An Introduction to the New Keynesian Framework

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Motivation: Traditional v.s New Keynesian

Common features shared with Keynes/Traditional Keynesian

- A framework to allow **demand** to partially determine output
- Phillips curve (short-run output-inflation tradeoff) generated by **nominal rigidity**
- Inefficiency arising from **aggregate demand externality**, which motivates M policies

Unique to New Keynesian

- Based on Walrasian equilibrium (market clearing imposed)
- Incorporate rational expectation

New Keynesian: a well-accepted, compact, and well-studied framework if you need above features

Outline

- Deriving the canonical three-equation models
- Properties of the canonical model (transmission mechanism; determinacy; dynamics)
- Important and New Topics

References: standard textbooks (Gali/Woodford); Mackay, Nakamura and Steinsson (16); Guerrieri et al. (2021)

The Canonical New-Keynesian Model

Overall Setup

- Discrete time; infinite horizon; Markov shocks. Time denoted by t and uncertainty by s^t
- Decision makers: Households; Firms (intermediate- and final-goods); Government
- Labor as only factor input
- Price stickiness (staggered price setting/price adjustment cost) faced by intermediate firms

Households' Decision Problem

- Households solve

$$\max_{C_t, L_t, B_t, M_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[U(C_t) + \Gamma(\frac{M_t}{P_t}) - V(L_t) \Big],$$

$$s.t. \quad \frac{B_t}{1+i_t} + M_t + P_t C_t = B_{t-1} + M_{t-1} + W_t L_t + T_t + D_t$$

  and a s standard No-Ponzi Game condition.

- Remarks
  - Money in the utility (MIU) function as a short-cut to generate money demand

Households' Decision Problem - Characterization

- The Lagrangian:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[U(C_t) + \Gamma(\frac{M_t}{P_t}) - V(L_t)$$
$$+\lambda_t \Big\{ \frac{B_{t-1} + M_{t-1} + W_t L_t + T_t + D_t - \frac{B_t}{1+i_t} - M_t - P_t C_t}{P_t} \Big\} \Big]$$

- First Order Conditions:

$$\{C_t\} : U'(C_t) = \lambda_t$$
$$\{L_t\} : -V'(L_t) + \lambda_t \frac{W_t}{P_t} = 0$$
$$\{B_t\} : -\lambda_t \frac{1}{(1+i_t)P_t} + \beta \mathbb{E}_t \lambda_{t+1} \frac{1}{P_{t+1}} = 0$$
$$\{M_t\} : \frac{1}{P_t} \Gamma'(\frac{M_t}{P_t}) - \lambda_t \frac{1}{P_t} + \beta \mathbb{E}_t \lambda_{t+1} \frac{1}{P_{t+1}} = 0$$

Households' Decision Problem - Characterization, cont'd

- The FOCs are simplified to:

$$
\begin{aligned}
V'(L_t) &= U'(C_t)\frac{W_t}{P_t} \\
U'(C_t) &= \beta\mathbb{E}_t\frac{P_t}{P_{t+1}}(1+i_t)U'(C_{t+1}) \quad \text{(IS curve)} \\
U'(C_t) &= \Gamma'(\frac{M_t}{P_t}) + \beta\mathbb{E}_t\frac{P_t}{P_{t+1}}U'(C_{t+1})
\end{aligned}
$$

  where combining the last two lines we can write

$$
\begin{aligned}
\Gamma'(\frac{M_t}{P_t}) &= \beta\mathbb{E}_t U'(C_{t+1}) i_t \frac{P_t}{P_{t+1}} \\
\Rightarrow \Gamma'(\frac{M_t}{P_t}) &= U'(C_t)\frac{i_t}{1+i_t} \quad \text{(LM curve)}
\end{aligned}
$$

- Interpretations: opportunity cost of holding money is nominal bond return i_t
- Household block summarizes by the 4 conditions: 3 FOCs + 1 budget constraint

Deriving the New Keynesian Phillips Curve - Outline

- Phillips curve: a relation that represents short-term output-inflation tradeoff
- Rough idea: due to price adjustment friction, firms are not able to adjust prices to the "ideal" level $\Rightarrow$ have to adjust quantities produced
- Such adjustments have to be accommodated by GE adjustment in labor supply
- Two formulation of price adjustment friction: (1) menu cost model a la Rotemberg (1982); (2) staggered price setting a la Calvo (1983)

Final-good Production Combining Varieties of Goods

- The representative final-good firm combines intermediate-good firm products according to the CES (Constant-Elasticity-of-Substitution) function:

$$Y = \left[\int_{i=0}^{1} Y_i^{(\eta-1)/\eta} di \right]^{\eta/(\eta-1)}, \quad \eta > 0$$

i: a variety of goods. The total mass of varieties is normalized to 1.
η: elasticity of substitution between varieties

- The final-good firm's profit maximization problem:

$$\max_{Y_i} PY - \int_0^1 P_i Y_i \; di$$
$$s.t. \quad Y = \left[\int_{i=0}^{1} Y_i^{(\eta-1)/\eta} di \right]^{\eta/(\eta-1)}$$

- The first order condition of the final-good firm gives

$$Y_i = Y\Big(\frac{P_i}{P}\Big)^{-\eta}$$

which gives a downward-sloping demand function for intermediate-good firms

- This also delivers a price level of the final good:

$$P = \Big(\int P_j^{1-\eta} dj\Big)^{1/(1-\eta)}$$

Price Setting Under Flexible Price

- Each variety i is produced by a firm.
- All firms engage in *monopolistic competition* in the sense that firm can set P_i but take P as given.
- Each firm uses labor as the only input and produces according to

$$Y_i = zL_i$$

- Firm's producing and price setting problem

$$\max_{L_i, Y_i, P_i} \frac{P_i Y_i}{P} - \frac{W}{P} L_i$$
$$s.t. \quad Y_i = zL_i \quad \text{(Production)}$$
$$Y_i = Y \left(\frac{P_i}{P} \right)^{-\eta} \quad \text{(Demand)}$$

where we've plugged in the demand function derived before for the second constraints.

- Plug in the two constraints to eliminate (L_i, Y_i) we have one single optimization problem for P_i:

$$\max_{P_i} Y\left(\frac{P_i}{P}\right)^{1-\eta} - \frac{W}{P}\frac{Y}{z}\left(\frac{P_i}{P}\right)^{-\eta}$$

- FOC:

$$Y(1-\eta)\frac{P_i^{-\eta}}{P^{1-\eta}} - \frac{W}{P^{1-\eta}}\frac{Y}{z}(-\eta)P_i^{-\eta-1}$$

- Simplify we have

$$\frac{P_i}{P} = \frac{\eta}{\eta-1}\frac{1}{z}\frac{W}{P}$$

Interpretations of Monopolistic Firm's Optimal Price Setting

$$\frac{P_i}{P} = \frac{\eta}{\eta - 1}\frac{1}{z}\frac{W}{P}$$

- Interpretations:
  - Monopoly firm charges a higher price than the marginal cost of production $(\eta/(\eta - 1) > 1)$.
  - The markup depends on the demand elasticity η.
  - This is the ideal price firm would charge responding to the marginal cost.
  - **Nominal rigidity**: the inability of firm to set the ideal price due to frictions such as not always get the chance to adjust price or adjustment is costly.
- At symmetric equilibrium with $P_i = P$, the marginal cost would be

$$\frac{1}{z}\frac{W}{P} = \frac{\eta - 1}{\eta}$$

- Next we see how nominal rigidity breaks the constant markup results

Price Setting with Price Adjustment Cost

- Consider firm's pricing problem modified to include price adjustment cost.

$$\max_{L_i, Y_i, P_i} \frac{P_i Y_i}{P} - \frac{W}{P} L_i - \frac{\theta}{2} \left(\frac{P_i - P_{i,-1}}{P_{i,-1}} \right)^2 Y$$

$$s.t. \quad Y_i = z L_i \quad \text{(Production)}$$

$$Y_i = Y \left(\frac{P_i}{P} \right)^{-\eta} \quad \text{(Demand)}$$

where $P_{i,-1}$ is the firm's last period price.

- Interpretations:
  - It incurs cost to set inflation not equal to a target inflation (0 here).
  - The **marginal** cost is higher when the deviation is larger (the quadratic function is convex)

Price Setting with Price Adjustment Cost

- Again, plug in the two constraints, rewrite the problem as

$$\max_{P_i} Y\Big(\frac{P_i}{P}\Big)^{1-\eta} - \frac{W}{P}\frac{Y}{z}\Big(\frac{P_i}{P}\Big)^{-\eta} - \frac{\theta}{2}\Big(\frac{P_i - P_{i,-1}}{P_{i,-1}}\Big)^2 Y$$

- First order condition

$$Y(1-\eta)\frac{P_i^{-\eta}}{P^{1-\eta}} - \frac{W}{P^{1-\eta}}\frac{Y}{z}(-\eta)P_i^{-\eta-1} - \theta\frac{(P_i - P_{i,-1})}{P_{i,-1}^2}Y$$

- At symmetric equilibrium, every firm uses the same pricing strategy and $P_i = P$, $P_{i,-1} = P_{-1}$. The first order condition simplifies to

$$(1-\eta)\frac{P^{-\eta}}{P^{1-\eta}} - \frac{W}{P^{1-\eta}}\frac{1}{z}(-\eta)P^{-\eta-1} - \theta\frac{P - P_{-1}}{P_{-1}^2} = 0$$

$$\Rightarrow \Big(\frac{P}{P_{-1}} - 1\Big)\frac{P}{P_{-1}} = \frac{\eta}{\theta}\Big[\frac{1}{z}\frac{W}{P} - \frac{(\eta-1)}{\eta}\Big]$$

Non-constant Markups due to Price Adjustment Cost

- Replace $\left(\frac{P}{P_{-1}} - 1\right)\frac{P}{P_{-1}}$ by $(1+\pi)\pi$ we have:

$$(1+\pi)\pi = \frac{\eta}{\theta}\Big[\frac{1}{z}\frac{W}{P} - \frac{(\eta-1)}{\eta}\Big]$$

With π small, this is

$$\pi \approx \frac{\eta}{\theta}\Big[\frac{1}{z}\frac{W}{P} - \frac{(\eta-1)}{\eta}\Big]$$

- Responding to increase in marginal cost $\frac{1}{z}\frac{W}{P}$, the firm would like to raise price. The price adjustment cost limits the degree of raising.
- The elasticity of π w.r.t. marginal cost
  - is higher if price adjustment cost θ is lower
  - is higher if the demand elasticity η is higher.

How Aggregate Demand Affects Output Due to Price Stickiness

$$\pi \approx \frac{\eta}{\theta}\Big[\frac{1}{z}\frac{W}{P} - \frac{(\eta-1)}{\eta}\Big]$$

How aggregate demand affects output due to sticky price:

- Aggregate demand$\uparrow$ pushes for higher output and higher W
- As marginal cost of production $\frac{1}{z}W$ rises, firms raise prices.
- If no sticky price, $P \uparrow \Rightarrow \frac{W}{P} \downarrow$, until real wage and output unchanged.
- Due to sticky price, $P \uparrow$ not enough, leaving $\frac{1}{z}\frac{W}{P} > \frac{\eta-1}{\eta}$
- ... and quantity produced higher than before.

In equilibrium, W/P is associated with labor supply (via HH's labor supply decision)

- giving a positive relation between inflation and output

Dynamic Price Setting

- Consider now the firm cares about future profits:

$$\max_{L_{it}, Y_{it}, P_{it}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\underbrace{\frac{P_{it} Y_{it}}{P_t}}_{\text{revenue}} - \underbrace{\frac{W_t}{P_t} L_{it}}_{\text{wage cost}} - \underbrace{\frac{\theta}{2} \left(\frac{P_{it}}{P_{i,t-1}} - (1+\bar{\pi}) \right)^2 Y_t}_{\text{price adjustment cost}} \right]$$

$$s.t. \quad Y_{it} = z_t L_{it} \quad \text{(Production constraints)}$$

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t} \right)^{-\eta} \quad \text{(Demand constraints)}$$

$$P_{i,-1} \text{ given}$$

- Future profit is discounted by β^t with $0 < \beta < 1$.
- The adjustment cost now increases in deviation from target inflation $(1 + \bar{\pi})$.

Dynamic Price Setting and NKPC

- First order condition w.r.t. P_{it}:

$$\underbrace{Y_t(1-\eta)\frac{P_{it}^{-\eta}}{P_t^{1-\eta}} - \frac{W_t}{P_t^{1-\eta}}\frac{Y_t}{z_t}(-\eta)P_{it}^{-\eta-1}}_{\text{marginal profit of rasing } P_{it}} - \underbrace{\theta\frac{1}{P_{i,t-1}}\Big(\frac{P_{it}}{P_{i,t-1}} - (1+\bar{\pi})\Big)Y_t}_{\text{marginal adjustment cost of rasing } P_{it}}$$

$$-\mathbb{E}_t \underbrace{\beta\theta\Big(-\frac{P_{i,t+1}}{P_{i,t}^2}\Big)\Big(\frac{P_{i,t+1}}{P_{i,t}} - (1+\bar{\pi})\Big)Y_{t+1}}_{\text{marginal effect of rasing } P_{it} \text{ on future adjustment cost}} = 0$$

- Interpretation:
  - Changes in current price affect future adjustment cost.
  - Due to convex cost function, firms want to adjust price smoothly.
- Impose symmetry s.t. $P_{it} = P_t$, we have the **New-Keyneisna Phillips Curve** (NKPC)

$$(1+\pi_t)(\pi_t - \bar{\pi}) = \frac{\eta}{\theta}\Big[\frac{1}{z_t}\frac{W_t}{P_t} - \frac{(\eta-1)}{\eta}\Big] + \beta\mathbb{E}_t(1+\pi_{t+1})(\pi_{t+1}-\bar{\pi})\frac{Y_{t+1}}{Y_t}.$$

Dynamic Price Setting and NKPC, discussions

- Derivation based on a constant discount factor β for simplicity
  - to be consistent with theory, should use households' stochastic discount factor $\beta \frac{u'(C_{t+1})}{u'(C_t)}$.
  - won't change the essence of the derivation or the results
- Firms' optimality summarized by (1) NKPC; (2) $Y_t = z_t L_t$; (3) $D_t = P_t Y_t - \frac{1}{2}\theta(\pi_t - \bar{\pi})^2 P_t Y_t - W_t L_t$
- Will leave the Calvo setting in the appendix

Government

- Government budget

$$P_t G_t + T_t + \frac{B_t}{1+i_t} = B_{t-1} + M_t - M_{t-1}$$

- Monetary policy rule. E.g., a **Taylor rule** for nominal interest rate:

$$(1+i_t) = (1+\bar{i})\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\phi_\pi}\left(\frac{Y_t}{Y^*}\right)^{\phi_y}\varepsilon_t^{MP},$$

  ε_t^{MP}: monetary policy shock.
  Strong inflation targeting: $\phi_\pi > 1$

- Fiscal policy rule. E.g., a constant bond supply $B_t = \overline{B}$.

Market-clearing

- Intermediate goods: incorporated into the NKPC
- Final goods:

$$C_t + G_t + AdjCost_t = Y_t,$$

  where $AdjCost_t \equiv \frac{1}{2}\theta(\pi_t - \bar{\pi})^2 Y_t$
- Labor market: $L_t = \int L_{it} di$.
- Money market

Equilibrium

Given $\{G_t\}$ and initial $(\frac{M_0}{P_0}, \frac{B_0}{P_0})$, a sequential competitive equilibrium is stochastic processes $\{C_t, L_t, \frac{M_t}{P_t}, \frac{B_t}{P_t}, Y_t\}$, $\{\pi_t, i_t, \frac{W_t}{P_t}, \frac{D_t}{P_t}, \frac{T_t}{P_t}\}$ such that

- Households' optimality conditions hold (3 FOCs + 1 BC)
- Firms' optimality conditions hold: NKPC, $Y_t = z_t L_t$, and $D_t/P_t = Y_t - W_t L_t - \frac{1}{2}\theta(\pi_t - \bar{\pi})^2 Y_t$
- Government budget satisfied. Monetary policy rule and fiscal policy rule hold
- Goods market and labor market clear

"The Cashless Limit"

- Parameterization:

$$\mathcal{U} = \frac{C^{1-\sigma}}{1-\sigma} + \psi\Gamma\left(\frac{M}{P}\right) - \frac{L^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$$

- Cashless limit: let $\psi \to 0$. We then have $M = 0$ in equilibrium under an interest rate policy rule
- Remaining equilibrium conditions, for variables $C_t, Y_t, i_t, L_t, w_t, \pi_t$

$$C_t^{-\sigma} = \beta\mathbb{E}_t \frac{1+i_t}{1+\pi_{t+1}} C_{t+1}^{-\sigma} \quad \text{(IS)}$$

$$Y_t = z_t L_t; \qquad L_t^{1/\nu} = w_t C_t^{-\sigma}$$

$$(1+\pi_t)(\pi_t - \bar{\pi}) = \frac{\eta}{\theta}\left[\frac{1}{z_t} w_t - \frac{(\eta-1)}{\eta}\right] + \beta\mathbb{E}_t(1+\pi_{t+1})(\pi_{t+1} - \bar{\pi})\frac{Y_{t+1}}{Y_t} \quad \text{(NKPC)}$$

$$(1+i_t) = (1+\bar{i})\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\phi_\pi}\left(\frac{Y_t}{Y^*}\right)^{\phi_y}\varepsilon_t^{MP} \quad \text{(Taylor rule)}$$

$$C_t + G_t + \frac{1}{2}\theta(\pi_t - \bar{\pi})^2 Y_t = Y_t$$

The Three-Equation System

First, for IS

$$C_t^{-\sigma} = \beta \mathbb{E}_t \frac{1+i_t}{1+\pi_{t+1}} C_{t+1}^{-\sigma} \quad \text{(IS)} \quad \textit{and} \quad C_t + G_t + \frac{1}{2}\theta(\pi_t - \bar{\pi})^2 Y_t = Y_t$$

$$\Rightarrow \left[Y_t(1 - \frac{1}{2}\theta(\pi_t - \bar{\pi})^2) - G_t\right]^{-\sigma} = \beta \mathbb{E}_t \frac{1+i_t}{1+\pi_{t+1}} \left[Y_{t+1}(1 - \frac{1}{2}\theta(\pi_{t+1} - \bar{\pi})^2) - G_t\right]^{-\sigma} \quad \text{(IS)}$$

Next, for NKPC

$$Y_t = z_t L_t; \; L_t^{1/\nu} = w_t C_t^{-\sigma} \; \textit{and} \; (1+\pi_t)(\pi_t - \bar{\pi}) = \frac{\eta}{\theta}\left[\frac{1}{z_t} w_t - \frac{(\eta-1)}{\eta}\right] + \beta \mathbb{E}_t (1+\pi_{t+1})(\pi_{t+1} - \bar{\pi}) \frac{Y_{t+1}}{Y_t}$$

$$\Rightarrow (1+\pi_t)(\pi_t - \bar{\pi}) = \frac{\eta}{\theta}\left[z_t^{-(1+\frac{1}{\nu})} Y_t^{\frac{1}{\nu}} (Y_t(1 - \frac{1}{2}\theta(\pi_t - \bar{\pi})^2) - G_t)^{\sigma} - \frac{(\eta-1)}{\eta}\right] + \beta \mathbb{E}_t (1+\pi_{t+1})(\pi_{t+1} - \bar{\pi}) \frac{Y_{t+1}}{Y_t}$$

And Finally,

$$(1+i_t) = (1+\bar{i})\left(\frac{1+\pi_t}{1+\pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y^*}\right)^{\phi_y} \varepsilon_t^{MP} \quad \text{(Taylor rule)}$$

The Log-linearized System

Log linearized with respect to Y_t, π_t, i_t and denote $\hat{y}_t = \log(Y_t/Y^*)$, $\hat{\pi}_t = \log(\frac{1+\pi_t}{1+\pi^*})$, $\hat{i}_t = \log(\frac{1+i_t}{1+\bar{i}})$:

$$
\begin{aligned}
\text{(IS)}: \quad & \hat{y}_t = \mathbb{E}_t\left[\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \hat{\pi}_{t+1})\right] + \varepsilon_t^{IS} \\
\text{(AS, NKPC)}: \quad & \hat{\pi}_t = \kappa \hat{y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} - \varepsilon_t^{AS} \\
\text{(MP, Taylor rule)}: \quad & \hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t + \varepsilon_t^{MP}
\end{aligned}
$$

where $\kappa = \frac{\eta-1}{(1+\bar{\pi})\theta}(\frac{1}{\nu} + \sigma)$, $\varepsilon_t^{IS} = \gamma^G \hat{g}_t$, $\varepsilon_t^{AS} = \gamma^Z \hat{z}_t$, with γ^G and γ^Z functions of parameters.

- ε_t^{IS} : demand shock (due to gov spending/confidence/trade/preference)
- ε_t^{AS} : supply shock (due to import price/preference/lockdown)
- Slope of NKPC $\kappa \downarrow \theta$

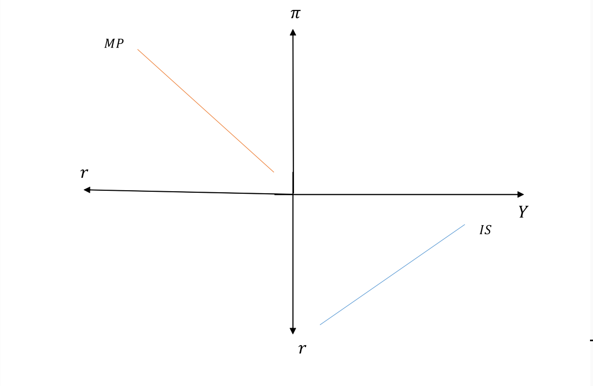
Transmission Mechanisms

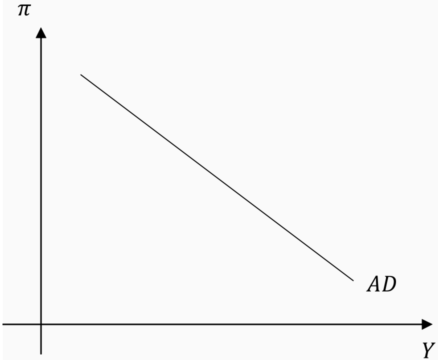
Graph-representation of the Impact Effect of Shocks

Ignoring uncertainty for now. Denote $r_t = i_t - \pi_{t+1}$.

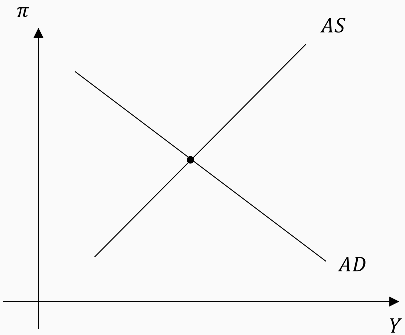
Strong inflation-targeting ($\phi_\pi > 1$ in the Taylor rule) + Sticky inflation $\Rightarrow r_t \uparrow \pi_t$

Plotting current-period (Y, π, r) in graphs. Combine *IS* and *MP* to arrive at *MP*-consistent Aggregate Demand (AD) curve:





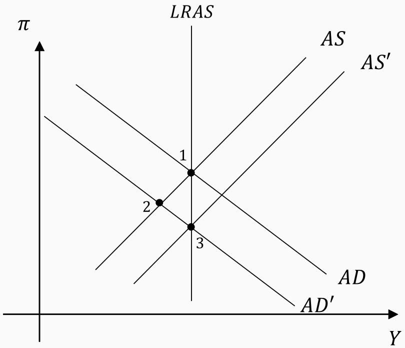
Graph-representation, cont'd



with r determined in the background.

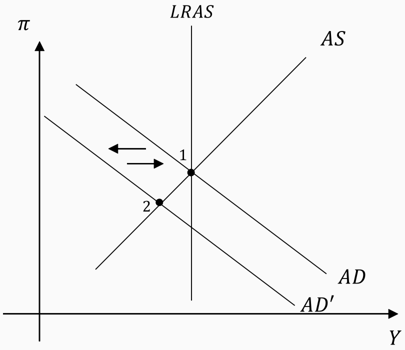
- Positive $\varepsilon_t^{IS}, \varepsilon_t^{MP}$ shifts AD upward
- Positive ε_t^{AS} shifts AS downward
- Expected future inflation $\uparrow$ shifts AS upward (recall $\hat{\pi}_t = \kappa \hat{y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} - \varepsilon_t^{AS}$)
- Expected future interest rate $\downarrow$ shifts AD rightward (recall $\hat{y}_t = \mathbb{E}_t \left[\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \hat{\pi}_{t+1}) \right] + \varepsilon_t^{IS}$)

Automatic Stabilization After A Negative Demand Shock



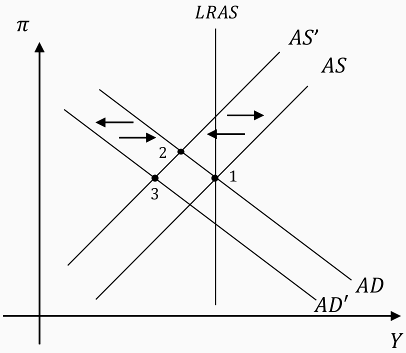
- The economy was initially at 1
- A negative ε_t^{IS} leads AD → AD', moving the economy to 2
- Expecting inflation would fall, AS shifts right
- With gradual price adjusting, AS eventually shifts to AS' with lower π, consistent with expectation
- However, AS → AS' may take multi periods, leading to a temporary recession

Countering a Negative Demand Shock: A First Look at Divine Coincidence



- The economy was initially at 1
- A negative demand shock AD → AD' moves the economy to 2
- Expansionary monetary policy (an outward shift of MP curve) shifts *AD* back and restores to 1
- Temporary recession is avoided
- **There is no inflation-output tradeoff when countering a demand shock with M policy**

Countering a Supply shock: Inflation and Output Tradeoffs



- The central bank can choose to stabilize inflation after the shock
- This can be done by autonomous *tightening* monetary policy, with AD shifting leftward
- This moves the economy from 2 to 3, with *further output loss*
- The long run the economy moves back to 1 if the shock is temporary

Dynamics - Solving the Linearized System by Hand

- Let's start with a simpler case with $\phi_y = 0$ in the Taylor rule:

$$\begin{aligned} \text{(IS)}: \quad & \hat{y}_t = \mathbb{E}_t\left[\hat{y}_{t+1} - \frac{1}{\sigma}(\hat{i}_t - \hat{\pi}_{t+1})\right] + \varepsilon_t^{IS} \\ \text{(NKPC)}: \quad & \hat{\pi}_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} - \varepsilon_t^{AS} \\ \text{(MP)}: \quad & \hat{i}_t = \phi_\pi\hat{\pi}_t + \varepsilon_t^{MP} \end{aligned}$$

- Solve $\hat{y}_t$ from the Phillips curve:

$$\hat{y}_t = \frac{1}{\kappa}\hat{\pi}_t - \frac{\beta}{\kappa}\hat{\pi}_{t+1} + \frac{1}{\kappa}\varepsilon_t^{AS}$$

- Plug $\hat{i}_t$ and $\hat{y}_t$ into the IS curve, we simplify to

$$A\hat{\pi}_t + B\hat{\pi}_{t+1} + D\hat{\pi}_{t+2} = \varepsilon_t$$

$A = \frac{1}{\kappa} + \frac{1}{\sigma}\phi_\pi$, $B = -\frac{1}{\kappa}(1+\beta) - \frac{1}{\sigma}$, $D = \frac{\beta}{\kappa}$, $\varepsilon_t = -\frac{1}{\sigma}\varepsilon_t^{MP} + \frac{1}{\kappa}\varepsilon_{t+1}^{AS} - \frac{1}{\kappa}\varepsilon_t^{AS} + \varepsilon_t^{IS}$.

$$A\hat{\pi}_t + B\hat{\pi}_{t+1} + D\hat{\pi}_{t+2} = \varepsilon_t \tag{1}$$

- a second-order difference equation for $\hat{\pi}_t$. Solve by the method of undetermined coefficients:
  - Guess the solution takes the form

$$\hat{\pi}_t = C_1\lambda_1^t + C_2\lambda_2^t + \sum_{s=0}^{\infty}\psi_s\varepsilon_{t+s}$$

  - Plug into equation (1):

$$C_1\lambda_1^t\left(D\lambda_1^2 + B\lambda_1 + A\right) + C_2\lambda_2^t\left(D\lambda_2^2 + B\lambda_2 + A\right)$$
$$+A\sum_{s=0}^{\infty}\psi_s\varepsilon_{t+s} + B\sum_{s=1}^{\infty}\psi_s\varepsilon_{t+s} + D\sum_{s=2}^{\infty}\psi_s\varepsilon_{t+s} = \varepsilon_t$$

  - The equation should hold for any t and θ. For non-trivial solution:

$$D\lambda_1^2 + B\lambda_1 + A = 0, \quad D\lambda_2^2 + B\lambda_2 + A = 0$$

Determinacy

$$\pi_t = C_1\lambda_1^t + C_2\lambda_2^t + \sum_{s=0}^{\infty} \psi_s \varepsilon_{t+s}$$

- λ_1 and λ_2 solve the *characteristic equation*

$$D\lambda^2 + B\lambda + A = 0 \tag{2}$$

- If equation (2) has both roots $\lambda_1, \lambda_2 \geq 1$,
  - Using condition $\lim_{t\to\infty} \pi_t = 0$, we get $C_1 = C_2 = 0$, leaving

$$\pi_t = \sum_{s=0}^{\infty} \psi_s \varepsilon_{t+s}$$

- Otherwise, if one root λ_1 of equation (2) is smaller than 1,
  - We can not use the condition $\lim_{t\to\infty} \pi_t = 0$ to set C_1 to 0.
  - Any C_1 corresponds to an equilibrium.
  - And the economy features *indeterminacy*.

Determinacy and the Taylor Principle

- Write the characteristic equation explicitly

$$\frac{\beta}{\kappa}\lambda^2 - [\frac{1}{\kappa}(1+\beta) + \frac{1}{\sigma}]\lambda + \frac{1}{\kappa} + \frac{1}{\sigma}\phi_\pi = 0$$
$$\Rightarrow \Big(\frac{\beta}{\kappa}\lambda - (\frac{1}{\kappa} + \frac{1}{\sigma})\Big)(\lambda - 1) + \frac{1}{\sigma}(\phi_\pi - 1) = 0$$

- The necessary and sufficient condition for both roots > 1 is

$$\phi_\pi > 1,$$

which is guaranteed by the *Taylor principle*.

- Intuition for why ϕ_π "stabilizes" the economy $\Leftrightarrow$ why *AD* curve is downward sloping

Solving DSGE Models with Perturbation Method, General Discussions

- Solve the model's steady state
- (Log) linearize the model around the steady state, you will get a linear difference equation system:

$$\Gamma_0 E_t y(t+1) = \Gamma_1 y(t) + C + \Psi z(t)$$

  - See Uhlig (95) for a guide on linearization

- Solutions can be characterized using general methods (e.g., Blanchard and Khan 80; Sims 02;)
- Toolbox Dynare is used to linearize and solve the model efficiently
- Discussions on the limitation of the local solution and the application of GDSGE (www.gdsge.com)

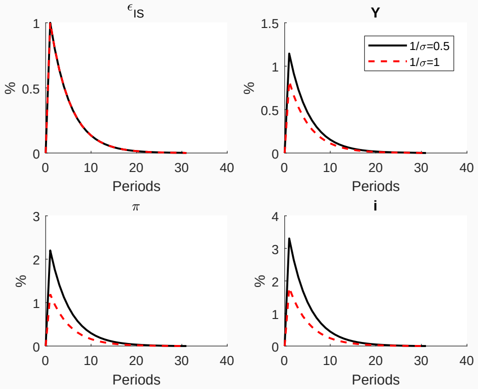
Assigning Values to Parameters

- Assume shocks are persistent

$$\varepsilon_t^{IS} = \rho^{IS}\varepsilon_{t-1}^{IS} + u_t^{IS}, \quad 0 < \rho^{IS} < 1$$

  and similar for ε_t^{AS} and ε_t^{MP}.
- Some parameters have well-founded *micro foundations* and can be estimated using micro data externally, e.g.,
  - Inter-temporal elasticity of substitution $1/\sigma = 0.5$.
  - $\kappa = \frac{\eta-1}{\theta}(\frac{1}{\nu} + \sigma)$:
  - Demand elasticity $\eta = 3$, adjustment cost $\theta = 10$, labor supply elasticity $\nu = 0.5$.
- Some parameters can be *calibrated* so that model quantitative properties agree with data.
  - e.g., $\beta = 1/(1 + \bar{r})$ to set the steady state interest rate to $\bar{r}$.
  - Persistence parameters ρ^{IS} etc. are estimated to match dynamic responses of the model.

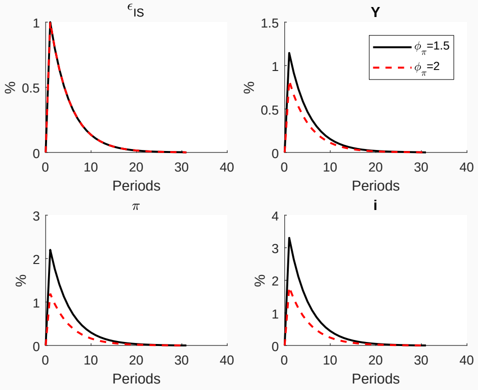
Impulse Response Functions to an *IS* Shock



Note: A shock $u_t^{IS} = 1\%$ hits at $t = 1$.

- After a positive *IS* shock, $Y \uparrow$ and $\pi \uparrow$. i responds more than one to as as π
- Persistence is entirely driven by shocks.
- Purely forward looking system: positive backward propagation but no forward propagation
- Response is larger for IES higher

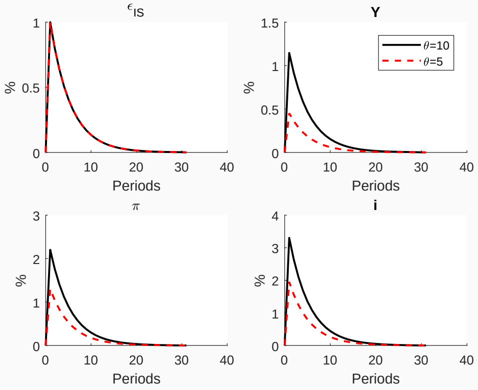
Impulse Response Functions to an *IS* Shock, Different Inflation Resp. Coefs.



Note: A shock $u_t^{IS} = 1\%$ hits at $t = 1$.

- Responses are weaker under a stronger inflation targeting (higher ϕ_π)

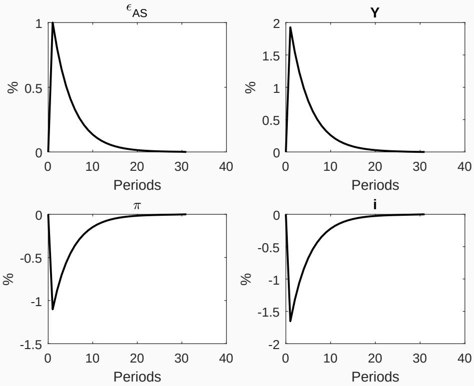
Impulse Response Functions to an *IS* Shock, Different Price Adj. Cost θ



Note: A shock $u_t^{IS} = 1\%$ hits at $t = 1$.

- Responses are larger with a higher price adjustment cost (higher θ)

Impulse Response Functions to an *AS* Shock



Note: A shock $u_t^{AS} = 1\%$ hits at $t = 1$.

- After a positive AS shock, $Y \uparrow$ and $\pi \downarrow$.
- Again, effects on π and i are different from those after a demand shock?

Efficiency Property of the Model

- Emphasize the role of demand in determining short-run output.
  - There would be *inefficient* slack in the absence of sufficient demand.
  - *Aggregate demand externality*. Keynes's narrative: Individual household does not take into account that his spending raises price, lowers real wage (due to sticky wage) and reduces unemployment
  - Modern formulations: Individual household does not take into account that his consumption demand lowers monopolistic markup / increases confidence / thickens market during a potential recession
  - *The Paradox of Thrift:* if everyone tries to save, the insufficient aggregate demand would actually lead to lower income and less saving.

Assessing the NK Framework

- A convenient building block to generate demand-shock driven business cycle correlation (positive consumption-inflation correlation), even you do not want to talk about M policy
  - then the con is that the behavior of the model depends on M policy
- Also *one* way to generate empirically plausible effect of demand stimulus (e.g., fiscal policy; see Baxter and King, 93; Huo and Rios-Rull, 13)
- Undesired properties due to indeterminacy at a nominal interest rate peg (e.g., implied large effects of forward guidance at ZLB)
- Empirical validity: nominal rigidity (some people have strong views against this...); counter-cyclical labor wedge

Some Milestones and Names of the Developments of the NK Framework

- John Taylor (80): introduced the idea of staggered wage and price contracts
- Calvo (83): staggered price-setting model
- Rotemberg (82): menu cost model
- Akerlof and Yellen (85), Mankiw (85): importance of real rigidities (monopolistic price setting) in driving nominal rigidities
- Mankiw and David Romer (91): the Book "New Keynesian Economics, Imperfect Competition and Sticky Prices Curve"
- Bernanke, Gertler and Gilchrist (99), Clarida, Gali and Gertler (99), Smets and Wouters (07): standardized and popularized the NK framework
- Woodford (2003): "Interest and Prices: Foundations of a Theory of Monetary Policy,"

Topic Summary - Empirical Validity of Key Assumptions of NK

- Nominal rigidity:
  - Frequency and patterns of price adjustment: Bils and Klenow (04); Nakamura and Steinsson (08); Kehoe and Midrigan (14)
  - Information friction: Mankiw and Reis (02), Reis (06), Alvarez et al., (15)
- Existence/Slopes/Stability of Phillips curve: Lucas (X,72); Gali and Gertler (99); Atkeson and Ohanian (X, 01); Stock and Watson (08); Hall (11); Coibion and Gorodnichenko (15); Del Negro et al. (15)
- The microfoundation of IS curve:
  - small inter-temporal elasticity of substitution: Campbell and Mankiw (89)
  - consumption Euler equations fail to hold: Canzoneri, Cumby and Diba (07)
  - large propensities to spend out of transitory income: Johnson et al. (06)
  - motivating a large new literature on Heterogeneous-Agent NK model (Kaplan, Moll and Violante, 18; Auclert, 19)

Topic Summary - Source of Business Cycle

- Canonical business cycle models combine the NK block and other features (financial-constrained firms/banks/households, investment, tons of shocks etc.; e.g., classical Bernanke, Gertler and Gilchrist (99); Smets and Wouters (07))
- Model-based decomposition highlight the importance of demand shock (Smets and Wouters, 07);
- A model-free wedge accounting (Chari, Kehoe and McGrattan, 07) also shows fluctuations in labor wedge are important
- In this sense, the NK framework has some success by simultaneously accounting for the two within a compact setting

Topic Summary - How Does HH Heterogeneity matter for M Policy Designs

- McKay et al. (2016): using liquidity-constrained consumers to resolve the forward guidance puzzle
- Kaplan et al. (2018): high propensities to spend due to liquidity-constrained HH
- Auclert (2019): redistributional effects contribute to aggregate demand responses
- Luetticke (2021): heterogeneous marginal propensity to *invest* + redistributional channel
    - fully-blown DSGE (i.e., with aggregate shocks), solved using Reiter
- Wong (2019): mortgage loan refinancing channel
    - a fixed cost for refinancing
    - used to explore the effect of aging on monetary policy effectiveness
    - partial equilibrium
- Cloyne et al. (2020): differential effects of monetary policy by whether households have debt
    - main contribution empirics
    - Iacoviello style finite-type agent models
- McKay and Wieland (2021): durable consumption that limits the persistence of monetary policy effects
    - durable consumption: a long-term asset with adjustment cost
- Ottonello and Winberry (2020): differential effects on firms with different existing leverages
    - firms with low leverages and high distance to defaults have larger response

Topic Summary - How Does HH Heterogeneity matter for Trans. Mech.

- McKay and Wolf (2023): rule-based policy; based on sequence-jacobian; find redistributive policy not important
- Yang (2022): rule-based policy; based on sequence-jacobian; extra redistributive motives (consumption basket; nominal wealth; earnings exposure); asymmetric response (more aggressive toward deflation; as inflation achieves redistributive roles); starting from inefficient eq
- Nuno and Thomas (2021): optimal redistributive inflation along the transition path
- Bhandari, Evans, Golosov, and Sargent (2021): unconstrained policy; perturbation approach
- Acharya, Challe and Dogra (2020): analytical tractable models
- Bilbiie and Ragot (2021): analytical tractable models
- Fernndez-Villaverde, et al. (2023): with ZLB; neural network approach; interplay between precautionary saving and future likelihood of ZLB