmark economy with defaultable debt builds on Chatterjee and Eyigungor (2012) who introduce long-term bonds. Within this literature, our work is closely related to Hatchondo et al. (2017), who consider the case of adding a non-defaultable bond into the otherwise standard defaultable bond economy, and show that there are welfare gains by swapping defaultable bonds into non-defaultable bonds. Our work also relates more closely to Roch and Uhlig (2018) who model a bailout agency with a *minimal intervention policy* but focus on self-fulfilling debt crises.

Besides this, our study addresses the literature on optimal contracts with limited enforcement constraints such as Kehoe and Levine (2001), Kocherlakota (1996) and, in particular, Kehoe and Perri (2002) and Restrepo-Echavarria (2019) who already applied the Lagrangian-recursive approach developed by Marcet and Marimon (2019). Unlike Aguiar et al. (2019) and Aguiar and Amador (2020), our planner's problem integrates two-sided limited enforcement constraints. Our focus is close to Thomas and Worrall (1994) who already studied international lending contracts, with one-sided limited commitment. We decentralize the Fund contract using state-contingent securities and endogenous debt constraints as in Alvarez and Jermann (2000) and show the First and Second Welfare Theorem hold. As a result, in the economy with the Fund, the competitive equilibrium implements the *unique* constrained efficient allocation. Callegari et al. (2023) extend our framework in the environment of Cole and Kehoe (2000) and show that the Fund continues to implement the unique constrained efficient allocation by eliminating self-fulfilling debt crises.

[9] The IMF together with the World Bank have a *de facto* seniority, but it is not a formal contractual feature (see Schlegl et al., 2019). In opposition, the ESM has a *de jure* seniority with respect to the market. The only exception to this is Spain. The Spanish programme was initially agreed with the EFSF with a standard *pari passu* clause and managed to extend this feature into the ESM loan.

[10] Recently, the IMF DSA analysis takes the form of a *Stochastic Debts Sustainability Analysis*, (SDSA), where risk paths are 'statistically calibrated.' There are two differences with our analysis. First, we calibrate the parameters of a stochastic dynamic model to the macro-history of the country, in order to generate an exogenous stochastic structure, which provides a risk assessment without the Fund's contract. Second, we compute the constrained efficient contract design, given our calibration. Furthermore, as it is also done with standard DSA or SDSA, we obtain our 'counterfactual' DSA accounting with the Fund contract.

[11] See also Aguiar and Amador (2014), Aguiar et al. (2016) and Aguiar and Amador (2021).

A more recent literature merges these last two strands of literature and it is the most closely related one to our work. In particular, Dovis (2019) decentralizes optimal contracts through partial default and an active debt maturity management, and Müller et al. (2019) through *ex post* state-conditionality given by default and renegotiation procedures. Our approach is not to 'rationalize' *ex post* observed behaviour, but to account for existing constraints. In view of this, we adopt a Nash specification in which the Fund takes the decision in the private bond market as given. We then characterize the constraint efficient allocation and assess it quantitatively in relation to a calibrated version of the benchmark defaultable debt economy.

Finally, as a theoretical foundation for the design of a – effectively running – fiscal fund, able to stabilize sovereign debt and expand the supply of safe assets, our work is related to a large literature regarding the IMF and other international institutions lending practices, and to the debate on the need to develop the Fiscal Union within the European Economic and Monetary Union (EMU) and expand its supply of eurobonds (as it has been done with the Next Generation EU (NGEU) programme) as safe assets.[12]

The paper is organized as follows. We lay down the environment in Section 2 and present the Fund contract in Section 3. We expose the decentralized economy in Section 4, which includes the sovereign's, the private lenders' and the Fund's problems. Section 5 develops the Fund's intervention with seniority. After this, we calibrate our model to Italy in Section 6 and present the underlying results in Section 7. Finally, we conclude in Section 8.

2. Environment

We assume an infinite-horizon small open economy with a single homogeneous consumption good in discrete time. There is a sovereign borrower acting as a representative agent and taking decisions on behalf of the small open economy, a Fund acting as official lender and a continuum of competitive private lenders.

2.1. The sovereign borrower

The sovereign's preference is represented by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$, where $\beta \in (0,1)$ is the discount factor, n_t is the labour, $1-n_t$ the leisure and c_t the consumption at time t. The sovereign is relatively impatient as $\beta < 1/(1+r)$. We adopt a specific form of utility function so as to obtain a (stochastic) balanced growth path and to simplify the detrended formulation of the problem: $U(c,n) = u(c) + h(1-n) = \log(c) + \xi \frac{(1-n)^{1-\zeta}}{1-\zeta}$.

The sovereign has access to a labour technology $y = \theta f(n)$ subject to decreasing returns to scale, where $f_n(n) > 0$, $f_{nn}(n) < 0$. Moreover, $\theta \in \Theta$ represents a trend shock to the productivity. It is the only source of uncertainty in the economy. The law of motion of the shock is given by $\theta_t = \gamma_t \theta_{t-1}$, where $\gamma_t \in \Gamma$ represents the growth rate at time t. We denote the history of θ up to time t by θ^t. The exact form of the shock is detailed in Section 6.[13]

Finally, the sovereign has access to a long-term state-contingent contract with the Fund – a credit-insurance line that we specify below – and long-term debt contracts with a continuum of competitive private lenders. However, it cannot commit to honour the terms of any contract. Given that contracts are all long term, this gives rise not only to *default risk* – i.e. non repayment – but also to *dilution risk* — i.e. devaluation of legacy debt.

In the first part of our analysis, we assume that the Fund contract has no seniority with respect to the private debt contracts. That is, every default is a *full* default as the sovereign reneges its entire debt position. Under such default, the sovereign receives a penalty in the form of a reduced output, $\theta^d \leq \theta$, and loses access to both the private bond market and the Fund. Later, it can reintegrate the private bond market with some probability, λ, but cannot obtain the assistance of the Fund anymore. In the second part of our analysis, we consider the case in which the Fund possesses seniority with respect to the private bond market.[14]

2.2. The private lenders

There is a continuum of competitive private lenders which have access to international financial markets. They are risk neutral and discount the future at $\frac{1}{1+r}$ where r is the risk-free rate. Private lenders' contracts are a continuum of simple long-term debt contracts.[15] We denote by $b_{l,t}$ the debt held by the private lenders, while we denote by b_t the debt issued by the sovereign at time t. By market clearing, $b_t = -b_{l,t}$ for all t. Furthermore, $b_t > 0$ denotes an asset and $b_t < 0$ denotes a debt from the point of view of the sovereign.

[12] See Marimon and Wicht (2021) for a discussion on how our Fund proposal relates to this literature and it can be implemented within EMU.

[13] We present in the main text the model with the stochastic trend and keep track of θ in the state space. The detrended version is presented in the Online Appendix B. There we only keep track of γ in the state space.

[14] We do not consider the case in which the Fund is junior relative to the private lenders as official multilateral lending institutions generally enjoy a preferred creditor status (see Schlegl et al., 2019).

[15] We assume that private lenders do not offer Arrow securities, basically for two reasons. One is factual: Arrow securities are more complex than, for example, insurance contracts against natural disasters, for which private insurance companies provide contracts to households and firms, but – not surprisingly – not to countries; the other is redundancy (and convenience): we expect the same results would hold but would require to always keep track of the fraction of Arrow securities in the private sector, which is not just a problem of more baroque notation, but also of properly restating some results.

2.3. The financial stability fund

Similar to the private lenders, the Fund has access to international financial markets, is risk neutral, discounts the future at $\frac{1}{1+r}$ and breaks even in expectation.

While private lenders are competitive, the design of the Fund contract is based on a risk assessment of the country which, as it is common practice in *debt sustainability analysis* (DSA), also accounts for the effect of the same Fund contract in enhancing the sustainability of the country's sovereign debt. In our Nash specification, the Fund takes the decisions in the private bond market as given. At the same time, the private lenders take the lending decisions of the Fund as given.

In addition, the Fund provides a state-contingent contract, whereas private lenders offer non-contingent debt contracts.[16] Particularly, the Fund contract is a state-contingent asset, $a_{l,t}$, which can be decomposed into a debt, $\bar{a}_{l,t}$, and an insurance components, $\hat{a}_{l,t}(\theta^t)$. As before, by market clearing, $a_t = -a_{l,t}$ for all t where $a_t > 0$ denotes an asset and $a_t < 0$ denotes a debt from the point of view of the sovereign. Importantly, liabilities in the Fund contract cannot be arbitrary. There is a limit on the extent of losses the Fund can make given by $\theta_{t-1}Z \leq 0$. The reason is that any contract with permanent losses has to be compensated with other contracts having permanent gains. For instance, in a union of sovereign countries, expected losses must be mutualized if the Fund is only backed with the union's primary surplus. Thus assuming that $Z = 0$ means that there is no permanent transfer across the different Fund's contracts.[17]

Finally, the Fund's withdrawal in the case of a *full* default is permanent, whereas the private bond market's exclusion is temporary.

2.4. Timing of actions

The timing of actions within the period is:

1. Given $(\theta_{t-1}Z, a_{l,t}, b_{l,t})$, after the realization of the growth shock θ_t, the Fund announces what is the (state-contingent) sustainable debt capacity of the sovereign country for next period: $\{\omega_l'(\theta^{t+1})\}_{\theta^{t+1}|\theta^t}$ which can be decomposed into a debt component, $\bar{\omega}_l'(\theta^t) = b_{l,t} + \bar{a}_{l,t}$, to be allocated between the private lenders and the Fund, and insurance components, $\omega_l'(\theta^{t+1}) - \bar{\omega}_l'(\theta^t) = \hat{a}_{l,t}(\theta^{t+1})$, which must be part of the Fund contract.
2. The sovereign decides whether to default or not and, in the latter case, the sovereign then determines its borrowing with the private bond market before going to the Fund.
3. Conditional on no default, the Fund and the sovereign implement the corresponding debt and insurance part of their contract.[18]

3. The financial stability fund

We specify the Fund contract in a Nash specification where the actions in the private bond market are taken as given.

3.1. Debt and sustainability

The private lenders' and Fund's contracts establish that at time t and state-history θ^t the country must transfer $\tau_f(\theta^t)$ for its state-contingent liabilities with the Fund and $\tau_p(\theta^t)$ for its non-contingent debt liabilities with the private lenders. We denote $\tau(\theta^t) \equiv \tau_f(\theta^t) + \tau_p(\theta^t)$ as the total transfer the country pays. That is, given a consumption and employment plan $\{c(\theta^t), n(\theta^t)\}_{t=0}^{\infty}$, in period-state (t, θ^t) feasibility implies that

$$\tau(\theta^t) = \theta_t f(n(\theta^t)) - c(\theta^t); \tag{1}$$

that is, $\tau(\theta^t)$ is the *primary surplus* in state-history θ^t. Therefore, if the country's debt with private lenders is $-b_{l,t}$ and its asset position with the Fund is $-a_{l,t}$ for a total amount of $-\omega_{l,t} = -(b_{l,t} + a_{l,t})$, *debt sustainability* requires that the expected present value of future transfers discounted with the risk free rate r should cover the outstanding amount of debt:

$$\mathbb{E}_t \sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^{j-t} \tau(\theta^j) \geq \omega_{l,t}.$$

In particular, there is a decomposition of total transfers such that:

$$\mathbb{E}_t \sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^{j-t} \tau_p(\theta^j) \geq b_{l,t} \quad \text{and} \quad \mathbb{E}_t \sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^{j-t} \tau_f(\theta^j) \geq a_{l,t}. \tag{2}$$

[16] See notably Hatchondo and Martinez (2012), Mateos-Planas and Seccia (2014) and Kirpalani (2017) for models with private state-contingent contracts.

[17] As already noted in footnote 1, we could also assume more generally that $Z(\theta^t)$ meaning that there can be bounded solidarity transfers among union countries depending on the realization of θ^t. For example, the 'grant component' of the NGEU recovery plan mentioned in footnote 1.

[18] This timing rules out self-fulfilling debt crises (Ayres et al., 2018). See Callegari et al. (2023) for a version of the model with self-fulfilling debt crises.

Without loss of generality, we assume that $a_{l,0} = 0$, therefore the initial state is given by $(\theta_0, b_{l,0})$. In contrast with private lenders which only issue non-contingent debt contracts, the Fund provides a state-contingent contract, i.e. $a_{l,t} = \bar{a}_{l,t} + \hat{a}_{l,t}(\theta^t)$. More precisely, it defines contingent transfers for $(t+1, \theta^{t+1})$ at (t, θ^t); i.e. $\tau'_f(\theta^{t+1}) = \tau_f(\theta^t) + \hat{\tau}'_f(\theta^{t+1})$, with $\sum_{\theta^{t+1}|\theta^t} \hat{\tau}'_f(\theta^{t+1}) = 0$ and

$$\tau_f(\theta^t) = \sum_{\theta^{t+1}|\theta^t} \tau'_f(\theta^{t+1}). \tag{3}$$

We later specify the form that these transfers have in a decentralized economy.

However, for the debt to be sustainable two other factors must be taken into account. First, a *sovereign country can default* on its liabilities. Therefore, if in state θ_t the value of the outside default option is $V^{af}(\theta_t)$, to prevent *full* default the Fund contract must satisfy:

$$\mathbb{E}\Big[\sum_{j=t}^{\infty} \beta^{j-t} U(c(\theta^j), n(\theta^j)) \Big| \theta^t\Big] \geq V^{af}(\theta_t). \tag{4}$$

Second, the Fund contract must account that the *liabilities with the Fund cannot be arbitrary*. Therefore, since the Fund takes into account the private debt liabilities $b_{l,t}$, and both debt liabilities are treated *at par* (a feature we analyse in detail in Section 5) the Fund contract must satisfy:[19]

$$\mathbb{E}\Big[\sum_{j=t}^{\infty} \Big(\frac{1}{1+r}\Big)^{j-t} \tau(\theta^j) \Big| \theta^t\Big] \geq \theta_{t-1} Z + b_{l,t}. \tag{5}$$

The above constraint depends on $Z \leq 0$ and b_l. The former variable indicates the level of redistribution of the Fund. In order to prevent that the Fund provides permanent transfers to the sovereign we will assume that $Z = 0$, i.e. that in no state the Fund contract has expected losses. Similarly, b_l indicates the level of outstanding private debt the sovereign needs to repay. Larger b_l tightens the constraint. We therefore interpret (5) as a DSA as it corresponds to an evaluation of the present value of the sovereign's future surpluses.

The literature has mainly considered one-sided limited enforcement contracts in which (4) is the standard constraint. We focus on a two-sided limited enforcement contract in which we introduce (5) alongside (4). Without (5), the Fund prevents defaults on equilibrium path and is unconcerned by the extent of losses in the contract. In opposition, with (5), the Fund actively monitors the sovereign's capacity to generate surpluses — i.e. τ. In particular, in states where the sovereign's future surpluses might not appropriately cover additional amount of debt – say, when (5) is binding at θ^t – there is a lending 'sudden stop' to avoid losses that would go beyond the contract's terms. Thus, with (5), the Fund internalizes the fact that marginal lending can be excessive.

The design of the Fund contract has two distinct features. First, it establishes the levels of debt which are sustainable next period, $\{\omega'_l(\theta^{t+1})\}_{\theta^{t+1}|\theta^t}$, according to a DSA. The announcement of $\{\omega'_l(\theta^{t+1})\}_{\theta^{t+1}|\theta^t}$ by the Fund makes such levels of debt common knowledge at the beginning of the period and therefore coordinates the private lenders' beliefs. Second, the Fund defines the long-term contract between the Fund and the sovereign, which here takes the form of financial transfers. In particular, it commits to a debt and insurance level $\{a_{l,t+1}(\theta^{t+1}, b_{l,t+1})\}_{\theta^{t+1}|\theta^t}$, where $a_{l,t+1}(\theta^{t+1}, b_{l,t+1}) = \omega_{l,t+1}(\theta^{t+1}) - b_{l,t+1}$.[20]

3.2. The fund contract problem

We now turn to the specific design of the Fund's announcement and contract. Once the corresponding country's risk assessment regarding $\{\theta_t\}_{t=0}^{\infty}$ has been done, the Fund solves a planner's problem with two agents – the sovereign and the Fund itself – taking into account the participation of a continuum of private lenders in absorbing credit needs. This defines an allocation, of consumption and employment, which the Fund takes as the benchmark policy the sovereign will follow, and the corresponding transfers of the sovereign to the lenders.

We say that $\{c(\theta^t), n(\theta^t)\}_{t=0}^{\infty}$ is a Fund's *constrained efficient allocation* in *sequential form*, given $b_{l,0}$, if there exist sequences of transfers $\{\tau_p(\theta^t), \tau'_f(\theta^{t+1})\}_{t=0}^{\infty}$, with associate $\{b_{l,t}\}_{t=0}^{\infty}$ satisfying (2), such that:

$$\max_{\{c(\theta^t), n(\theta^t)\}_{t=0}^{\infty}} \mathbb{E}\Big[\mu_{b,0} \sum_{t=0}^{\infty} \beta^t U(c(\theta^t), n(\theta^t)) + \mu_{l,0} \sum_{t=0}^{\infty} \Big(\frac{1}{1+r}\Big)^t \tau(\theta^t) \Big| \theta_0\Big] \tag{6}$$
$$\text{s.t. } (5), (4), (3) \text{ and } (1), \text{ for all } (t, \theta^t), t \geq 0.$$

[19] To obtain Eq. (5), observe that, conditional on θ^t,

$$\mathbb{E}\Big[\sum_{j=0}^{\infty} \Big(\frac{1}{1+r}\Big)^j \tau(\theta^{t+j}) \Big| \theta^t\Big] = \mathbb{E}\Big[\sum_{j=0}^{\infty} \Big(\frac{1}{1+r}\Big)^j (\tau_f(\theta^{t+j}) + \tau_p(\theta^{t+j})) \Big| \theta^t\Big].$$

Using the valuation formula in (2), the previous equation simplifies into

$$\mathbb{E}\Big[\sum_{j=0}^{\infty} \Big(\frac{1}{1+r}\Big)^j \tau(\theta^{t+j}) \Big| \theta^t\Big] \geq \mathbb{E}\Big[\sum_{j=0}^{\infty} \Big(\frac{1}{1+r}\Big)^j \tau_f(\theta^{t+j}) \Big| \theta^t\Big] + b_l(\theta^t).$$

The present value constraint on Fund's lending is $\mathbb{E}\big[\sum_{j=0}^{\infty} \big(\frac{1}{1+r}\big)^j \tau_f(\theta^{t+j}) \big| \theta^t\big] \geq \theta_{t-1} Z$, thus the overall participation constraint of the Fund reduces to (5). Note that we cannot consider $\mathbb{E}\big[\sum_{j=0}^{\infty} \big(\frac{1}{1+r}\big)^j \tau_f(\theta^{t+j}) \big| \theta^t\big] \geq \theta_{t-1} Z$ and $\mathbb{E}\big[\sum_{j=0}^{\infty} \big(\frac{1}{1+r}\big)^j \tau_p(\theta^{t+j}) \big| \theta^t\big] \geq b_l(\theta^t)$ as two separate constraints because the Fund takes the actions in the private bond market as given.

[20] Note that the Fund contract is state contingent with respect to the productivity shocks θ_{t+1} but also with respect to $V^{af}(\theta_{t+1})$ and $\theta_{t-1} Z + b_{l,t+1}$ being binding.

The *constrained efficient allocation* prescribes that, in period t, the sovereign consumes $c(\theta^t)$ and provides labour $n(\theta^t)$.[21] Furthermore, the Fund's break-even assumption determines the initial weights $(\mu_{b,0}, \mu_{l,0})$. If without private debt there is an interior solution to the Fund's contracting problem, then an optimal solution exists and there are feasible paths of private debt, starting at $b_{l,0}$, subject to an upper bound on how large the initial debt $b_{l,0}$ can be. We come back to this later.

Using the recursive contracts approach of Marcet and Marimon (2019), we can formulate the Fund's problem (6) in *recursive form*. Defining $s \equiv \{\theta^-, \gamma\}$ and $\eta \equiv \beta(1+r) < 1$,

$$FV(s,x,b_l) = \mathcal{SP} \min_{\{v_b,v_l\}} \max_{\{c,n\}} x\left[(1+v_b)U(c,n) - v_b V^{af}(\theta)\right] + \left[(1+v_l)\tau - v_l(\theta^- Z + b_l)\right] + \frac{1+v_l}{1+r}\mathbb{E}\left[FV(s',x',b_l')\middle|\theta\right] \tag{7}$$

$$\text{s.t. } \tau = \theta f(n) - c,$$

$$x' = \frac{1+v_b}{1+v_l}\eta x \quad \text{with } x_0 \text{ given.} \tag{8}$$

The Online Appendix A presents all the details of such exposition. We denote by x' the prospective Pareto weight of the sovereign relative to the Fund where $v_b \geq 0$ and $v_l \geq 0$ are the normalized multipliers attached to the sovereign's and the Fund's participation constraints, respectively. The value function of the contracting problem satisfies:

$$FV(s,x,b_l) = xV^b(\theta,x,b_l) + V^l(s,x,b_l), \text{ with}$$

$$V^b(\theta,x,b_l) = U(c,n) + \beta\mathbb{E}[V^b(\theta',x',b_l')|\theta] \quad \text{and} \quad V^l(s,x,b_l) = \tau + \frac{1}{1+r}\mathbb{E}[V^l(s',x',b_l')|\theta].$$

It should be underlined that we take a specific planner's perspective in solving for the Fund contract. The Fund is designing a constrained efficient contract with the sovereign borrower while taking as given the lending policies of the private lenders in the market, and at the same time, the private lenders are aware of the lending decisions of the Fund in the contract. However, the Fund does not play the role of a Ramsey planner in our framework, since it lacks the authority to fully control the market transactions between the private lenders and the sovereign borrower, neither directly through planned allocations nor indirectly via policy instruments. In other words, as we will see more explicitly when, in the next section, we characterize the decentralized economy, the equilibrium between the Fund, the sovereign and the continuum of private lenders has a Nash-competitive equilibrium characterization, not a Ramsey policy implementation.

Our present formulation is close to the current rules of international lending institutions such as the IMF or the ESM. The Fund takes into account all the sovereign's debt liabilities – within and outside the Fund – that satisfy the DSA in every possible state. The difference with current practices is that the DSA is usually only conducted at the beginning of the contract, or at certain time intervals, while in our characterization of the Fund contract, DSA, i.e. (5), is contingent to all states that the contract specifies, including those where participation constraints are binding. This means that our DSA has a different definition of 'sustainability' than existing official multilateral lending institutions. Particularly, sovereign liabilities have to remain sustainable *ex ante* and *ex post* in all considered paths.

Another difference is that in this framework, the Fund provides state-contingent transfers, key component that averts default on equilibrium path as we will see. Finally, the Fund has no seniority over privately owned debt. This is in general not the case when official multilateral lenders intervene (cf. footnote 1). In Section 5, we consider an alternative formulation where the Fund liabilities has seniority over privately held sovereign debt. We show that the seniority structure of the Fund might affect the Fund's intervention.

3.3. The sovereign's outside option

The autarky value of the standard incomplete market model with default represents the sovereign's outside option (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008). Since the Fund has no seniority with respect to the privately held sovereign debt, the sovereign reneges its entire debt position if it decides to default. This is what we call a *full* default. The Bellman equation in such situation reads

$$V^{af}(\theta) = \max_n\left\{U(\theta^d f(n), n)\right\} + \beta\mathbb{E}\left[(1-\lambda)V^{af}(\theta') + \lambda J(\theta',0)\middle|\theta\right], \tag{9}$$

where $\theta^d \leq \theta$ contains the penalty for defaulting. Furthermore, V^{af} corresponds to the value under financial autarky and J to the value of reintegrating the private bond market without the Fund. More precisely, $J(\theta,b) = \max\{V^o(\theta,b), V^{af}(\theta)\}$, with

$$V^o(\theta,b) = \max_{\{c,n,b'\}} U(c,n) + \beta\mathbb{E}\left[J(\theta',b')\middle|\theta\right] \tag{10}$$

$$\text{s.t. } c + \tau_p(b') \leq \theta f(n).$$

Given Eq. (2), the sequence of private transfers $\{\tau_p(\theta^t)\}_{t=0}^{\infty}$ directly relates to a sequence of private debt $\{b(\theta^t)\}_{t=0}^{\infty}$. Hence, for a given b, by picking b', the borrower directly chooses a certain level of transfer τ_p. By a slight abuse of notation, we write τ_p as a function of b'.

[21] Our contract accounts for all the lenders on equal footing in the maximization. While the Fund *directly* specifies contingent transfers $\tau_f'(\theta^{t+1})$ taking as given $\tau_p(\theta^t)$, effectively the contract accounts for the total surplus, $\tau(\theta^t)$.

3.4. Properties of the fund contract

This subsection demonstrates the main properties of the Fund contract. Other properties such as the inverse Euler equation and the steady state are presented in the Online Appendix C. Proofs are in the Online Appendix D.

We start with the existence of the Fund contract and, for this, we need the following interiority assumption (Marcet and Marimon, 2019).

Assumption 1 (*Interiority*). There is an $\epsilon > 0$, such that, for all $\theta^t \in \Theta^t$ with associate $\{b_{l,t}\}_{t=0}^{\infty}$ satisfying (2), there is a sequence $\{\ddot{c}(\theta^t), \ddot{n}(\theta^t)\}$ satisfying for all $t \geq 0$,

$$\mathbb{E}\Big[\sum_{j=t}^{\infty} \beta^{j-t} U(\ddot{c}(\theta^j), \ddot{n}(\theta^j))\Big|\theta^t\Big] \geq V^{af}(\theta_t) + \epsilon,$$

$$\mathbb{E}\Big[\sum_{j=t}^{\infty} \Big(\frac{1}{1+r}\Big)^{j-t} \big(\theta_j f(\ddot{n}(\theta^j)) - \ddot{c}(\theta^j)\big)\Big|\theta^t\Big] \geq \theta_{t-1} Z + b_{l,t} + \epsilon.$$

This assumption ensures the uniform boundedness of the Lagrange multipliers. For Eqs. (4) and (5), it requires that, in spite of the enforcement constraints, there are strictly positive rents to be shared among the contracting parties. In our environment, since rents to be shared are positively correlated with productivity shocks, this assumption is easily satisfied given that default is costly. Otherwise, there may not exist a constrained efficient risk-sharing agreement.

Proposition 1 (*Existence and Uniqueness*). *In the specified environment,*[22] *if Assumption 1 is satisfied, for every θ there is a $\underline{b}_l(\theta) > 0$ such that if $b_{l,0}(\theta) \leq \underline{b}_l(\theta)$, then there exists a unique Fund's allocation with initial condition $(\theta, b_{l,0}(\theta))$. Furthermore, there is a $\underline{t}(\theta, \underline{b}_l(\theta))$ such that for $t > \underline{t}(\theta, \underline{b}_l(\theta))$ the detrended Fund contracts are at the steady state.*

Proposition 1 is made of three parts. First, a Fund contract exists if – among other requirements – the initial level of private indebtedness is not too high, as to Assumption 1 to be satisfied. However, if an economy is in an initial state $(\theta, b_{l,0}(\theta))$ but $b_{l,0}(\theta) > \underline{b}_l(\theta)$ then the private debt will need to be restructured — i.e. to a $\check{b}_{l,0}(\theta) \leq \underline{b}_l(\theta)$ — for a Fund contract to exist. In other words, there is a strict risk-assessment of the sovereign and, provided that the existing level of private liabilities is sustainable, if there is a Fund contract then no other *ex ante* conditionality is needed. Second, the Fund contract allocation in terms of consumption and employment is unique and, third, it is characterized by an ergodic distribution which we detail in the Online Appendix C.

Corollary 1 (*No Full Default*). *In a Fund contract, there is no full default.*

The sovereign's participation constraint (4) implies no (*full*) default on equilibrium path. The Fund always provides state-contingent transfers to the sovereign. This sustains the chosen sequence of private liabilities, $\{\tau_p(\theta^t)\}_{t=0}^{\infty}$, and ensures that the sovereign finds optimal not to default. This shows the importance of the state contingency of the Fund's transfer. Without this feature, the Fund would not be capable of accounting for the possibility of default in each state $\theta' \in \Theta$ specifically.

4. The decentralized economy

The previous section derived the Fund contract from the perspective of a mixed centralized-private economy. It had the advantage that it allowed a full characterization of the Fund contract, but the disadvantage of having the sovereign in the shadow, with its actions being decided by the Fund contract. We now consider the decentralized version of the economy in which the Fund and the private lenders trade securities with the sovereign.

The financial market is composed of private lenders and the Fund. The sovereign has therefore two funding opportunities. On the one hand, it can borrow long-term defaultable bonds, b', on the private bond market at a unit price of $q_p(\theta, \bar{\omega}')$, where $\bar{\omega}$ is defined momentarily. On the other hand, it can trade $|\Theta|$ state contingent securities $a'(\theta')$ at a unit price of $q_f(\theta', \omega'(\theta')|\theta)$. A fraction $1-\delta$ of each financial asset matures today and the remaining fraction δ is rolled-over and pays a coupon κ. Given this, the transfer to the Fund and the private lenders are, respectively

$$\tau_f(\theta) = \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)(a'(\theta') - \delta a(\theta)) - (1 - \delta + \delta\kappa)a(\theta), \tag{11}$$

$$\tau_p(\theta) = q_p(\theta, \bar{\omega}')(b'(\theta) - \delta b(\theta)) - (1 - \delta + \delta\kappa)b(\theta). \tag{12}$$

The assets provided by the Fund are state contingent, while private bonds are not. More precisely, the portfolio $a'(\theta')$ can be decomposed into a common bond $\bar{a}'$ that is independent of the next period state, traded at the implicit bond price $q_f(\theta, \bar{\omega}') \equiv \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)$, and an insurance portfolio of $|\Theta|$ Arrow securities $\hat{a}'(\theta')$. Thus we have that $a'(\theta') = \bar{a}' + \hat{a}'(\theta')$ with $\bar{a}' = \frac{\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta) a'(\theta')}{q_f(\theta, \bar{\omega}')}$ and $\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)\hat{a}'(\theta') = 0$ which represents the market clearing condition of Arrow securities.[23]

[22] We define a specific functional form for $U(c, n)$, but it is enough to assume that U is continuous, strictly monotone and concave in $\mathbb{R}^+ \times [0, 1]$.

[23] Note that $\tau_f'(\theta') = \tau_f(\theta) + \hat{\tau}_f(\theta')$, where $\tau_f(\theta) = q_f(\theta, \bar{\omega}')(\bar{a}'(\theta) - \delta a(\theta)) - (1 - \delta + \delta\kappa)a(\theta)$ and $\hat{\tau}_f(\theta') = q_f(\theta', \omega'(\theta')|\theta)\hat{a}'(\theta')$.

Given that the Fund has no seniority with respect to the private lenders, the bond prices are a function of the total liabilities next period. We denote the entire position – including insurance and debt – by $\omega = a + b$ and a total *debt* position by $\bar{\omega} = \bar{a} + b$.

The Fund takes the decisions in the private bond market as given and *vice versa*. In addition, as long as there are no spreads – positive or negative – on the debt, private lenders are willing to provide all the debt the sovereign asks for.

The timing of actions is the one presented in Section 2. Given $(\theta^{-}Z, a_l, b_l)$, after the realization of the growth shock θ, the Fund announces what is the (state-contingent) sustainable debt capacity of the sovereign country for next period: $\{\omega_l'(\theta')\}_{\theta'|\theta}$. The sovereign decides whether to default or not and, in the latter case, the sovereign then determines its borrowing with the private bond market (i.e. b_l') before going to the Fund (i.e. $\{a_l'(\theta')\}_{\theta'|\theta}$). Conditional on no default, the Fund and the sovereign implement the corresponding debt and insurance part of their contract.

4.1. The sovereign's and private lender's problems

The economy is decentralized as a competitive equilibrium with endogenous borrowing and lending constraints following Alvarez and Jermann (2000) and Krueger et al. (2008). Under the above market structure, the sovereign's problem reads

$$W^b(\theta, a, b) = \max_{\{c,n,b',\{a'(\theta',b')\}_{\theta'\in\Theta}\}} U(c,n) + \beta \mathbb{E}\left[W^b(\theta', a'(\theta',b'), b')\middle|\theta\right] \tag{13}$$

$$\text{s.t. } c + \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)(a'(\theta',b') - \delta a) + q_p(\theta, \bar{\omega}')(b' - \delta b) \tag{14}$$

$$\leq \theta f(n) + (1 - \delta + \delta\kappa)(a + b), \text{ and}$$

$$\omega'(\theta') = a'(\theta', b') + b' \geq \mathcal{A}_b(\theta'). \tag{15}$$

Eq. (15) is the equivalent to the participation constraint (4), which prevents defaults. The endogenous borrowing limit $\mathcal{A}_b(\theta')$ is such that

$$W^b(\theta', \ddot{a}'(\theta', \ddot{b}'), \ddot{b}') = V^{af}(\theta') \text{ for all } \ddot{a}'(\theta', \ddot{b}') + \ddot{b}' = \mathcal{A}_b(\theta'). \tag{16}$$

In words, the endogenous borrowing limit is such that the sovereign's expected lifetime utility from repaying its debts is at least as high as that of defaulting. It is therefore a no-default borrowing constraint (Zhang, 1997). Particularly, it is tight enough in the sense of Alvarez and Jermann (2000) to prevent default but allows as much risk sharing as possible. We explain the dependence of $a'(\theta', b')$ on b' when we derive the decentralized Fund's problem.

Private lenders solve a static problem. However, we express it in recursive form to later formulate the DSA of the Fund. We have

$$W^p(\theta, a_l, \bar{a}_p, b_l) = \max_{\{c_p, b_l', \bar{a}_p'\}} c_p + \frac{1}{1+r}\mathbb{E}\left[W^p(\theta', a_l', \bar{a}_p', b_l')\middle|\theta\right] \tag{17}$$

$$\text{s.t. } c_p + q_p(\theta, \bar{\omega}')(b_l' - \delta b_l) + q_f(\theta, \bar{a}_p')(\bar{a}_p' - \delta \bar{a}_p) \leq (1 - \delta + \delta\kappa)(b_p + \bar{a}_p).$$

An important object which emanates from this problem is the private lending policy, $b_l' = \boldsymbol{B}_l(\theta, a_l, b_l)$ which is taken as given by the Fund.

The private lenders also have access to the bonds issued by the Fund. This enables that the bond price in the Fund and in the private bond market coincide through arbitrage. We will consider equilibria where, without loss of generality, $\bar{a}_p = 0$, therefore we simplify notation by eliminating $\bar{a}_p$ if not necessary. Notably, we write $W^p(\theta, a_l, 0, b_l) \equiv W^p(\theta, a_l, b_l)$. Besides this, the trade of private bonds satisfies the following transversality condition:

$$\lim_{n\to\infty} \mathbb{E}\left\{\left[\prod_{j=0}^{n} Q_p\left(\theta^{t+j}, \bar{\omega}\left(\theta^{t+j}\right)\right)\right] b_l\left(\theta^{t+j}\right)\middle|\theta^t\right\} = 0, \quad \text{with} \tag{18}$$

$$Q_p\left(\theta^{t+j}, \bar{\omega}\left(\theta^{t+j}\right)\right) = \frac{q_p\left(\theta^{t+j}, \bar{\omega}\left(\theta^{t+j}\right)\right)}{1 - \delta + \delta\kappa + \delta q_p\left(\theta^{t+j+1}, \bar{\omega}\left(\theta^{t+j+1}\right)\right)}. \tag{19}$$

The implicit interest rate in the private bond market is $r_p(\theta, \bar{\omega}') \equiv \frac{1}{Q_p(\theta, \bar{\omega}')} - 1$. As we will see, it is possible that $r_p(\theta, \bar{\omega}') < r$ generating a wedge between the lenders' discount factor and the pricing kernel. That is why the valuation Eq. (2) holds with inequality.

4.2. The decentralized fund contract

We can further decentralize the Fund contract. We show that, given the realization of the state, the Fund formulates an announcement stating the level of indebtedness that remains sustainable in all future states. The maximization problem of the Fund is given by

$$W^f(s, a_l, b_l) = \max_{\{c_f, \{a_l'(\theta', b_l')\}_{\theta'\in\Theta}\}} c_f + \frac{1}{1+r}\mathbb{E}\left[W^f(s', a_l'(\theta', b_l'), b_l')\middle|\theta\right] \tag{20}$$

$$\text{s.t. } c_f + \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)(a_l'(\theta', b_l') - \delta a_l) \leq (1 - \delta + \delta\kappa) a_l, \tag{21}$$

$$\omega_l'(\theta') = a_l'(\theta', b_l') + b_l' \geq \mathcal{A}_f(\theta', b_l'), \tag{22}$$

$$\text{with } b_l' = \boldsymbol{B}_l(\theta, a_l, b_l) \text{ given},$$

where $B_l(\theta, a, b)$ is the lending policy of the private lenders and $s \equiv \{\theta^-, \gamma\}$. Again, we remove $\bar{a}_p$ to simplify notation as $\bar{a}_p = 0$ in equilibrium.

Note that in (22), $\omega_l'(\theta')$ and $a_l'(\theta', b_l')$ are simultaneously determined for a given b_l'.[24] That is, the Fund, as a security trader choosing $a_l'(\theta', b_l')$, determines $\omega_l'(\theta')$ by (22); alternatively, the Fund, as capacity announcer, could have chosen $\omega_l'(\theta')$ and use (22) to determine $a_l'(\theta', b_l')$. The variable $\mathcal{A}_f(\theta', b_l)$ represents an endogenous limit defined as

$$W^f(s', \mathcal{A}_f(\theta', b_l') - b_l', b_l') = \theta Z. \tag{23}$$

This condition restricts the extent of losses. Particularly, it ensures that the present discounted value of the Fund's assets are at least equal to $\theta^- Z \leq 0$. Specifically, when $Z = 0$, $\mathcal{A}_f(\theta', b_l')$ ensures that the sovereign's liabilities can be absorbed by the Fund without incurring permanent losses. Adding Eqs. (23) to the value of the lender (17) and applying the transversality condition (18), we obtain

$$W^f(s', \mathcal{A}_f(\theta', b_l') - b_l', b_l') + W^p(\theta', \mathcal{A}_f(\theta', b_l') - b_l', b_l') = \theta Z + b_l'.$$

This gives the decentralized counterpart of the Fund's participation constraint in (5),

$$W^l(s', a_l'(\theta', b_l'), b_l') \equiv W^f(s', a_l'(\theta', b_l'), b_l') + W^p(\theta', a_l'(\theta', b_l'), b_l') \geq \theta Z + b_l', \tag{24}$$

We interpret condition (24) as a proper DSA since it links the value of the current lending with its prospective stream of transfers. This DSA takes into account the sovereign's entire debt position – within and outside the Fund – in every possible state. Moreover, owing to the trade of Arrow securities, it is contingent on all the states that the contract specifies, including those states where participation constraints are binding.

Note that, with $\bar{a}_p(\theta) = 0$, the market clearing condition in the Fund is given by $a(\theta, b) + a_l(\theta, b) = 0$ for all (θ, b). In addition, the initial asset holdings of the sovereign in the Fund, $a(\theta_0, b_0) = -a_l(\theta_0, b_0) = 0$, are given.

4.3. Properties of the competitive equilibrium

We first define a (recursive) competitive equilibrium in this environment and then characterize the price dynamic and the optimal holdings of assets.

Definition 1 (*Recursive Competitive Equilibrium (RCE)*)**.** Given the outside options of the sovereign, $V^{af}(\theta')$, and of the lenders, $\theta^- Z + b_l$, a *Recursive Competitive Equilibrium* (RCE) consists of: prices $q_f(\theta', \omega'(\theta')|\theta)$ and $q_p(\theta, \bar{\omega}')$; value functions $W^b(\theta, a, b)$, $W^f(s, a_l, b_l)$, and $W^p(\theta, a_l, b_l)$; endogenous limits, $\mathcal{A}_b(\theta')$ and $\mathcal{A}_f(\theta', b_l')$; and policy functions $c(\theta, a, b)$, $c_f(\theta, a_l, b_l)$, $c_p(\theta, a_l, b_l)$, $n(\theta, a, b)$, $a'(\theta', b') = A(\theta', \theta, a, b, b')$, $a_l'(\theta', b_l') = A_l(\theta', \theta, a_l, b_l, b_l')$, $b' = B(\theta, a, b)$ and $b_l' = B_l(\theta, a_l, b_l)$, which are solutions to the problems of the sovereign, the private lenders and the Fund, and all markets clear. Particularly, the announcement $\omega_l'(\theta')$ is equal to its equilibrium value, i.e. $\omega_l'(\theta') = a_l'(\theta', b_l') + b_l' = -\omega'(\theta')$.

The definition of the RCE is made of two parts. The first part follows Alvarez and Jermann (2000) requiring optimality and markets clearing with the endogenous limits $\mathcal{A}_b(\theta')$ and $\mathcal{A}_f(\theta', b_l')$ defined as equilibrium objects. The second part of the definition makes it clear that the RCE has a Nash specification. On the one hand, the Fund takes the private lending policy as given in (20). On the other hand, the Fund's announcement $\{\omega_l'(\theta')\}_{\theta'|\theta}$ is not a constraint in either (13) or (17), while, as we said, it is part of (20).

We now characterize the price dynamic and the optimal holdings of assets in the decentralized environment. Using the fact that the borrowing constraints of the sovereign and the Fund do not bind at the same time, the price is determined by the agent whose constraint is not binding (Krueger et al., 2008).[25] Defining $\eta \equiv \beta(1 + r)$, it follows that

$$q_f(\theta', \omega'(\theta')|\theta) = \frac{\pi(\theta'|\theta)}{1 + r}\left[(1 - \delta + \delta\kappa) + \delta \sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right] \max\left\{\frac{u_c(c(\theta', a', b'))}{u_c(c(\theta, a, b))}\eta, 1\right\}. \tag{25}$$

Given the above price schedule, the intertemporal discount factor is defined by

$$Q_f\left(\theta', \omega'(\theta')|\theta\right) \equiv \frac{q_f\left(\theta', \omega'(\theta')|\theta\right)}{1 - \delta + \delta\kappa + \delta \sum_{\theta''|\theta'} q_f\left(\theta'', \omega''(\theta'')|\theta'\right)}. \tag{26}$$

The implicit interest rate in the Fund is then defined by $r_f(\theta, \bar{\omega}') \equiv \frac{1}{Q_f(\theta, \bar{\omega}')} - 1$ with $Q_f(\theta, \bar{\omega}') \equiv \sum_{\theta'|\theta} Q_f\left(\theta', \omega'(\theta')|\theta\right)$.

Provided that the private lenders have access to the Fund's securities, no arbitrage is possible between the Fund and the private bond market for the borrower. Hence, the bond prices in the Fund and the private bond market are alike.

[24] Even if it will not happen in equilibrium, the Fund must have a policy for the case that the interaction between the sovereign and private lenders ends with an over-lending which makes the continuation of the contract unfeasible. Then, its policy is to do as it does at the beginning of the Fund contract: discontinue the contract unless there is a debt restructuring that makes its intervention possible and credible.

[25] If both constraints would bind at the same time, Assumption 1 would be violated.

Proposition 2 (*Bond Price*). *In an RCE, for all* $(\theta, \omega'(\theta'))$,

$$\sum_{\theta'|\theta} q_f\left(\theta', \omega'(\theta')|\theta\right) = q_p(\theta, \bar{\omega}').$$

Moreover, whenever (24) *binds,* $\sum_{\theta'|\theta} q_f\left(\theta', \omega'(\theta')|\theta\right) > \frac{1-\delta+\delta\kappa}{1+r-\delta}$.

Given the definition of the price in (25), if the Fund's DSA is binding, the price of a bond reads $q_f(\theta, \bar{\omega}') > \frac{1}{1+r}\sum_{\theta'|\theta}\pi(\theta'|\theta)\left[(1-\delta+\delta\kappa)+\delta q_f(\theta', \bar{\omega}'')\right]$, or equivalently $Q_f(\theta, \bar{\omega}') > \frac{1}{1+r}$ implying that $r_f(\theta, \bar{\omega}') < r$. In words, when the Fund's DSA binds, a *negative spread* appears. The Fund's binding DSA has therefore two opposite effects. On the one hand, accumulating debt, $\bar{a}'_l > 0$, is cheaper owing to the fact that $q_f(\theta, \bar{\omega}')$ is above the risk-free price. On the other hand, buying insurance, $\hat{a}'_l(\theta') < 0$, becomes more expensive.

The negative spread is a strong signal that the Fund refrains from further lending and causes private lenders to stop lending to the sovereign as the rate of return settles below r. At this rate, the private lenders are willing to *borrow* from the Fund in terms of a portfolio of securities which constitutes risk free asset a_p, and investing the funds to earn a risk free rate r. Nevertheless, the binding DSA of the Fund also prevents such trading activities. As a result, a binding DSA in (24) not only restricts the provision of the Fund's insurance to the sovereign, it also sustains a no-trade equilibrium in the private bond market: there is a private lending 'sudden stop'.

From Proposition 2, without (24), there is no negative spread and the private lenders are willing to provide all the debt the sovereign asks for. Thus, without negative spread, the private lenders fail to realize that the present value of future transfers does not cover additional lending. As a result, they would lend "too much" in the sense that the sovereign becomes a permanent net debtor to the rest of the world. The negative spread prevents such excessive lending. In particular, it ensures any amount lent is appropriately covered by future transfers and therefore guarantees no permanent positive transfers to the sovereign when $Z = 0$. Thus, (24) internalizes *the pecuniary externality of a negative spread* that competitive private lenders do not: the fact that marginal lending can be excessive.

When (24) binds, the maximal amount of debt the Fund may have to absorb is $\theta^- Z + \delta b_l - \min\left\{\hat{a}'_l(\theta') : (24) \text{ binds} \wedge \pi(\theta'|\theta) > 0\right\}$. On the one hand, the Fund provides the transfer component $\theta^- Z \leq 0$ and the complement to the maximal amount of insurance the sovereign may receive with positive probability. On the other hand, from the perspective of the private lenders, the Fund has to guarantee a maximal absorption of δb_l. In other words, the Fund must stand ready to guarantee just enough lending for the sovereign to honour its long-term liabilities. This is because the private lending sudden stop endangers the ability of the sovereign to maintain the value of its long-term debt, either directly — under the counterfactual interpretation that each period the sovereign buys and sells the long-term debt — since it may not be able to borrow from the private lenders to cover it; or, indirectly since private lenders may want to sell their holdings of over-priced, low-return, long-term debt in exchange for safe assets. The Fund's guarantee is therefore a form of prudential policy which is active as long as debt is long term (i.e. $\delta > 0$).

Proposition 3 (*Private Debt*). *In a RCE, in the states in which* (24) *binds,* $b'_l \leq \delta b_l$. *Conversely, in the states in which* (24) *does not bind, the division of* $\bar{\omega}'_l$ *between* b'_l *and* $\bar{a}'_l$ *is indeterminate.*

However, when the Fund's DSA in (24) does not bind, the sovereign can equally access the private bond market and the Fund. In this case, given Proposition 2, debt is as expensive in the Fund as in the private bond market and the sovereign can accumulate debt in both locations. Therefore, the sovereign is indifferent between holding debt in the private bond market or in the Fund. It is then without loss of generality that we can set $\bar{a}'_l = 0$ whenever (24) does not bind. As we have said, our underlying assumption is that as long as there are no spreads (positive or negative) on the debt's interest rates, private lenders are willing to buy all the debt being offered by the sovereign. We can therefore define the Fund's *minimal intervention policy* (MIP) in the following terms.

Definition 2 (*The Fund's Minimal Intervention Policy (MIP)*). For a given state (θ, b_l), we say that the Fund implements a Minimal Intervention Policy (MIP) if $\bar{a}'_l = \underline{a}(\theta, b_l)$ where, if (24) binds $\underline{a}(\theta, b_l) \in [\check{a}_l, \check{a}_l + \delta b_l]$ with $\check{a}_l \equiv \theta^- Z - \min\left\{\hat{a}'_l(\theta') : (24) \text{ binds} \wedge \pi(\theta'|\theta) > 0\right\}$ and $\underline{a}(\theta, b_l) = 0$ otherwise.

Having characterized the bond price and the Fund's MIP, we first show that the Second Welfare Theorem (SWT) holds before turning to the First Welfare Theorem (FWT).

Proposition 4 (*Second Welfare Theorem (SWT)*). *Given initial conditions* $\{\theta_0, b_0, x_0\}$, *the Fund's constrained efficient allocation can be decentralized as a RCE with endogenous borrowing and lending limits.*

In solving the Fund contract, the Fund takes the strategy of the private lenders and the sovereign as given. In particular, it solves for consumption and leisure and the corresponding transfers, which, given the lending strategy of the private agents, split between the private lenders and the Fund, with the latter also providing insurance. The proof of the SWT requires to map the structure of the Fund's contract, accounting for its lending from competitive private lenders, into the more decentralized market structure of the RCE. A key step is to map the state of the Fund (θ, x) to the state of the RCE (θ, a_l, b_l), given the lending strategy of the lenders; that is, to map from (θ, x) to (θ, ω_l) and, giving b_l, a_l is determined. This map is given by the identification of the consumption policies and the Fund's consumption first-order condition

$$u_c(c(\theta, a, b)) = u_c(c(\theta, x, b_l)) = \frac{1 + \nu_l(\theta, x, b_l)}{1 + \nu_b(\theta, x, b_l)}\frac{1}{x},$$

and, since by (8) the right hand side is equal to η/x', the law of motion of the co-state variable x maps into the borrower's Euler equation. Using this, we define the Fund contract as a long-term state-contingent asset and derive the corresponding asset prices. Then, we map policies and value functions and show that they satisfy the RCE conditions of Definition 1. Furthermore, by Proposition 1, the constrained efficient allocation is unique (when Assumption 1 is satisfied), therefore the RCE of Proposition 4 can take different forms (e.g. different asset structures), but the corresponding RCE allocation is also unique.

The SWT is satisfied in many environments. This is not the case for the FWT since multiplicity of equilibria usually prevails; in particular, inefficient equilibria, such as autarky. We first introduce an assumption that, similar to Assumption 1, ensures the uniform boundedness of the Lagrange multipliers in the decentralized economy.

Assumption 2 (*Decentralized Interiority*). There is an $\epsilon > 0$, such that, for all equilibrium states (θ, a, b) the sovereign, lenders and Fund problems —(13), (17) and, (20) — have a solution when the right hand sides of constraints (15) and (22) are replaced by $\mathcal{A}_b(\theta') + \epsilon$ and $\mathcal{A}_f(\theta', b'_l) + \epsilon$, respectively.

In general, equilibrium boundedness follows from standard monotonicity of preferences and an interiority, *free disposal*, assumption. Assumption 2 introduces the equivalent to *free disposal* when there are endogenous limit constraints. In particular, it dismisses autarky as the Lagrange multiplier of the budget constraint is unbounded in the autarkic allocation.[26]

Proposition 5 (*First Welfare Theorem (FWT)*). *Given initial conditions $\{\theta_0, b_0, a_0\}$, a RCE with endogenous borrowing and lending limits, satisfying Assumption 2 implements the constrained efficient allocation of the Fund.*

In the decentralized economy it is even more explicit that the Fund takes the strategy of the private lenders and the sovereign as given, as well as asset prices contingent on the sovereign's liabilities. The proof of the FWT requires the (inverse) map from the market structure — given by problems (13), (17) and (20), and the corresponding equilibrium conditions — to the structure of the Fund contract problem. The starting point is the first-order condition of the sovereign's problem (13): $u_c(c(\theta, a, b)) = \varkappa(\theta, a, b)$, where $\varkappa(\theta, a, b)$ is the Lagrange multiplier of the budget constraint (14). From the sovereign's and Fund's Euler equations we obtain the following intertemporal relation between these multipliers:

$$\varkappa(\theta, a, b) = \eta \frac{1 + \mathring{v}_b(\theta', a', b')}{1 + \mathring{v}_l(\theta', a', b')} \varkappa'(\theta', a', b'),$$

where $\mathring{v}_b(\theta', a', b')$ and $\mathring{v}_l(\theta', a', b')$ are normalized Lagrange multipliers of the endogenous limit constraints (15) and (22). As it can be seen, this intertemporal relation mirrors the law of motion of the co-state variable (8), which is at the core of Fund's problem. With the (inverse) map of value and policy functions it follows that the RCE allocation is a solution to the Fund's problem. Furthermore since, again by Proposition 1, the solution is unique the RCE allocation (when Assumption 2 is satisfied) must also be unique.

5. The seniority structure of the fund

So far, we assume that the Fund has no seniority with respect to the privately held sovereign debt. We therefore consider that a default always implicates both the Fund and the private lenders. We now relax this assumption allowing for a *partial* default in which the sovereign defaults only on its private liabilities while remaining in the Fund.

5.1. The sovereign and the private lenders under seniority

Compared to the case without seniority, the sovereign possesses two outside options. On the one hand, it can default on both the private lenders and the Fund. This represents the case of *full* default considered previously. On the other hand, the sovereign can repudiate its private debt while remaining in the Fund. We refer to this situation as a *partial* default because the sovereign solely defaults on the private lenders. That is, if the sovereign has an outstanding debt of $\omega = a + b$, it defaults on b and repays a. We assume that the default penalty and the re-access probability are the same in *partial* and *full* defaults.

There is a clear tradeoff when deciding whether to enter *partial* default. On the one hand, in *partial* default, the sovereign is less productive — i.e. $\theta^d \le \theta$ — for some time. On the other hand, the sovereign repudiates is private liabilities — i.e. $b = 0$ — and continues to receive support from the Fund. That is, unlike in *full* default, it can still trade bonds and insurance with the Fund. Given this, the state space in the decentralized economy is now (θ, a, b, d_p) where $d_p = 1$ if the sovereign is in *partial* default and $d_p = 0$ otherwise. Hence, in a given state (θ', a', b'), the sovereign does not enter in *partial* default if

$$W^b(\theta', a'(\theta', b', 0), b', 0) \ge W^b(\theta', a'(\theta', 0, 1), 0, 1), \tag{27}$$

where the value upon *partial* default reads

[26] With Assumption 2 we restrict our attention to allocations enabling risk sharing between the contracting parties, which rules out an autarky equilibrium and, by Proposition 4.10 in Alvarez and Jermann (2000), their *high implied interest rates* condition is satisfied in our constrained efficient equilibrium; i.e. our Assumption 2 implies their condition.

$$W^b(\theta,a,0,1) = \max_{\{c,n,\{a'(\theta',0,d_p')\}_{\theta',d_p'}\}} U(c,n) + \beta \mathbb{E}\left[(1-\lambda)W^b(\theta',a'(\theta',0,1),0,1) + \lambda W^b(\theta',a'(\theta',0,0),0,0)\Big|\theta\right]$$

$$\text{s.t.}\quad c + \sum_{\theta'|\theta,d_p'} q_f(\theta',a'(\theta',0,d_p'),0|\theta)(a'(\theta',0,d_p') - \delta a) \le \theta^d f(n) + (1-\delta+\delta\kappa)a,$$

$$a'(\theta',0,d_p') = \bar{a}'(0) + \hat{a}'(\theta',d_p') \ge \mathcal{A}_b(\theta',d_p').$$

We then define $V^{ap}(\theta,a) \equiv W^b(\theta,a,0,1)$.[27] In the case of *partial* default, the endogenous borrowing limit is defined as $W^b(\theta',\mathcal{A}_b(\theta',1),0,1) = V^{af}(\theta')$, while in the case of repayment $W^b(\theta',\ddot{a}'(\theta',\ddot{b}',0),\ddot{b}',0) = V^{af}(\theta')$ for all $\ddot{a}'(\theta',\ddot{b}',0)+\ddot{b}' = \mathcal{A}_b(\theta',0)$.

Compared to the case without seniority, a' is now a function of the *partial* default status next period, d_p'. As the bond component $\bar{a}'$ is not contingent, it is the Arrow component, $\hat{a}'$, that depends on d_p'. This is because the sovereign is less productive in *partial* default — i.e. $\theta^d \le \theta$ — and repudiates its liabilities towards private lenders — i.e. $b = 0$. The Fund's insurance component must therefore discriminate whether the sovereign is in default as the sovereign's risk profile changes.

The private lenders' problem remains static as in Section 4. Thus, when the DSA does not bind, the private bond price is given by

$$q_p(\theta,\bar{a}',b') = \frac{\mathbb{E}\{(1-D(\theta',a',b'))[1-\delta+\delta\kappa+\delta q_p(\theta',\bar{a}'',b'')]|\theta\}}{1+r}, \tag{28}$$

where $D(\theta,a,b) = D_p(\theta,a,b) + D_f(\theta,a,b)$ with $D_p(\theta,a,b) = 1$ if $V^{ap}(\theta,a) > W^b(\theta,a,b,0)$ and $V^{ap}(\theta,a) \ge V^{af}(\theta)$ and $D_p(\theta,a,b) = 0$ otherwise, while $D_f(\theta,a,b) = 1$ if $V^{af}(\theta,a) > W^b(\theta,a,b,0)$ and $V^{af}(\theta,a) > V^{ap}(\theta)$ and $D_f(\theta,a,b) = 0$ otherwise. The value under *full* default might coincide with the value under *partial* default. Hence, if the sovereign is indifferent between the two types of default, we assume it selects the *partial* default.

However, the price may not depend on the total level of debt $\bar{\omega}'$ anymore but on $\bar{a}'$ and b' separately. As we will see, the split of $\bar{\omega}'$ between $\bar{a}'$ and b' becomes relevant as in *partial* default the sovereign defaults on b' but repays $\bar{a}'$.

5.2. The fund under seniority

The Fund still aims at making the sovereign's debt safe. Thus, even though it possesses seniority, its announcement continues to relate to the sovereign's entire indebtedness as in the case without seniority. However, in addition to (θ,a,b), the announcement now includes the default status d_p. That is, depending on the *partial* default decision, the sovereign does not necessarily receive the same amount of resource from the Fund. Again, this is because a *partial* default affects the sovereign's risk profile.

As we have seen previously, the sovereign's participation constraint continues to relate to the case of *full* default. The rationale is that, in the contract with seniority, the sovereign defaults on the Fund only in the case of a *full* default. A *partial* default solely affects private lenders. This means that if the value under *partial* default is greater than the value of *full* default in some states, *partial* defaults can occur. In other words, the sovereign's participation constraint alone is not sufficient to prevent *partial* defaults.

The Funds's participation constraint may change in the case of seniority. Particularly, the transfer to the private lenders is now given by $\tau_{p,t} = q_p(\theta_t,\bar{a}_{t+1},b_{t+1})b_{t+1} - (1-\delta+\delta\kappa+\delta q_p(\theta_t,\bar{a}_{t+1},b_{t+1}))b_t(1-D_t)$. Hence, depending on whether there are *partial* defaults, given (2), b_t might not be the same as in the Fund's participation constraint without seniority. The difference comes from the private bond market exclusion and the haircut following a default. Furthermore, given that $\theta^d \le \theta$, a *partial* default impacts the sovereign's output which also affects the transfer to the Fund, τ_f. Hence, only without *partial* default does the Fund's participation constraint with and without seniority coincide.

5.3. The fund's minimal intervention policy under seniority

To evaluate the importance of the seniority assumption, we need to check whether the sovereign is willing to follow the Fund's announcement when we impose seniority. For this purpose, we define the Fund's MIP under seniority such that a *partial* default is never optimal.

First, observe that a *partial* default can occur only when the sovereign holds private debt. In other words, if $\bar{a}_l' = \bar{\omega}_l'$ and $b' = 0$, there is no *partial* default. In opposition, if *partial* default is optimal next period, the sovereign would like to set $b' = \bar{\omega}'$ and $\bar{a}' = 0$. Moreover, if $\theta^d = \theta$ for all θ, there is no penalty upon *partial* default meaning that it is not possible to sustain debt in the private bond market. This follows from the standard result in Bulow and Rogoff (1989). In the same logic, if $\theta^d < \theta$ for at least one θ, then the sovereign can hold some level of private debt without being willing to enter *partial* default. Thus, the MIP is the minimal level of debt the Fund should absorb such that (27) holds for all θ' for which $\pi(\theta'|\theta) > 0$.

Definition 3 (*The Fund's Minimal Intervention Policy (MIP) Under Seniority*)**.** For a given state θ, we say that the Fund implements a Minimal Intervention Policy (MIP) under seniority if $\bar{a}_l' = \max\{\underline{a}(\theta,b_l),\underline{\underline{a}}(\theta)\}$ where $\underline{a}(\theta,b_l)$ is given in Definition 2 and $\underline{\underline{a}}(\theta) = \max\big\{\{-\bar{a}' > 0 : (27) \text{ binds } \wedge\ \pi(\theta'|\theta) > 0\} \cup \{0\}\big\}$.

[27] The value under repayment is a simple extension of (13) with the additional state variable $d_p = 0$.

The sovereign does not have any incentive to enter *partial* default if Definition 3 is satisfied. Thus we come up with the following proposition.

Proposition 6 (*MIP and Partial Default*). *In equilibrium, if $\bar{a}'_l \geq \underline{\underline{a}}(\theta)$, the sovereign never enters in partial default. Conversely, if $0 < \bar{a}'_l < \underline{\underline{a}}(\theta)$, the sovereign is willing to enter in partial default in at least one θ'.*

The first part of the proposition directly follows from Definition 3: the MIP under seniority is such that there is no *partial* default. Note that depending on the severity of the output penalty and the duration of the private bond market exclusion, (27) may hold in all states with $\underline{\underline{a}}(\theta) = 0$. In other words, under the Fund's seniority, the Fund's MIP can be identical to the one without seniority given in Definition 2. This is the case in our calibration below. The second part of the proposition states that if the MIP is violated there is a strictly positive probability of a *partial* default next period.

Nevertheless, the next proposition shows that the sovereign cannot deviate from the Fund's MIP. If the DSA binds with strictly positive probability next period, as we have seen in Section 4, a negative spread arises and the private lenders prefer borrowing from the Fund rather than lending to the sovereign. In other words, private lenders would like that the Fund absorbs all the sovereign debt. In this case, there is no possibility for the sovereign to deviate — say by accumulating more private debt — from the Fund's MIP.

In contrast, if the DSA does not bind next period, without the MIP (Definition 3), there is no private lending sudden stop and the sovereign can freely accumulate debt in the private bond market. In addition, the Fund adapts the insurance it provides to the sovereign according to the *partial* default status. Arrow securities therefore aim at equating wealth not only across productivity states but also across repayment states. Thus, for a given level of debt in the Fund, the repayment decision is not state contingent when the DSA does not bind. However, with the MIP (Definition 3), the private lenders anticipate that, giving the Fund debt holdings, partial default will occur with probability one, and, therefore, set the bond price to zero consistent with (28). In this case, there is a *lending sudden stop*, not because the private lenders are trying to borrow from the Fund but to escape from a *partial* default. Again, the sovereign cannot deviate from the Fund's MIP.

Proposition 7 (*No Partial Default*). *For a given Fund's announcement $\bar{\omega}'_l$, the sovereign cannot deviate from the Fund's MIP given in Definition 3:*

I If the DSA binds in at least one θ', the private lenders do not lend as of Proposition 3.
II If the DSA does not bind, then for all $b' < -(\bar{\omega}'_l - \underline{\underline{a}}(\theta))$ we have that $\mathbb{E}[D_p(\theta', -\bar{\omega}'_l - b', b')|\theta] = 1$ implying that $q_p(\theta, -\bar{\omega}'_l - b', b') = 0$.

All in all, depending on the output penalty upon default and the re-access probability, the Fund's MIP might differ in the case with and without seniority. In the former, the Fund may need to absorb relatively more debt. However, in equilibrium, the sovereign cannot profitably deviate from the Fund's MIP. Thus, the entire debt position remains safe as no default — either *partial* or *full* — arise on equilibrium path.[28] Thus, the seniority only affects the Fund and, in that view, a *pari passu* clause is preferable to seniority.

6. Calibration

We calibrate the parameters of the model economy by fitting the sovereign debt model (9)–(10), i.e. the one without the Fund, to quarterly data of Italy over the period 1992Q1 to 2019Q4.[29] Table 1 summarizes the value of each parameters.

We calibrate the productivity growth rate shock γ_t with a Markov regime switching AR(1) process to the sample productivity series of Italy. We choose a specification of 2 regimes that we denote by $\varsigma \in \{1, 2\}$, with the first regime capturing the crisis period (i.e. the Great Financial Crisis) observed in the data. Specifically, we estimate the following model for the (net) growth rate $\gamma_t - 1$ with the expectation maximization (EM) algorithm of Hamilton (1990):

$$\gamma_t - 1 = (1 - \rho(\varsigma_t))\mu(\varsigma_t) + \rho(\varsigma_t)(\gamma_{t-1} - 1) + \sigma(\varsigma_t)\epsilon_t, \tag{29}$$

where ς_t denotes the regime at t, $\rho(\varsigma_t)$, $\mu(\varsigma_t)$, $\sigma(\varsigma_t)$ are the regime-specific autocorrelation, mean and variance of the process, respectively, and ϵ_t follows an i.i.d. standard normal distribution. As shown in the Online Appendix E, such a regime switching process can capture the sudden drop in productivity dynamics around crisis periods. In the computation, we further discretize the shock process using the method of Liu (2017) with 15 grid points for each regime. Aguiar and Gopinath (2006) show that given a CRRA utility in consumption $\frac{c^{1-\sigma}}{1-\sigma}$, one requires that $\lim_{t\to\infty} \mathbb{E}_0 \beta^t(\theta_{t-1}^{1-\sigma} - 1)/(1-\sigma) = 0$, so that the discount utility can be well defined with stochastic trend. For the case of log utility, this amounts to $\lim_{t\to\infty} \mathbb{E}_0 \beta^t \log \theta_{t-1} = 0$, which holds automatically in our setup. We subsequently detrend an 'allocation' variable x_t by θ_{t-1}: $\tilde{x}_t = x_t/\theta_{t-1}$.

The preference parameters for labour supply are set to $\zeta = 0.29$ and $\xi = 1.265$. These are used to match the average fraction of working hours and its correlation with GDP, together with the volatility of consumption relative to GDP. The risk free interest rate is fixed to $r = 1.32\%$, the average real short-term interest rates of the Euro area. We further set $\delta = 0.9297$ and $\kappa = 0.0543$ to match the average Italian bond maturity and coupon rate (coupon payment to debt ratio), respectively. Finally, we fix $\beta = 0.96$ to match

[28] Wicht (2023) shows that in an environment without state-contingent contracts, seniority is actually preferable to a *pari passu* regime.

[29] The calibration starts in 1992 due to data availability and ends in 2019 owing to the pandemic. The Online Appendix E contains detailed explanations on data sources, measurement, and additional information on shock process estimation.

Table 1
Parameter values.

Parameter	Value	Definition	Targeted moment
A. Direct measures from data			
α	0.5295	labour share	labour share
λ	0.032	return probability	average exclusion period
r	0.0132	risk-free rate	annual real short-term rate
δ	0.9297	bond maturity	bond maturity
κ	0.0543	bond coupon rate	bond coupon rate
B. Based on model solution			
β	0.96	discount factor	average b'/y
ψ	0.746	productivity penalty	corr(spread, y)
ζ	0.29	labour elasticity	average n, $\sigma(c)/\sigma(y)$ and corr(n, y)
ξ	1.265	labour utility weight	
C. By assumption			
Z	0	Fund's outside option	

Note: The variable $\sigma(\cdot)$ denotes the volatility.

Table 2
Data and models.

Targeted moments				Non-Targeted moments			
Variable	Data	Without Fund	With Fund	Variable	Data	Without Fund	With Fund
			A. First Moments				
b'/y%	117.64	116.20	221.00	τ/y%	2.09	6.49	9.54
n%	38.64	38.23	39.93	spread%	2.50	0.43	0.00
			B. Second Moments				
$\sigma(c)/\sigma(y)$	1.27	1.25	0.28	σ(spread)	0.96	0.11	0.00
corr(n, y)	0.68	0.63	0.99	$\sigma(n)/\sigma(y)$	0.75	1.42	0.62
corr(spread, y)	−0.16	−0.25	0.00	corr(c, y)	0.53	0.04	0.95
				$\sigma(\tau/y)/\sigma(y)$	1.09	2.32	0.72
				corr$(\tau/y, y)$	0.29	0.71	0.98

Note: The variable $\sigma(\cdot)$ denotes the volatility and τ/y denotes the primary surplus (i.e $\theta f(n) - c$) over output. We simulate 5,000 economies with 600 periods each, and we discard the first 200. For the volatilities and correlation statistics, we filter the simulated data — except the spread — through the HP filter with a smoothness parameter of 1600.

the average indebtedness relative to annual output. The production function is Cobb–Douglas $f(n) = n^{\alpha}$, and we set $\alpha = 0.5295$ to match the average labour share in Italy.

The default penalty is asymmetric as in Arellano (2008). To ensure that we can properly detrend the penalty, we consider

$$\theta_t^d = \theta_{t-1}\psi\mathbb{E}\gamma_t \text{ if } \theta_t \geq \theta_{t-1}\psi\mathbb{E}\gamma_t \quad \text{and} \quad \theta_t^d = \theta_t \text{ if } \theta_t < \theta_{t-1}\psi\mathbb{E}\gamma_t.$$

One sets $\psi = 0.746$ to match the correlation of spread with respect to output. Furthermore, we fix $\lambda = 0.032$ which corresponds to an average default duration between 7 and 8 years. This is consistent with the average default length Italy recorded during its defaults on external debt in the 1930s and the 1940s (Reinhart and Rogoff, 2011). Note that under such parameter values, the MIP is the same with and without seniority.

7. Quantitative analysis

In this section, we first assess the fit of the model to the data. We then compare the economy with and without the Fund through various exercises.

7.1. Model fit and comparison

The fit of the model with respect to the data is depicted in Table 2. As we calibrate the model to Italy, the relevant benchmark is the economy without the Fund. To compute the moments we run 5,000 simulations of the model with 600 periods each, and we discard the first 200. For the volatilities and correlation statistics, we filter the simulated data — except the spread — through the HP filter with a smoothness parameter of 1600.

As one can see, the model replicates well the average indebtedness of Italy owing to the long-term debt structure (Chatterjee and Eyigungor, 2012). We are also matching the share of hours worked and its correlation with output given the specification of the shocks. The same holds true for the volatility of consumption. In addition, the model replicates well the correlation of the spread

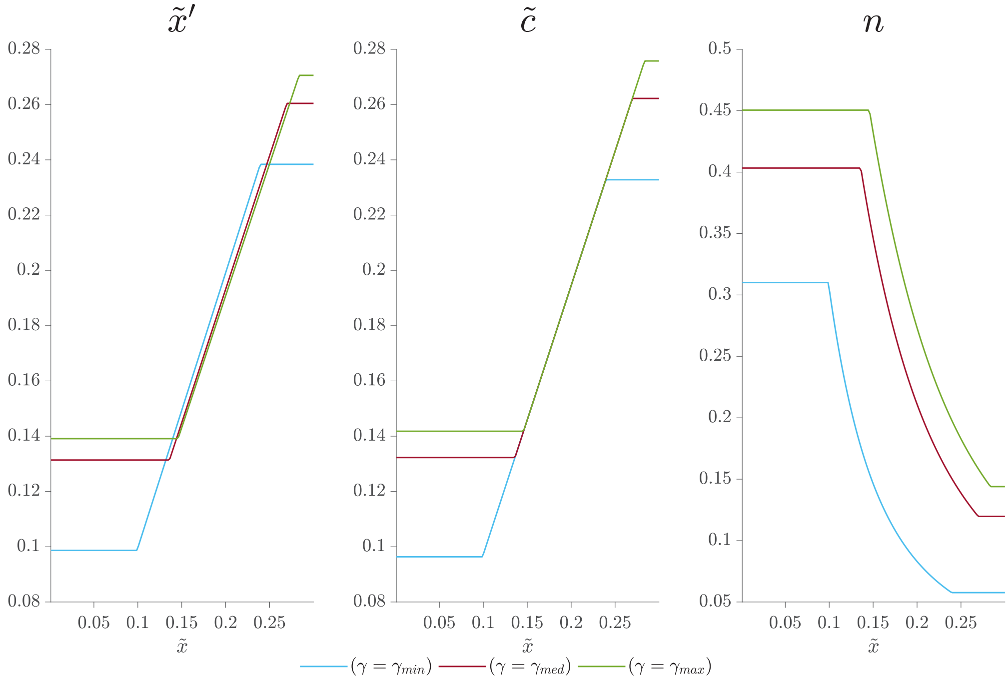

Fig. 1. Optimal policies with zero private debt as function of $(\gamma, \tilde{x})$.
Note: The Figure depicts the optimal policies for the relative Pareto weight, $\tilde{x}'$, consumption, $\tilde{c}$, and labour, $\tilde{n}$ as a function of $(\gamma, \tilde{x})$. We fix $\tilde{b} = 0$ and consider three main values of the growth rate: the smallest one, γ_{min}, the median one, γ_{med}, and the highest one, γ_{max}.

with output. However, it cannot match the average spread observed in the data.[30] In terms of other non-targeted moments, the model also exaggerates the volatility and the correlation of the primary surplus.

In addition, Table 2 compares the economy with and without the Fund. The difference between the two is important. First, the Fund enables a greater accumulation of debt in total. Particularly, the Fund almost doubles the debt capacity of the economy. Nevertheless, with the MIP, the Fund's debt holdings is nil given that the Fund's DSA never binds in steady state as we will see. Second, there is no spread with the Fund, while the spread attains 0.43% without the Fund. Hence, the Fund achieves the goal of making sovereign debt safe — i.e. without default risk. Third, consumption is much less volatile in the presence of the Fund. This means that there is a greater risk sharing across states. This comes from the highly pro-cyclical surplus. In other words, in periods of distress, the Fund provides resources to sustain consumption. Such mechanism is less marked in the economy without the Fund owing to the risk premium attached on the debt and the lack of state contingency.

7.2. Policy functions and financial variables

To gain better understanding of the working of the Fund, we first present the numerical solutions of the policy functions of the Fund under our calibration. Fig. 1 depicts the different policy functions for zero private debt as a function of $(\gamma, \tilde{x})$, while Fig. 2 depicts the main financial variables. All figures relate to the detrended version of the model presented in the Online Appendix B. We focus on three main values of the growth rate: the smallest one, γ_{min}, the median one, γ_{med}, and the highest one, γ_{max}. We denote the annual output by $\tilde{y}$.

Fig. 1 presents the optimal policies with respect to the future relative Pareto weight, consumption and labour as function of $(\gamma, \tilde{x})$. As explained in Section 3 and in the Online Appendix A, the recursive formulation of the Fund relies on the relative Pareto weight $\tilde{x}$ which keeps track of the binding constraints. With a logarithmic utility, one has that $\tilde{c} = \tilde{x}' \frac{\gamma}{\eta}$. Both $\tilde{c}$ and $\tilde{x}'$ are increasing, while n is decreasing in the current relative Pareto weight $\tilde{x}$. In each panel, the horizontal line on the left hand side is determined by the sovereign's binding participation constraint, while the horizontal line on the right hand side is determined by the Fund's binding participation constraint. The line rejoining both horizontal lines is determined by the first best allocation and has a slope of $\eta < 1$.

[30] Models of sovereign defaults following Aguiar and Gopinath (2006) and Arellano (2008), with stochastic growth shocks and risk-neutral lenders, have similar difficulty to match the average spreads typical for emerging economies. Chatterjee and Eyigungor (2012) manage to match an average spread of 8% by means of long-term debt and quadratic output penalty but do not use growth shocks. Bocola and Dovis (2019) also match the average spread using multiple maturities but target an average spread of 0.61%; see also Bocola et al. (2019).

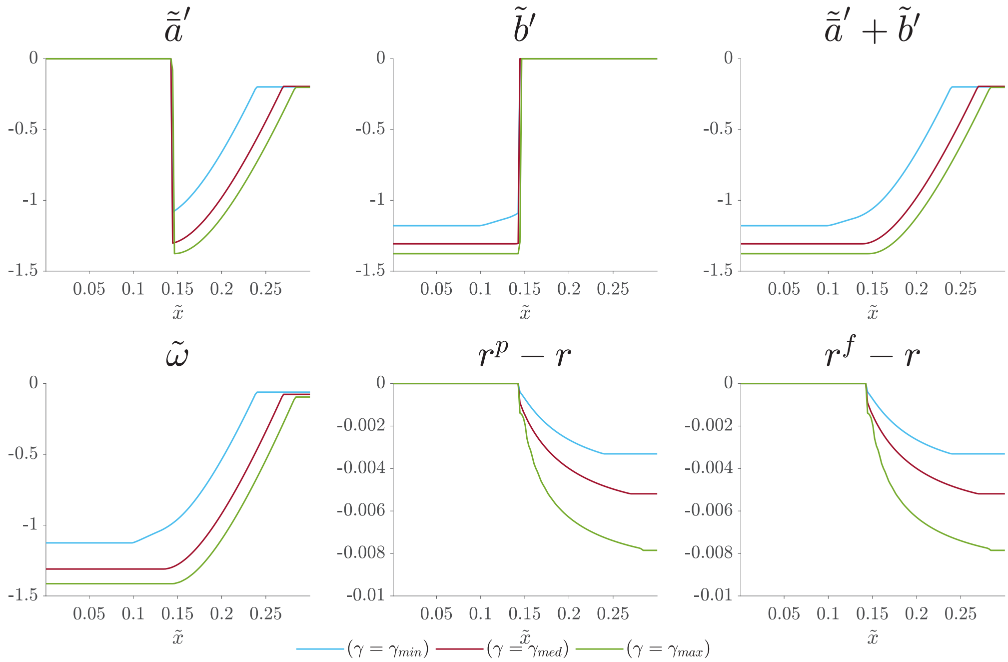

Fig. 2. Financial variables with zero private debt as function of $(\gamma, \tilde{x})$.
Note: The Figure depicts the optimal policies for the debt in the Fund, $\tilde{\tilde{a}}'$, the debt in the private bond market, $\tilde{b}'$, together with the outstanding total liabilities, $\tilde{\omega}$, the spread in the Fund, $r^f - r$, and the spread in the private bond market, $r^p - r$. We fix $\tilde{b} = 0$ and consider three main values of the growth rate: the smallest one, γ_{min}, the median one, γ_{med}, and the highest one, γ_{max}.

We now turn to the financial variables depicted in Fig. 2. The first row of the figure represents the prospective debt holdings of the sovereign. Consistent with the definition of MIP, when the Fund's DSA does not bind, the credit line of the Fund is nil. Conversely, when the Fund's DSA binds, there is a private lending sudden stop. With zero initial private debt this translates into a complete stop of private lending activities. In this case, the debt accumulation is largely reduced.

The second row of Fig. 2 depicts the current asset holdings and the interest spreads. One sees that when the Fund's DSA is binding, $\tilde{\omega}$ is very close to zero because of Definition 2 and the fact that $Z = 0$ and $\tilde{b} = 0$. As $\tilde{\omega} = \tilde{\tilde{\omega}} + \tilde{\tilde{a}}(\gamma)$, this tells us that if the Fund's DSA is binding today then the value of the sovereign's debt is in great part offset by the value of the realized Arrow security. Hence, when the Fund's DSA binds, the sovereign is limited in the trade of both Arrow securities and bonds.

Regarding interest rates, the Fund's and private bonds market's spreads are nil when the Fund's DSA is not binding consistent with Corollary 1. In contrast, spreads are negative when the Fund's DSA is binding consistent with Proposition 2. As one can see, the negative spread remains relatively modest in terms of magnitude.

7.3. *Steady state analysis*

As detailed in Appendices A and C, the relative Pareto weight, $\tilde{x}$, is key to the dynamics of the model economy as it represents a sufficient statistic of the contract's binding constraints. We first explain the dynamic of the relative Pareto weight before simulating the economy with and without the Fund in steady state.

Fig. 3 displays the law of motion of the relative Pareto weight. The dark grey region represents the ergodic set given in Definition C.1. The light grey region represents the basin of attraction of the ergodic set. As one can see, the convergence path to the steady state depends on the level of privately held debt. Especially, the larger is the level of private debt, the closer the economy gets to the ergodic set. This is different than in Ábrahám et al. (2022) where the convergence path solely depends on $\tilde{x}$ ad γ.

Most importantly, we see that the Fund's DSA does not bind in steady state. This has two main consequences. First, the private lending sudden stop exposed in Proposition 3 does not arise in the long run. Second, in line with Definition 2, the Fund's holding of sovereign debt is nil — i.e. $\tilde{a}' = 0$. In other words, the Fund solely provides insurance.[31]

We simulate the economy within the ergodic set of relative Pareto weights. For this purpose, we generate one history of shocks for 500 periods in steady state starting with the lowest Pareto weight in the ergodic set. To avoid that the initial conditions blur the results, the first 200 periods are discarded. To gauge the impact of the Fund's intervention in this exercise, we simulate both the economy with and without the Fund in parallel.

[31] Also, consistent with Corollary C.1, the average bond maturity is irrelevant.

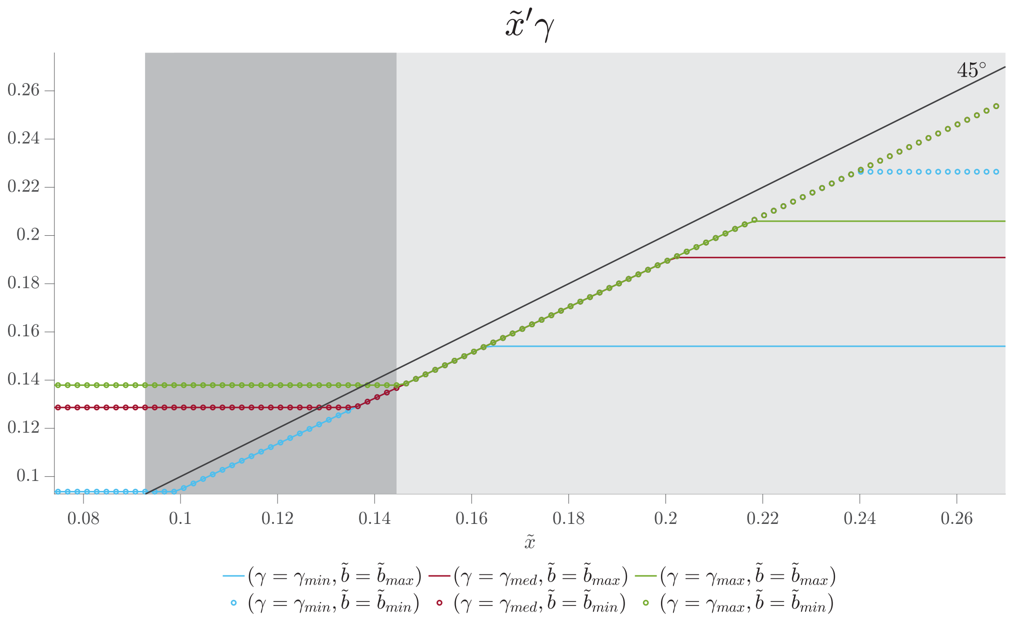

Fig. 3. Evolution of the relative pareto weight in steady state as a function of $(\gamma, \tilde{b}, \tilde{x})$.
Note: The figure depicts the law of motion of the relative Pareto weight for different growth states and different private debt levels. The dark grey x-axis region is the ergodic set which defines the steady state of the economy. The light grey region is the basin of attraction of the ergodic set. We take $\tilde{b} = \tilde{b}_{min} = 0$ and $\tilde{b} = \tilde{b}_{max}$ and consider three main values of the growth rate: the smallest one, γ_{min}, the median one, γ_{med}, and the highest one, γ_{max}.

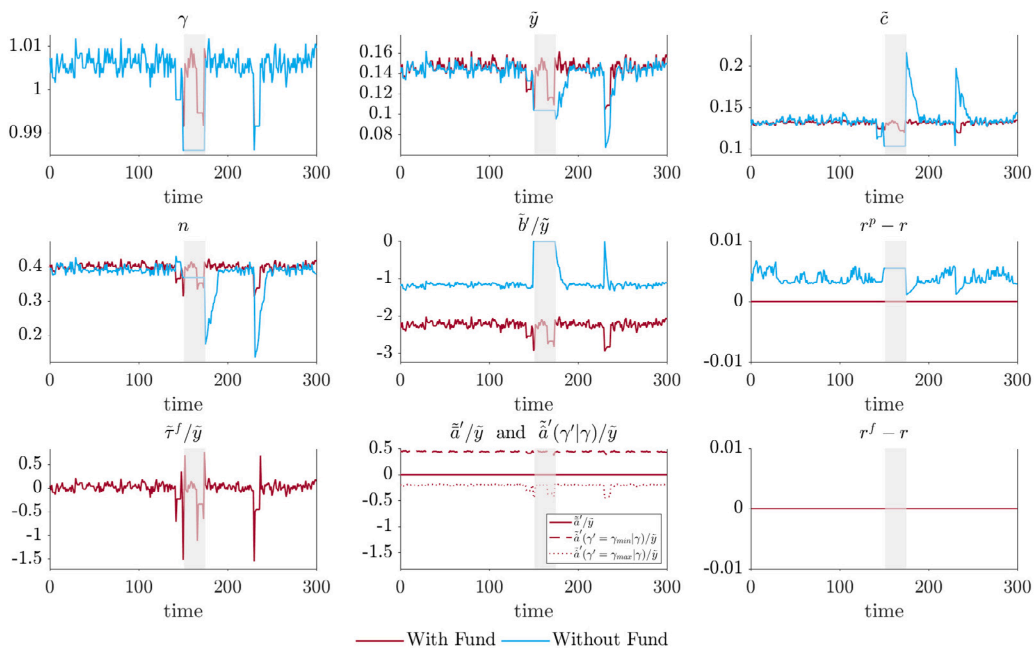

Fig. 4. Simulation of a specific steady state path. *Note*: The figure depicts the simulation of a specific steady state path on selected key variables. The economy without (with) the Fund is in blue (red). The grey area represents the region in which the economy without the Fund is in default. We simulate one history of shocks for 500 periods in steady state starting with the lowest Pareto weight in the ergodic set. To avoid that the initial conditions blur the results, the first 200 periods are discarded.

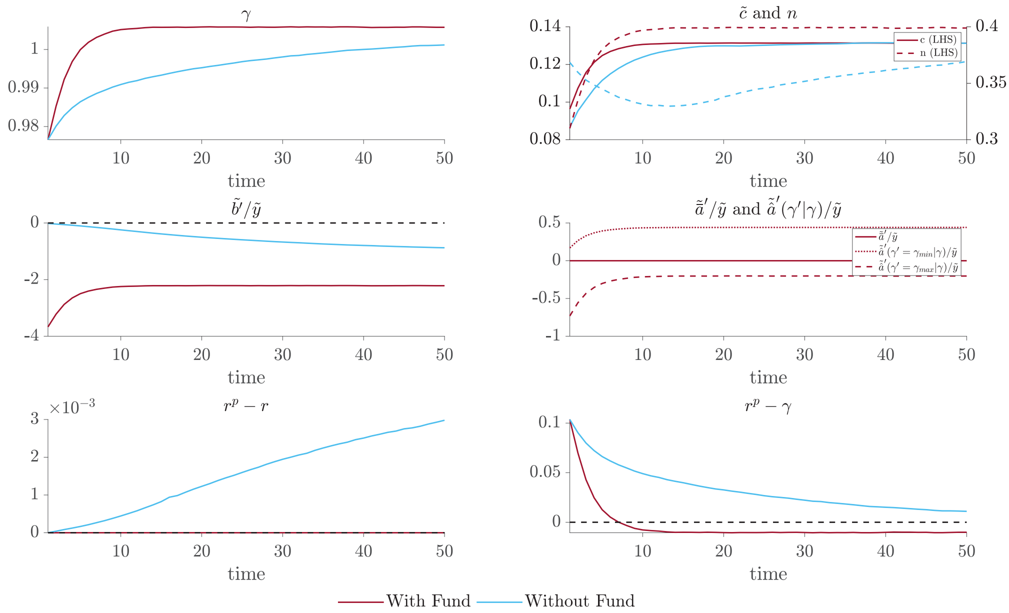

Fig. 5. Impulse response functions to a negative γ shock.
Note: The figure depicts impulse response functions following a negative growth shock on selected key variables. The economy without (with) the Fund is in blue (red). Impulse response functions are obtained by averaging the simulation of 5,000 independent shock histories for 50 periods starting with $\gamma = \gamma_{min}$ and initial debt holding and relative Pareto weight drawn from the ergodic set.

Fig. 4 depicts the simulation result with the grey region representing the periods in which the economy without the Fund is in default. With the Fund's intervention, the economy has a more stable consumption path over time. The sovereign avoids the major fluctuations of consumption that characterize the standard incomplete market economy with defaults. Moreover, the sovereign is able to accumulate private debt at the risk-free rate in regions where it would normally default without the Fund. This is entirely due to the fact that the entire debt position is hedged by Arrow securities. To get a sense of the insurance component, we display the Arrow securities purchased today for the highest and the lowest states tomorrow. Two points deserve to be noted. First, the portfolio of Arrow securities is procyclical as it closely follows the shock process. Second, the positions taken in Arrow securities are substantial. If one focuses on $\tilde{a}'(\gamma'|\gamma)$ for $\gamma' = \gamma_{min}$, we see that it amounts on average to roughly 50% of annual GDP. Instead of looking at the Arrow securities one can observe the Fund's primary surplus, $\tilde{\tau}_f$, which also moves procyclically and largely oscillates around zero since $Z = 0$.

Fig. 5 depicts the impulse response functions resulting from a stark negative growth shock on selected key variables.[32] The responses are computed as the mean of 5,000 independent shock histories starting with the lowest growth shock as well as initial debt holdings and relative Pareto weights drawn from the ergodic set. In the very first periods following the negative shock's realization, the Fund provides additional insurance to the sovereign. This sustains the existing level of debt and prevents a large decrease in consumption and a large increase in labour supply. Hence, without the Fund's intervention, the sovereign repudiates its debt and is obliged to provide more labour to avoid a massive reduction in consumption. Thus, the immediate impact of a sudden low growth shock is more severe in the absence of the Fund. In the long run, the sovereign without the Fund is likely to repudiate debt again and therefore reaches a lower level of steady state indebtedness. Besides this, the economy with the Fund avoids the positive spread in the private bond market. It can therefore reach more quickly a low level of $r^p - \gamma$ easing debt management.

7.4. Welfare analysis

Sharp difference in the dynamics of the economy with and without Fund translates into superior welfare implications of the Fund. The first column of Table 3 represents the welfare gains of the Fund's intervention in consumption equivalent terms at zero initial debt holdings. Recall that the sovereign which has access to the Fund can hold debt in the Fund or in the private bond market. Thus, to adequately compare the two economies, we compare them for the same *total* debt holdings. That is, the welfare comparisons are computed at the points where $\tilde{\omega} = 0$ for the economy in the Fund and at $\tilde{b} = 0$ for the economy outside the Fund. The welfare computation is presented in the Online Appendix F.

[32] Figures G.2 and G.3 in the Online Appendix G present the impulse response functions to a negative and to a positive shock of all relevant variables in the model, respectively.

Table 3
Welfare comparison at zero initial debt.

State	Welfare gains (%)	Maximal debt absorption (% of GDP)	
		With fund	Without fund
$\gamma = \gamma_{min}$	15.27	496	224
$\gamma = \gamma_{med}$	14.01	244	125
$\gamma = \gamma_{max}$	13.82	204	104
Average	14.05		

Note: The table reports welfare gains of the Fund's intervention at zero initial debt in consumption equivalent terms. The welfare computation is presented in the Online Appendix F.

Welfare gains are significant with the Fund's intervention. With zero initial debt, the consumption-equivalent welfare gains are on average 14%. Moreover, the largest welfare gains are recorded in low growth states. Thus, the Fund's intervention is mostly valued when the sovereign is in a difficult economic situation. As mentioned above, welfare gains are the consequence of two main features of the Fund's intervention. First, the Fund provides state-contingent transfers and therefore enhances consumption smoothing. Second, it enables a greater accumulation of debt in general. As one can see in the last two columns of Table 3, the maximal debt absorption of the economy is almost always twice larger with the Fund than without.

To be more precise on the source of the aforementioned welfare gains, in the Online Appendix F, we provide a decomposition of the welfare gains. We show that they are mostly due (i.e. above 90%) to the greater debt capacity and the insurance component; among these two factors debt capacity represents the largest share of total gains (i.e. circa 85%).

7.5. Debt dynamic decomposition

We further decompose the evolution of the debt according to Cochrane (2020, 2022): sovereign debt at the end of the year, v_{t+1}, is equal to its value at the beginning of the year, v_t, plus the net cost of keeping debt, $r_t^p - \gamma_t$, and the year's primary deficit (excluding interest payment), $-s_t$, so that $v_{t+1} = v_t + r_t^p - \gamma_t - s_t$, assuming no discounting for simplification. In our environment, the primary surplus without interest payment corresponds to $\tilde{b}_{t+1} - \tilde{b}_t$ for the economy without the Fund and $\tilde{\tilde{\omega}}_{t+1} - \tilde{\omega}_t$ for the economy with the Fund.

Fig. 6 depicts the decomposition for Italy as well as the model economy with and without the Fund in logarithmic scale. We generate the two panels for the model economy by feeding the smoothed growth path of Italy over 2000Q1–2019Q4 into the model and start with the same level of debt of Italy in 2000Q1.[33] We then obtain the path of debt and interest rate through the optimal policy functions. The blue line represents the evolution of the value of debt which is the combination of the green line (i.e. $r^p - \gamma$) and the red line (i.e. $-s$). In view of this, had the accumulation of debt been costless (i.e. $r^p - \gamma = 0$), then the blue line would coincide with the red line.

We observe that the evolution of Italy's debt is the result of two conflicting forces: a remarkable history of increasing accumulated primary surpluses and two decades of growth decline resulting in accumulated costs $r^p - \gamma$. The model without the Fund replicates well the dynamic of the Italian public indebtedness. It nonetheless minimizes the positive impact of primary surpluses and the negative impact of the interest rate-growth differential.

Turning to the economy with the Fund, we see that the evolution of debt is flatter than in the economy without. This comes from two components. On the one hand, the rate at which the sovereign issues debt is at most risk free. This therefore largely reduces the $r^p - \gamma$ cost compared to the economy without the Fund. On the other hand, the Fund provides insurance through Arrow securities. This eases debt management by making fiscal policy countercyclical as shown previously. As a result, the debt path is more smooth. Particularly, the model predicts that the Italian indebtedness by the end of 2019 would have been around 80% of GDP rather than 135% if Italy could have joined the Fund in 2000.[34]

This shows that the path followed by the Italian economy in the last two decades was highly inefficient. The Italian government's perseverance in maintaining positive primary surpluses, in spite of growth reversals, can be seen as a commitment to debt sustainability, in line with the European Union's fiscal policy. Indeed, the accumulation of large primary surpluses dampened the increase in Italian indebtedness, but this was a highly inefficient path to have been followed compared with the path that could have been followed with the Fund.

8. Conclusion

A starting point of this research has been the recognition that in a monetary union, such as the Euro area and as the result of the 21st Century crises, not only sovereign debt is very high, but also that a large fraction of the union-countries' sovereign debt is being held in Euro area institutions. This has helped 'stressed countries', reducing sovereign debt spreads — for example, in the Euro crisis in 2012. However, a simple maturity transformation or a long-term holding of sovereign debts may not be the most

[33] We consider a smoothed version of the growth path to avoid defaults in the economy without the Fund.
[34] We obtain this figure by computing the model implied debt-to-GDP ratio at the end of the sample period using the decomposition of Cochrane (2020, 2022).

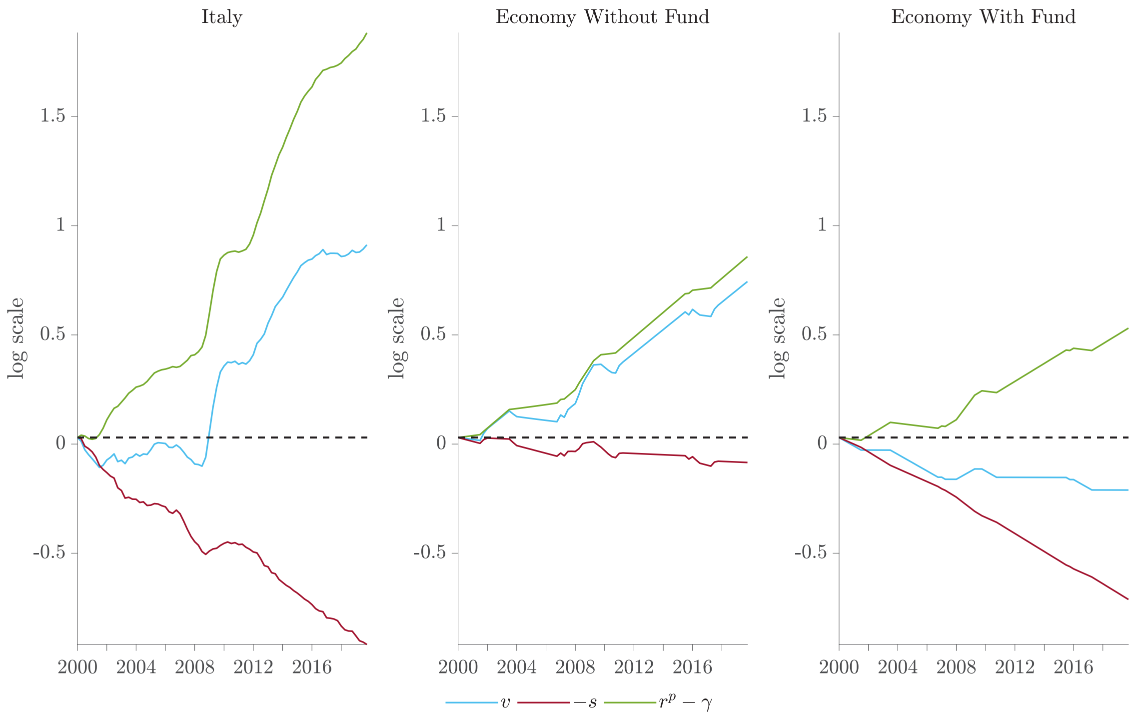

Fig. 6. Cochrane decomposition.
Note: depicts the decomposition for Italy as well as the model economy with and without the Fund in logarithmic scale using the decomposition proposed by Cochrane (2020, 2022). The variable v corresponds to the value of debt, $-s$ is the primary deficit and $r^p - \gamma$ is the interest-growth differential. A line above zero contributes positively to debt accumulation, while the opposite holds for a line below zero.

efficient debt management policy for the union. In fact, Ábrahám et al. (2022) has already shown that there can be high efficiency and welfare gains from having a *Financial Stability Fund,* with the proviso that the Fund absorbs all the sovereign debt of a country. We remove this proviso and show that the gains are still very high. Particularly, we show that Fund's intervention needs only to be minimal. Such *minimal intervention policy* (MIP) consists of an insurance component with an additional guarantee on long-term debt holdings by private lenders when the DSA binds, as *prudential policy* to prevent 'excessive lending' when sovereign debt is safe, we call it *the pecuniary externality of a negative spread.*

In sum, there are many interesting features to our results but we want to emphasize the two key elements that give the Fund a leading role in 'making sovereign debt safe' even with a MIP. The two elements also require innovation with respect of existing official lender's practices. First, the existence of a proper country risk-assessment, accounting for the effect of the constrained efficient Fund contract. Second, the role of the Fund state-contingent contract in defining a thick contingent (and contention) wall between the level of liabilities which is sustainable and the level which is not. And, linking the two, its role in coordinating lenders' and sovereign's beliefs with its announcements. As we said, most of the sovereign literature has focused on default problems, but in a mature union, outright default or exit may be rare events.[35] However, with the uncertainty and challenges that even advanced economies face, debt sustainability can remain a persistent concern for years to come and, even if sovereign debt is perceived to be safe, excessive lending can be a problem that private lenders may not internalize. We hope our work will not only contribute to the existing literature but also to face these challenges.

Finally, we show in our calibration to the Italian economy and subsequent simulations and computations, how important welfare gains can be achieved by improving existing official lending practices offering long-term state-contingent Fund contracts, even when there is debt accumulation or $r - g$ uncertainty, as most countries nowadays face.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

[35] The recent Brexit shows that exit can happen or, alternatively, that the union was still immature.

Data availability

https://data.mendeley.com/datasets/b5skz5g88z/1.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jinteco.2023.103834.

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