# Money as a Medium of Exchange

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# The roles of money

- Unit of Account: Money acts as a common measure of value, providing a consistent method of quantifying value and allowing different goods and services to be compared against each other.

- Store of Value: Money can be saved and used for future purchases, providing a way to store wealth.

- Medium of Exchange: Money is used as an intermediary in trade to avoid the inconveniences of a barter system, which requires a coincidence of wants between two parties for trade to occur.

- Barter economy and double coincidence of wants
- Credit and record keeping
- Money holding as the memory of trading history
- A Unified Framework for Monetary Theory and Policy Analysis

# Barter economy and double coincidence of wants

- Exchange in a barter economy relies on the double coincidence of wants

- Barter：以物易物

- "double coincidence of wants": It refers to the situation where two individuals each have a good or service that the other wants. For example, if person A has apples and wants bananas, and person B has bananas and wants apples, then there is a double coincidence of wants, and they can trade directly.

- The double coincidence of wants is a significant limitation in a barter economy. It's often difficult to find two people who each have something the other person wants. For example, if person A has apples but wants bananas, and person B has bananas but doesn't want apples, then a direct trade can't happen. This makes barter transactions complicated and inefficient.

- Money (and credit) solves the problem of the double coincidence of wants by serving as a medium of exchange.

- As a medium of exchange, money is accepted by all participants in an economy for the exchange of goods and services. This means that person A can sell their apples to anyone for money, and then use that money to buy bananas from anyone who is selling them. Money facilitates transactions when there is a single coincidence of want.

- This makes transactions much simpler and more efficient, as you don't need to find a trading partner who wants what you have and has what you want. Instead, you just need to find someone who is selling what you want, and they don't need to want what you have because they will accept money in exchange.

# Credit and record keeping
# Asset as money

# Environment

- Goods are nonstorable and produced on the spot for immediate consumption, at cost $c > 0$. Hence, they cannot be retraded.
- An agent can only consume 1 unit of good at a time
- If an agent likes a product, she derives utility $u$.
- Agents still specialize: when $i$ and $j$ meet (with probability $\alpha$)
  - The probability of a double coincidence is $\delta$
  - The probability of a single coincidence of want where $i(j)$ likes $j(i)$'s output is $\sigma$
- Without durable asset or credit, only barter can take place. An agent's value in the barter economy

$$rV^B = \alpha\delta(u - c)$$

better than staying in autarky where their value is $V^A = 0$

# A credit system

- Now suppose agent $i$ produces for $j$ whenever $j$ likes $i$'s output
- This is credit because agents produce while receiving nothing by way of quid pro quo, with the understanding—call it a promise—that someone will do the same for them in the future.
- The flow payoff to this arrangement is

$$\begin{aligned} rV^c &= \alpha\delta(u-c) + \alpha\sigma u - \alpha\sigma c \\ &= \alpha(\delta+\sigma)(u-c) \\ &> rV^B \end{aligned}$$

- Is this incentive compatible?
  Would an agent default and not produce output in a single coincidence of want?

# Limited commitment and imperfect information

- If agents could commit, therefore, they would promise to abide by this arrangement, and this maximizes ex ante utility conditional on the matching process
- If they cannot commit, we must check that said promise is credible
- Incentive compatibility condition when an agent is supposed to produce in a single-coincidence meeting. It depends on the consequences of deviating:

  Denote $V^D$ the deviation payoff, IC is

  $$-c + V^C \geq \mu V^D + (1-\mu)V^C$$

  where $\mu$ is the probability deviators are caught and punished.

  - $\mu < 1$ captures imperfect monitoring, or record keeping, so that deviations are only probabilistically detected and communicated to the population at large.
  - $V^C - V^D$: the punishment when deviation is detected.
  - One possibility is $V^D = V^B$. In this case, IC implies $r \leq \hat{r}_C \equiv \mu\alpha\sigma(u-c)/c$
- The credit system is incentive compatible under limited commitment and imperfect information

  when agents are patient and the monitoring/communication technology is sufficiently good

# Asset as a token for transaction

- Suppose that credit is not feasible.
- There is a fixed supply of asset $A \in (0,1)$. An asset generates flow dividend $\rho$.
- Agents can hold either 0 or 1 unit of the asset.
- Let $V_a$ be the value function for agents with $a \in \{0,1\}$ asset and call those with $a = 1$ ($a = 0$) buyers (sellers).

$$rV_1 = \alpha\delta(u - c) + \alpha\sigma(1 - A)\tau_0\tau_1(u + V_0 - V_1) + \rho,$$

$$rV_0 = \alpha\delta(u - c) + \alpha\sigma A\tau_0\tau_1(V_1 - V_0 - c)$$

where $\tau_0$ is the probability sellers are willing to produce for the asset;

$\tau_1$ is the probability that buyers are willing to give up assets to consume

# Asset as a token for transaction

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$$rV_0 = \alpha\delta(u - c) + \alpha\sigma A\tau_0\tau_1(V_1 - V_0 - c)$$

Let $\Delta = V_1 - V_0$, $\quad r\Delta = \cancel{\alpha\delta(u - c)} + \alpha\sigma(1 - A)\tau_0\tau_1(u - \Delta) + \rho$
$\quad -\cancel{\alpha\delta(u - c)} - \alpha\sigma A\tau_0\tau_1(\Delta - c)$

$\rightarrow (r + \alpha\sigma\tau_0\tau_1)\Delta = \alpha\sigma(1 - A)\tau_0\tau_1 u + \alpha\sigma A\tau_0\tau_1 c + \rho$

The best responses are $\tau_0 = \begin{cases} 1 & \text{if } \Delta > c \\ [0,1] & \text{if } \Delta = c \\ 0 & \text{if } \Delta < c \end{cases}$ $\quad \tau_1 = \begin{cases} 1 & \text{if } u > \Delta \\ [0,1] & \text{if } u = \Delta \\ 0 & \text{if } u < \Delta \end{cases}.$

# Asset as a token for transaction

$$\boxed{\begin{aligned} rV_0 = {} & \alpha\delta(u-c) \\ & + \alpha\sigma A\,\tau_0\tau_1(V_1 - V_0 - c) \end{aligned}}
\quad
\boxed{\begin{aligned} rV_1 = {} & \alpha\delta(u-c) \\ & + \alpha\sigma(1-A)\,\tau_0\tau_1(u + V_0 - V_1) \\ & + \rho, \end{aligned}}$$

Let $\Delta = V_1 - V_0$,

Equilibrium: $\mathbf{V} = (V_0, V_1)$ and $\boldsymbol{\tau} = (\tau_0, \tau_1)$.

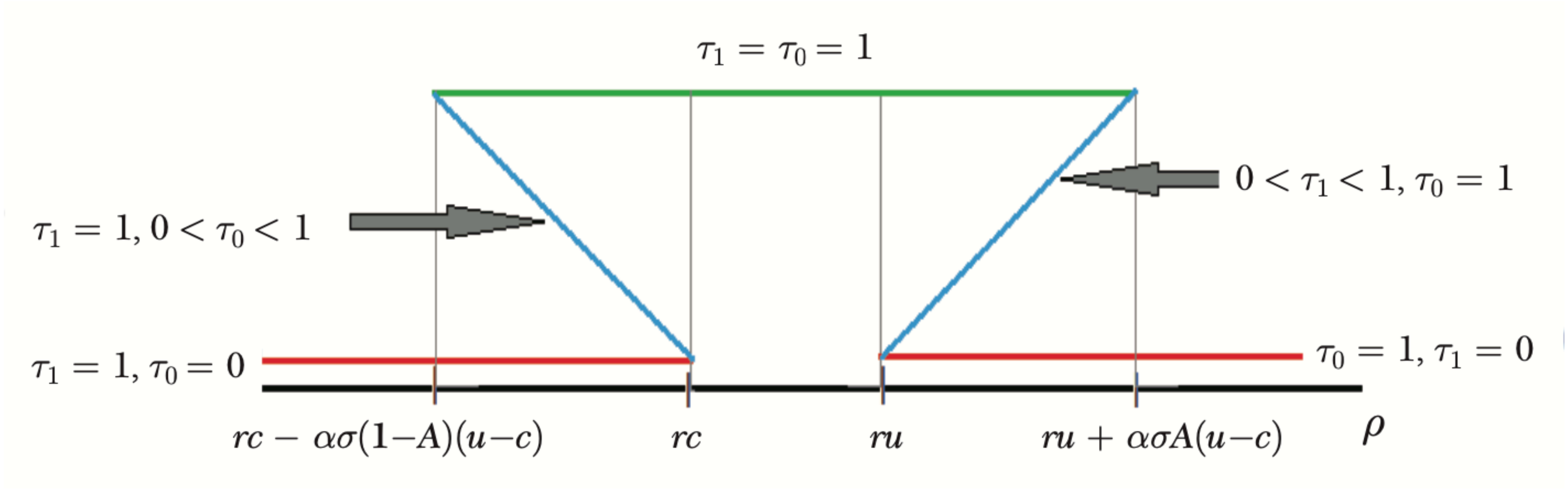
When $\rho$ is not too big so that $\tau_1 = 1$, $\tau_0 = 1$ iff
$r \le \hat{r}_M \equiv \alpha\sigma(1-A)(u-c)/c$

$$(r + \alpha\sigma\tau_0\tau_1)\Delta = \alpha\sigma(1-A)\tau_0\tau_1 u + \alpha\sigma A\tau_0\tau_1 c + \rho$$

The best responses are $\boxed{\tau_0 = \begin{cases} 1 & \text{if } \Delta > c \\ [0,1] & \text{if } \Delta = c \\ 0 & \text{if } \Delta < c \end{cases}}$ $\boxed{\tau_1 = \begin{cases} 1 & \text{if } u > \Delta \\ [0,1] & \text{if } u = \Delta \\ 0 & \text{if } u < \Delta \end{cases}.}$

# Equilibrium characterization

Whether an asset circulates is not always pinned down by primitives: for some $\rho$ there coexist equilibria where it does and where it does not.



# Comparing the credit system and the monetary system

- In the monetary system. a monetary equilibrium exists when

$$r \leq \hat{r}_M \equiv \alpha\sigma(1 - A)(u - c)/c$$

- In the credit system, a credit equilibrium exists when

$$r \leq \hat{r}_C \equiv \mu\alpha\sigma(u - c)/c$$

- $\hat{r}_C > \hat{r}_M$ if and only iff $\mu > 1 - A$
- "Money is memory" (Kocherlakota, 1998):
- If the monitoring and record-keeping technology is perfect ($\mu = 1$), money cannot be essential. Money exchange is viable whenever credit is, too. And the later is better.

# The essentiality of money

- Necessary ingredients for essentiality are:
  - not all gains from trade are exhausted by barter
  - lack of commitment
  - imperfect monitoring
- While it can be a useful institution, fiat money is in a sense also tenuous: there is always an equilibrium where it is not valued, plus sunspot equilibria where $\tau_0$ fluctuates
- Yet in another sense, money is robust: equilibria with $\tau_0 = 1$ exist for $\rho < 0$ as long as it is not too costly to hold money

# A Unified Framework for Monetary Theory and Policy Analysis

# Monetary model -- cash-in-advance

A representative household chooses consumption, labour supply, and money holding to maximize

$$\sum_{t=1}^{\infty} \beta^t \left[U(c(t)) - v(h(t))\right]$$

subject to a budget constraint for each period

$$W_t h_t + m_t \geq P_t c_t + m_{t+1}$$

and a cash-in-advance (CIA) constraint

$$m_t \geq P_t c_t$$

- Derive the optimality condition
- Derive the pricing equation for money from the first order condition for money
- What is the condition on inflation, $\frac{P_{t+1}}{P_t}$, under which the CIA condition is binding?
- $$\frac{u'(c_{t+1})}{v'(n_t)} \frac{W_t}{P_t} = \frac{1}{\beta} \frac{P_{t+1}}{P_t}$$

# Two issues with the CIA model

- It is a model "with" money, not a model "of" money
- Frictions that make money essential matter for the welfare consequence of inflation

# A unified framework for monetary theory and policy analysis

- Many monetary models in macroeconomics are reduced-form models, putting money in the utility function or imposing cash-in-advance constraints.
  - The reduced form models of money do not model frictions that make money essential: spacial, temporal, or informational frictions.
- Kiyotaki-Wright model is not suited for monetary policy analysis because it imposes extreme restrictions on how much cash agents can hold
- Lagos-Wright model makes monetary policy analysis more tractable
- It puts search and bargaining friction in the centre stage of the analysis

# Environment

- Time is discrete and continues forever.
- There is [0,1] continuum of infinitely lived agents with discount factor $0 < \beta < 1$
- Each period is divided to two subperiods, say day and night.
- Agents consume and supply labor in both subperiods.
- Agents' preferences in a period are
- $\mathcal{U}(x, h, X, H) = u(x) - c(h) + U(X) - H$
- $x, h$ $(X, H)$: consumption and labor during the day (night)
- $u(0) = c(0) = 0,$
- $u, U$ $(c)$: concave (convex), increasing, twice differentiable

# Day market

- During the day, agents interact in a decentralized market with anonymous bilateral matching, where each agent meets another with probability $\alpha$
- The day good $x$ comes in many varieties, of which each agent consumes a subset
- Each agent can transform labor one for one into one of these special goods that he himself does not consume
- Four possible types of meeting between agents $i$ and $j$
  - Double coincidence of wants: both consume what the other can produce (probability $\delta$)
  - Single coincidence of want
    - $i$ consumes what $j$ produces but not vice versa (probability $\sigma$)
    - $j$ consumes what $i$ produces but not vice versa (probability $\sigma$)
  - Neither wants what the other produces (probability $1 - \delta - 2\sigma$)

# Night market

- At night agents trade in a centralized Walrasian market
- With centralized trade, specialization does not lead to a double-coincidence problem. It is irrelevant whether the night good $X$ comes in many varieties
- So assume that the goods traded in the night market is a general good.
- The general good is produced and consumed in the night. It is divisible and perishable
- Another object that can be traded in the night is money
  - Money is divisible and storable in any nonnegative quantity

# Trade in the day and night

- During the day, the only feasible trades are
  - barter in special goods and
  - the exchange of special goods for money
  - terms of trade are negotiated using bargaining
- During the night, the only feasible trades involve general goods and money
- Money is essential because
  - Meetings in the day market are anonymous, there is no scope for trading future promises in this market, so exchange must be quid pro quo

# Equilibrium

- $F_t(\tilde{m})$: measure of agents starting the decentralized day market at $t$ holding $m \leq \tilde{m}$
- $G_t(\tilde{m})$: measure of agents starting the centralized night market at $t$ holding $m \leq \tilde{m}$
- The total money stock is $M_t = \int \tilde{m} dF_t(\tilde{m}) = \int \tilde{m} dG_t(\tilde{m})$
- $\phi_t$: the price of money in general goods in the centralized market
- $V_t(m)$: the value function for an agent with $m$ dollars when he enters the decentralized market
- $W_t(m)$: the value function for an agent with $m$ dollars when he enters the decentralized market

- Since trade is bilateral in the day market, a seller's production $h$ must equal a buyer's consumption $x$.
- We denote their common value by $q_t(m, \tilde{m})$ and use $d_t(m, \tilde{m})$to denote the dollars the buyer pays. They may depend on the money holdings of the buyer $m$ and seller $\tilde{m}$
- In double-coincidence meetings, $B_t(m, \tilde{m})$ denotes the payoff of an agent holding $m$ who meets someone with $\tilde{m}$
- The value in the decentralized market is

$$\begin{aligned} V_t(m) = \alpha\sigma \int & \{u[q_t(m, \tilde{m})] + W_t[m - d_t(m, \tilde{m})]\} dF_t(\tilde{m}) \\ + \alpha\sigma \int & \{-c[q_t(\tilde{m}, m)] + W_t[m + d_t(\tilde{m}, m)]\} dF_t(\tilde{m}) \\ + \alpha\delta \int & B_t(m, \tilde{m}) dF_t(\tilde{m}) + (1 - 2\alpha\sigma - \alpha\delta) W_t(m), \end{aligned}$$

DEFINITION. An equilibrium is a list $\{V_t, W_t, X_t, H_t, m'_t, q_t, d_t, \phi_t, F_t, G_t\}$, where, for all $t$, $V_t(m)$ and $W_t(m)$ are the value functions; $X_t(m)$, $H_t(m)$, and $m'_t(m)$ are the decision rules in the centralized market; $q_t(m, \tilde{m})$ and $d_t(m, \tilde{m})$ are the terms of trade in the decentralized market; $\phi_t$ is the price in the centralized market; and $F_t$ and $G_t$ are the distributions of money holdings before and after decentralized trade. The equilibrium conditions are as follows. For all $t$, (i) given prices and distributions, the value functions and decision rules satisfy (2) and (3); (ii) the terms of trade in the decentralized market maximize (4), given the value functions; (iii) $\phi_t > 0$ (i.e., we focus on monetary equilibria); (iv) centralized money markets clear, $\int m'(m) dG_t(m) = M$ (goods markets clear by Walras' law); and (v) $\{F_t, G_t\}$ is consistent with initial conditions and the evolution of money holdings implied by trades in the centralized and decentralized markets.[4]

# Equilibrium characterization

To begin, substitute for $H$ from the budget equation to write (3) as

$$W_t(m) = \phi_t m + \max_{X,m'} \{U(X) - X - \phi_t m' + \beta V_{t+1}(m')\}. \tag{5}$$

This immediately implies several things. First, $X_t(m) = X^*$, where $U'(X^*) = 1$. Also, $m_t'(m)$ does not depend on $m$.[5] Third, $W_t$ is linear in $m$ with slope $\phi_t$.

# Equilibrium characterization

Given this linearity, the bargaining problem (4) simplifies to

$$\max_{q,d} \, [u(q) - \phi_t d]^{\theta}[-c(q) + \phi_t d]^{1-\theta} \tag{6}$$

subject to $d \leq m$ and $q \geq 0$.

# Equilibrium characterization

$$q_t(m,\ \tilde{m}) \ = \begin{cases} \hat{q}_t(m) & \text{if } m < m_t^* \\ q^* & \text{if } m \geq m_t^*, \end{cases}$$

$$d_t(m,\ \tilde{m}) \ = \begin{cases} m & \text{if } m < m_t^* \\ m^* & \text{if } m \geq m_t^*, \end{cases} \tag{7}$$

where $\hat{q}_t(m)$ is the $q_t$ that solves $\phi_t m \ = \ z(q_t)$, with

$$z(q) \ \equiv \ \frac{\theta c(q) u'(q) \ + \ (1 - \theta) u(q) c'(q)}{\theta u'(q) \ + \ (1 - \theta) c'(q)}, \tag{8}$$

and $m_t^* \ = \ z(q^*)/\phi_t$.

Notice that the solution does not depend on the seller's money holdings $\tilde{m}$ at all, and so we write $q_t(m,\ \tilde{m}) \ = \ q_t(m)$ and $d_t(m,\ \tilde{m}) \ = \ d_t(m)$ in what follows. For all $m < m_t^*$, $q_t'(m) \ = \ \phi_t/z'(q_t)$, where

$$z' \ = \ \frac{u'c'[\theta u' \ + \ (1 - \theta) c'] \ + \ \theta(1 - \theta)(u - c)(u'c'' - c'u'')}{[\theta u' \ + \ (1 - \theta) c']^2} > 0. \tag{11}$$

# Equilibrium characterization

. Notice that the solution does not depend on the seller's money holdings $\tilde{m}$ at all, and so we write $q_t(m, \tilde{m}) = q_t(m)$ and $d_t(m, \tilde{m}) = d_t(m)$ in what follows.

$q_t(m) = \hat{q}_t(m)$ is strictly increasing for $m < m_t^*$, is continuous at $m_t^*$, and is constant at $q_t(m) = q^*$ for all $m \geq m_t^*$.

We can now use what we know about the bargaining solution and $W_t(m)$ to simplify (2) to

$$V_t(m) = v_t(m) + \phi_t m + \max_{m'} \{-\phi_t m' + \beta V_{t+1}(m')\}, \tag{12}$$

where

$$\begin{aligned} v_t(m) \equiv\ & \alpha\sigma\{u[q_t(m)] - \phi_t d_t(m)\} + \alpha\sigma \int \{\phi_t d_t(\tilde{m}) - c[q_t(\tilde{m})]\} dF_t(\tilde{m}) \\ & + \alpha\delta[u(q^*) - c(q^*)] + U(X^*) - X^*. \end{aligned} \tag{13}$$

# Equilibrium characterization

$$\phi_t = \beta[v'_{t+1}(M) + \phi_{t+1}]$$

$$\phi_t = \beta\{\alpha\sigma u'[q_{t+1}(M)]q'_{t+1}(M) + (1-\alpha\sigma)\phi_{t+1}\}.$$

Inserting $\phi_t = z(q_t)/M$ and $q'_t(M) = \phi/z'(q_t)$, from the bargaining solution, we arrive at

$$z(q_t) = \beta z(q_{t+1})\left[\alpha\sigma\frac{u'(q_{t+1})}{z'(q_{t+1})} + 1 - \alpha\sigma\right], \tag{18}$$

in steady state (18) yields

$$\frac{u'(q)}{z'(q)} = 1 + \frac{1-\beta}{\alpha\sigma\beta}.$$

# Monetary policy

The generalization of (18) is

$$\frac{z(q_t)}{M_t} = \beta \frac{z(q_{t+1})}{M_{t+1}} \left[ \alpha\sigma \frac{u'(q_{t+1})}{z'(q_{t+1})} + 1 - \alpha\sigma \right]. \tag{20}$$

If $M_{t+1} = (1+\tau)M_t$

$$\frac{u'(q)}{z'(q)} = 1 + \frac{1+\tau-\beta}{\alpha\sigma\beta}.$$

Define nominal rate $i$ by the Fisher equation, $1+i = (1+r)(1+\pi)$,
where $\pi = \tau$ in the equilibrium inflation rate, $r = (1-\beta)/\beta$ is the equilibrium real interest rate

$$\frac{u'(q)}{z'(q)} = 1 + \frac{i}{\alpha\sigma}.$$

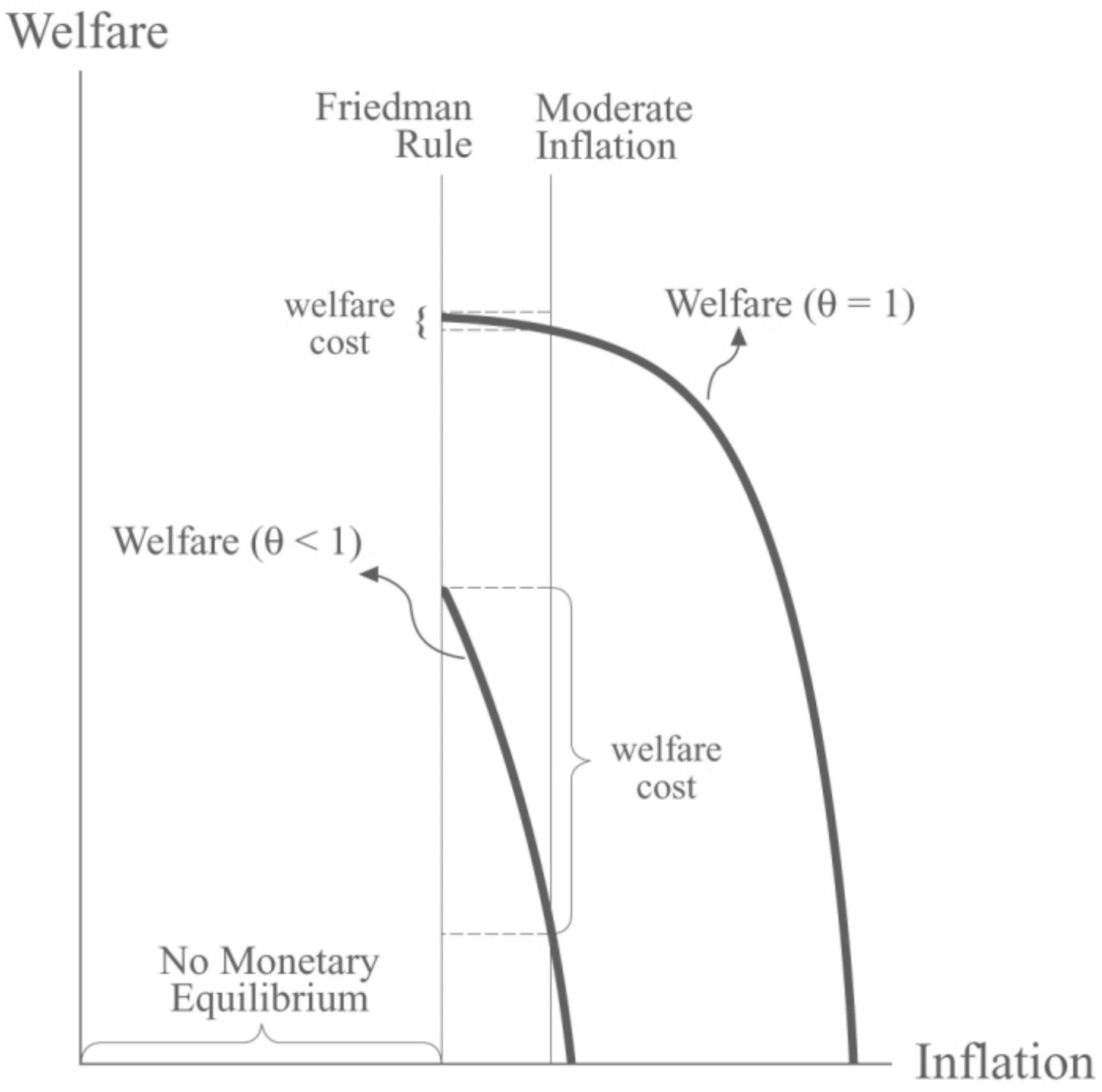



FIG. 1.—Welfare cost of moderate inflation

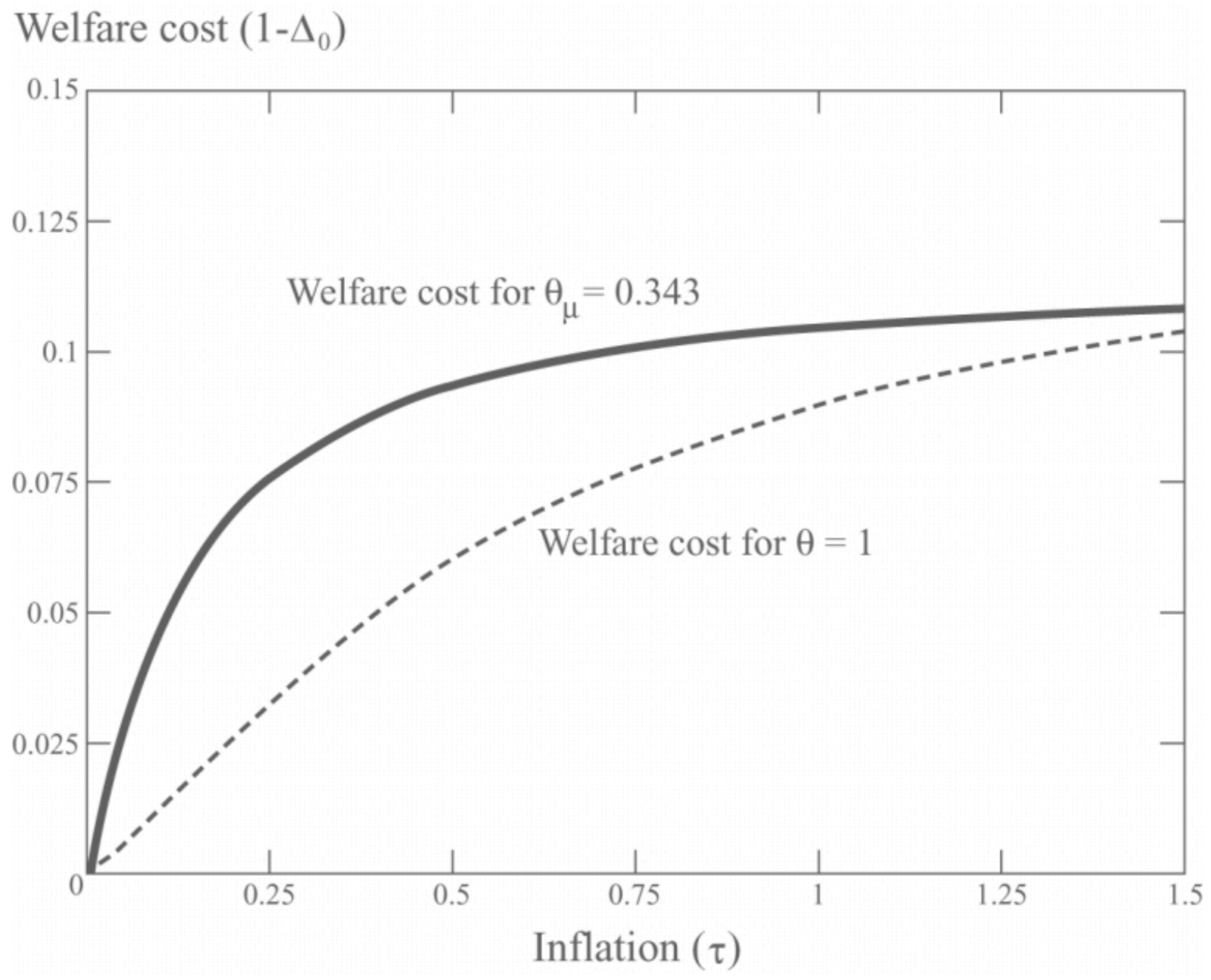



FIG. 3.—Welfare cost of higher inflation

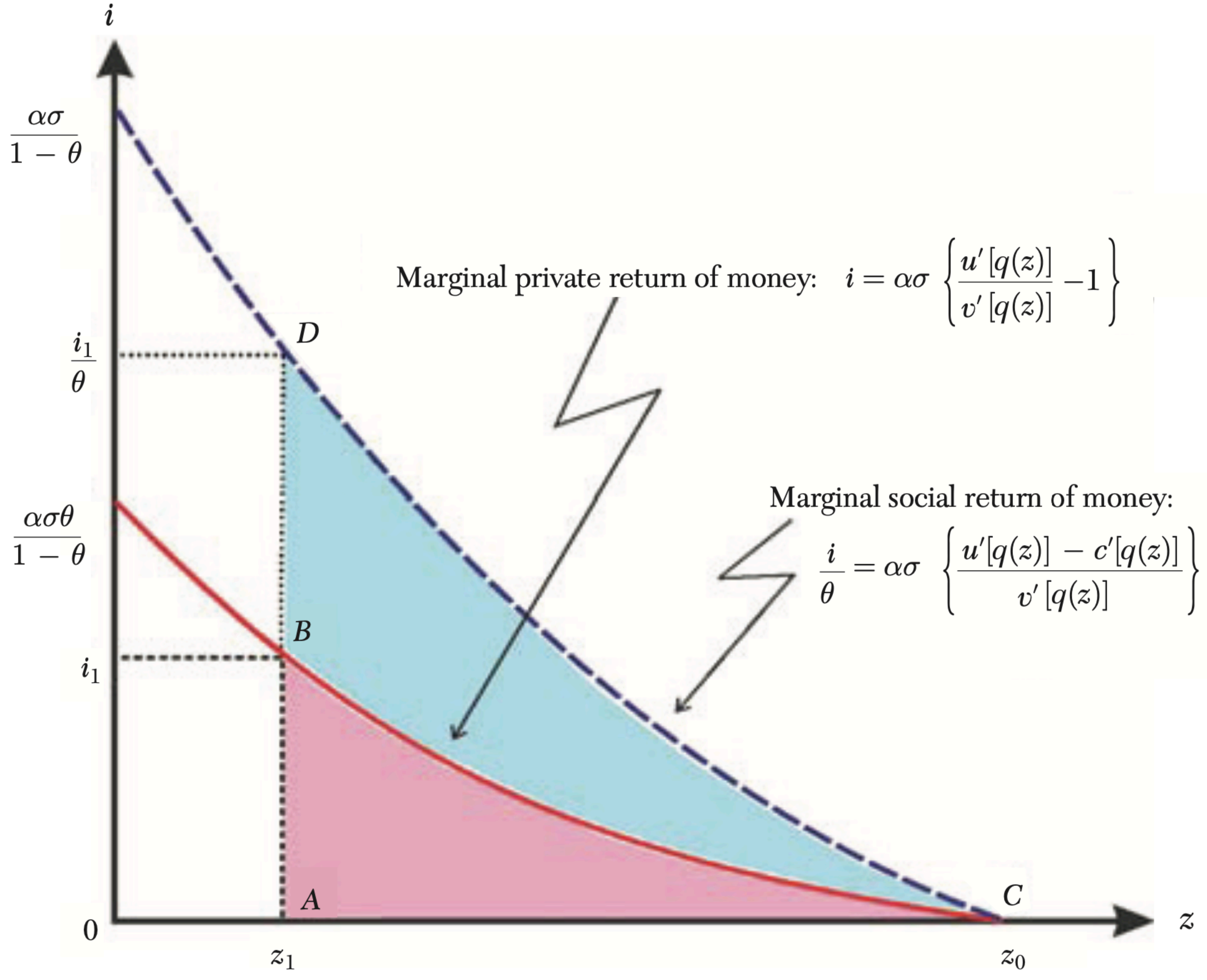



*Figure* 9. Welfare Triangles and the Cost of Inflation

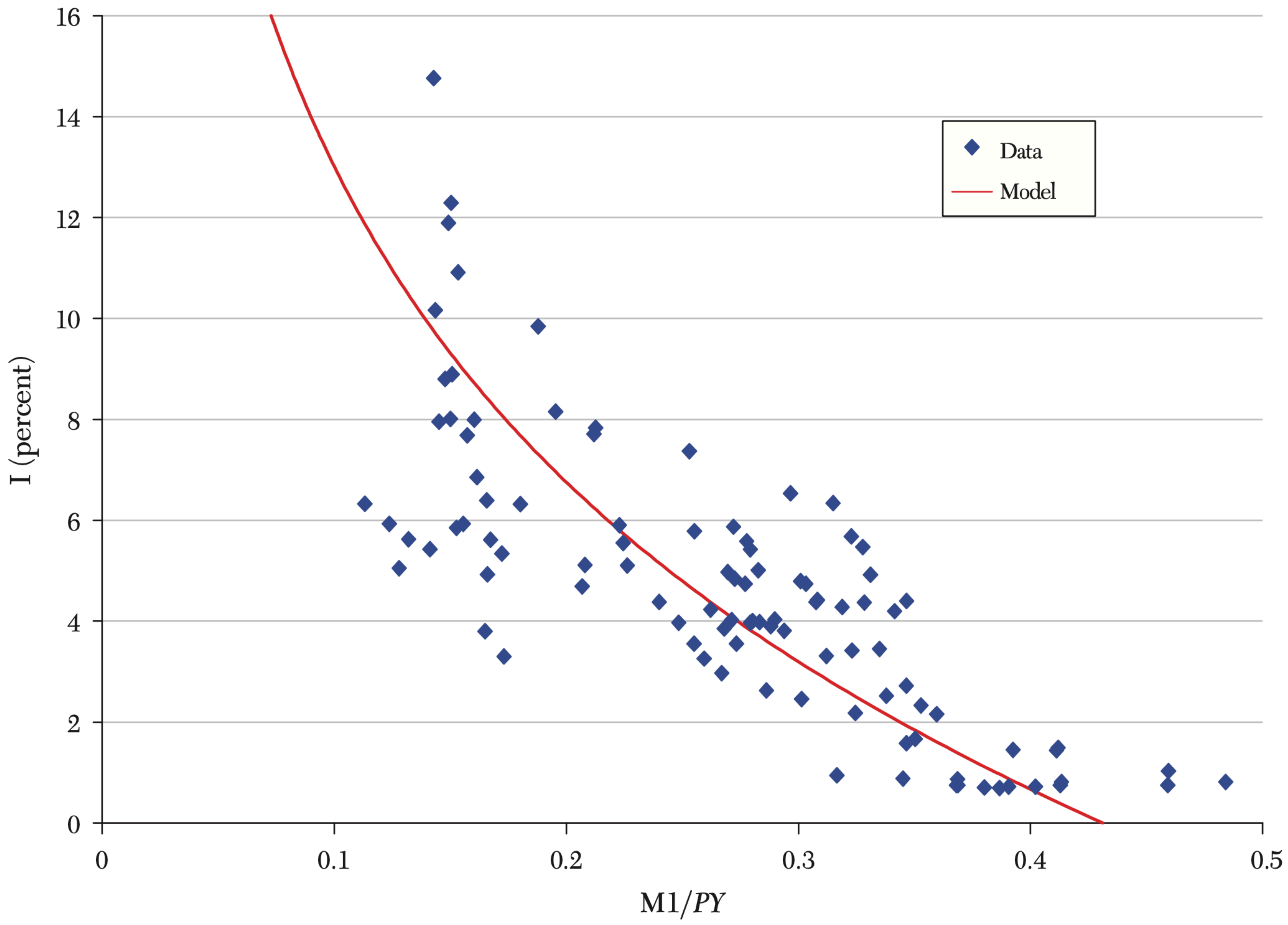



*Figure* 10. Money Demand: Theory and Data