Expectations and Stability

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Outline

- Expectations in macroeconomics and sources of business cycles
  - The role of expectations;
  - Sources of business cycles.
- Basic tools: difference equations and stability of nonlinear system
  - Concepts;
  - First-order difference equations;
  - Higher-order difference equations;
  - Simultaneous difference equations.
- RBC model
  - Solving for the recursive law of motion using the method of undetermined coefficients;
- RBC models with factor-generated externalities
- Monetary model with Taylor rule
- References

Expectations in macroeconomics

The role of expectations

- Central difference between economics and natural sciences: forward-looking decisions made by economic agents;
- Expectations play a key role;
  - Examples: consumption theory; investment decisions; asset prices, etc.
- The role of expectations: they influence the time path of the economy, and the time path of the economy influences expectations.
  - Rational expectation (RE): mathematical conditional expectation of the relevant variables;
  - The expectations are conditioned on all of the information available to the decision makers

Expectations in macroeconomics

Two examples

- Example 1. Cobweb model

$$\begin{aligned} d_t &= m_I - m_p p_t + v_{1t}, \\ s_t &= r_I + r_p p_t^e + v_{2t}, \\ s_t &= d_t, \end{aligned}$$

  where m_I, m_p, r_I and r_p are all positive constant.

- Example 2. Cagan model

$$m_t - p_t = -\psi \left(p_{t+1}^e - p_t \right), \; \psi > 0.$$

Sources of business cycles

News view of business cycles

- Business cycles are mainly the result of agents having incentives to continuously anticipate the economy's future demands.
  - If an agent can properly anticipate a future need...
  - If many agents adopt similar behavior...
  - However, errors are possible...
- Trace back to Pigou (1927)

The very source of fluctuations is the "*wave-like swings in the mind of the business world between errors of optimism and errors of pessimism*."

- Keynes' 1936 notion of animal spirits.
- Then what are optimism and pessimism in business cycles?
  - an entirely psychological phenomenon?
  - self-fulfilling fluctuations? The macroeconomy is inherently unstable
  - news view?

Concepts on difference equations

Definition

Discrete time: time is taken to be a discrete variable (integer number, like 1,2,3...)

Definition

First-order difference is

$$\Delta y_t = y_{t+1} - y_t,$$

where y_t is the value of y in the t^{th} period. Second-order difference is

$$\Delta^2 y_t = \Delta(\Delta y_t) = y_{t+2} - 2y_{t+1} + y_t.$$

Note 1: "period" - rather than point - of time.
Note 2: y has a unique value in each period of time.

Concepts

Definition

Difference Equation:

$$\Delta y_t = y_{t+1} - y_t = c,$$

or

$$y_{t+1} - y_t = ay_t + b.$$

Note: the choice of time subscripts is arbitrary, i.e. it does not make any difference if we write it as $y_{t+1} - y_t = c$ or as $y_{t+2} - y_{t+1} = c$.

Solving a first-order difference equation

- Iterative method
- General method

$$y_{t+1} + ay_t = c,$$

  The general solution y_t consists of y_p (particular solution) and y_c (complementary solution)

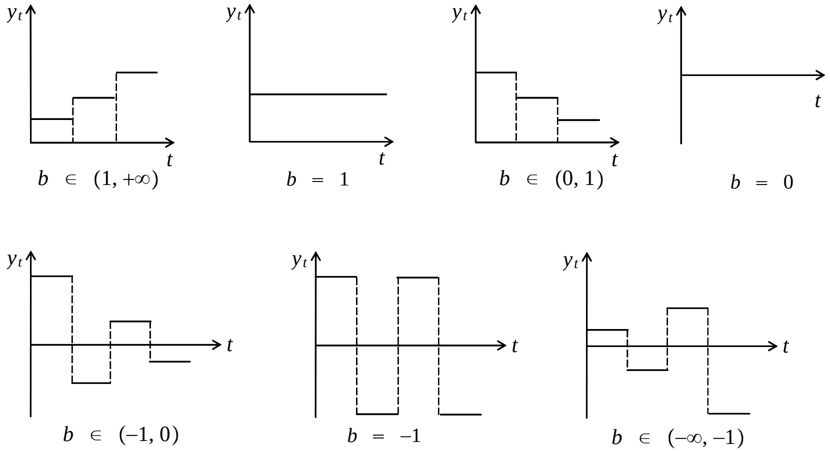
- Complementary solution: Try $y_t = Ab^t$ (A is arbitrary) and get $b = -a$.
- Particular solution: 1. if $a \neq -1$, solve $y_p = \frac{c}{1+a}$; 2. if $a = -1$, solve $y_p = ct$.
- Get y_t

$$y_t = y_c + y_p = \begin{cases} A\left(-a\right)^t + \frac{c}{1+a}, a \neq -1 \\ A + ct, a = -1 \end{cases}.$$

- How about A? - Determined by y_0.

Dynamic stability of equilibrium

- The significance of b:



$$y_t = b^t$$

- Nonoscillatory (oscillatory) if $b > (<) 0$;

Dynamic stability of equilibrium

- The role of A
  - Scale effect: magnitude
  - Mirror effect: sign
- Example: A market model with inventory

$$Q_{dt} = \alpha - \beta P_t,$$

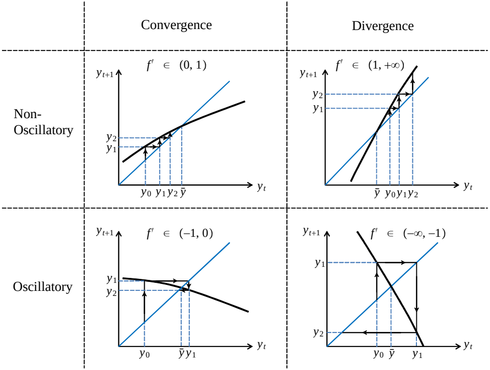
$$Q_{st} = -\gamma + \delta P_t,$$

$$P_{t+1} = P_t - \sigma \left(Q_{st} - Q_{dt} \right),$$

where $\alpha, \beta, \gamma, \delta, \sigma > 0$. Solve the time path of P_t.

Non-linear difference equations: qualitative-graphic approach

$$y_{t+1} = f(y_t).$$



Stability of Non-Linear System

Higher-order difference equations

- Second-order linear difference equations with constant coefficients and constant term

$$y_{t+2} + a_1 y_{t+1} + a_2 y_t = c.$$

- Particular solution y_p: the intertemporal equilibrium level of y.
  - If $a_1 + a_2 \neq -1$, try $y_p = k$ and get $k = \frac{c}{1+a_1+a_2}$;
  - If $a_1 + a_2 = -1$ and $a_1 \neq -2$, try $y_p = kt$ and get $k = \frac{c}{2+a_1}$;
  - If $a_1 + a_2 = -1$ and $a_1 = -2$, try $y_p = kt^2$ and get $k = \frac{c}{2}$.
- Complementary solution: the deviation from the equilibrium for every time period

$$y_{t+2} + a_1 y_{t+1} + a_2 y_t = 0.$$

Try $y_t = Ab^t$ and get the characteristic equation

$$b^2 + a_1 b + a_2 = 0.$$

Higher-order difference equations

- Case 1: two distinct real roots: $a_1^2 > 4a_2$

$$y_c = A_1 b_1^t + A_2 b_2^t.$$

- Case 2: repeated real roots: $a_1^2 = 4a_2$ and thus $b = b_1 = b_2 = -\frac{a_1}{2}$

$$y_c = A_3 b^t + A_4 t b^t.$$

- Case 3: complex roots: $a_1^2 < 4a_2$

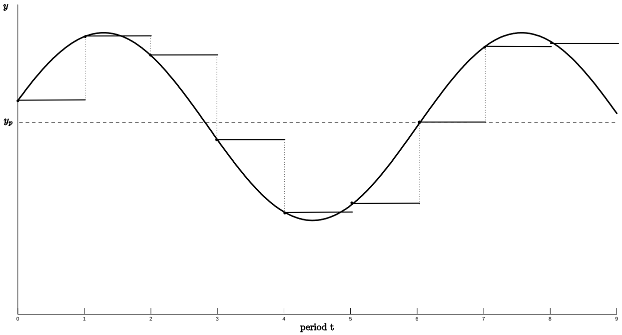
$$b_{1,2} = h \pm vi, \text{ where } h = -\frac{a_1}{2} \text{ and } v = \frac{\sqrt{4a_2 - a_1^2}}{2}.$$

$$y_c = A_1 b_1^t + A_2 b_2^t = A_1 (h + vi)^t + A_2 (h - vi)^t.$$

Higher-order difference equations

- Because $(h \pm vi)^t = R^t (\cos\theta t \pm i \sin\theta t)$ where $R = \sqrt{h^2 + v^2} = \sqrt{a_2}$, $\cos\theta = \frac{h}{R} = -\frac{a_1}{2\sqrt{a_2}}$ and $\sin\theta = \frac{v}{R} = \sqrt{1 - \frac{a_1^2}{4a_2}}$, we have

$$y_c = R^t \left(A_5 \cos\theta t + A_6 \sin\theta t\right).$$



Convergence of the time path

- Distinct roots
  - Dominant root: the root with the higher absolute value
  - A time path will be convergent iff the dominant root is less than 1 in absolute value
- Repeated roots: if $|b| < 1$, we have convergence
- Complex roots:
  - if $R < 1$, i.e. $|b| < 1$, we have damped stepped fluctuation
  - if $R > 1$, i.e. $|b| > 1$, we have explosive stepped fluctuation

Simultaneous difference equations

- Relation between higher-order difference equation and simultaneous difference equations

$$y_{t+2} + a_1 y_{t+1} + a_2 y_t = c.$$

Define $x_t = y_{t+1}$. We will have

$$\begin{aligned} x_{t+1} + a_1 x_t + a_2 y_t &= c, \\ y_{t+1} &= x_t. \end{aligned}$$

In matrix form, it is

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \end{pmatrix} = \begin{pmatrix} -a_1 & -a_2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix} + \begin{pmatrix} c \\ 0 \end{pmatrix}.$$

- Simultaneous difference equations

$$X_{t+1} = AX_t + b,$$

where $X_t = (x_{1t}, x_{2t}, ...x_{nt})'$, A is a constant matrix with coefficients a_{ij}, $i, j = 1...n$, and $b = (b_1, b_2, ...b_n)'$.

Simultaneous difference equations

- Particular solution: $x_{t+1} = x_t = x$ and $y_{t+1} = y_t = y$;
- Complementary solution: substituting $x_t = mb^t$ and $y_t = nb^t$, we have the characteristic equation and the characteristic roots b_1 and b_2.
- Characteristic equation

$$p(b) = |A - bI| = b^2 - \mathcal{T} b + \mathcal{D} = 0,$$

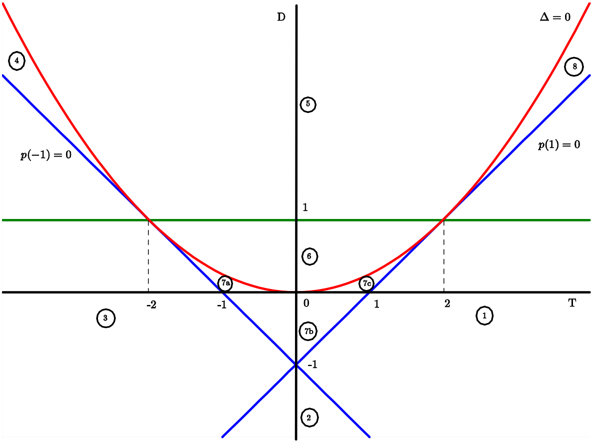
where $\mathcal{T}$ and $\mathcal{D}$ are the trace and determinant of matrix A.

- Moreover, we know that

$$\mathcal{T} = b_1 + b_2, \ \ \mathcal{D} = b_1 b_2,$$

$$p(b) = (b - b_1)(b - b_2).$$

Stability property



Stability Triangle

Stability property

- Red line: $\Delta = \mathcal{T}^2 - 4\mathcal{D} = 0$
  - Region above the red line: $\Delta < 0 \Rightarrow$ complex roots;
  - Region below the red line: $\Delta > 0 \Rightarrow$ real roots;
- Blue lines: $p(1) = 1 - \mathcal{T} + \mathcal{D} = 0$ and $p(-1) = 1 + \mathcal{T} + \mathcal{D} = 0$
  - Region above (below) the right blue line: $p(1) > (<) 0$;
  - Region above (below) the left blue line: $p(-1) > (<) 0$;
- Green line: $D = 1$
  - Region above the green line: $|b| > 1$;
  - Region below the green line: $|b| < 1$.

Stability property

Region	$p(b_i)$	b_i	Stability
1	$p(1) < 0,\ p(-1) > 0$	$\lvert b_1 \rvert < 1,\ b_2 > 1$	saddle
2	$p(1) < 0,\ p(-1) < 0$	$b_1 b_2 < 0,\ \lvert b_i \rvert > 1$	explosive
3	$p(1) > 0,\ p(-1) < 0$	$\lvert b_1 \rvert < 1,\ b_2 < -1$	saddle
4	$p(1) > 0,\ p(-1) > 0$ $\mathcal{D} > 1,\ \mathcal{T} < -2$	$b_i < -1$	explosive
5	$\Delta < 0,\ \mathcal{D} > 1$	$\lvert b_i \rvert > 1$	explosive
6	$\Delta < 0,\ \mathcal{D} < 1$	$\lvert b_i \rvert < 1$	stable
7	$p(1) > 0,\ p(-1) > 0$ $\mathcal{D} < 1$	$\lvert b_i \rvert < 1$	stable
8	$p(1) > 0,\ p(-1) > 0$ $\mathcal{D} > 1,\ \mathcal{T} > 2$	$b_i > 1$	explosive

Stability property

- Conditions for a saddle: $p(1) < 0$, $p(-1) > 0$ or $p(1) > 0$, $p(-1) < 0$

$$\begin{aligned} 1 - \mathcal{T} + \mathcal{D} &< 0 \text{ and } 1 + \mathcal{T} + \mathcal{D} > 0; \text{ or} \\ 1 - \mathcal{T} + \mathcal{D} &> 0 \text{ and } 1 + \mathcal{T} + \mathcal{D} < 0. \end{aligned}$$

  Or equivalent as

$$|\mathcal{T}| > |1 + \mathcal{D}| .$$

- Conditions for two stable roots: $\Delta < 0$, $\mathcal{D} < 1$ or $p(1) > 0$, $p(-1) > 0$, $\mathcal{D} < 1$

$$\mathcal{D} < 1,\ 1 - \mathcal{T} + \mathcal{D} > 0 \text{ and } 1 + \mathcal{T} + \mathcal{D} > 0,$$

  i.e.

$$\mathcal{D} < 1 \text{ and } |\mathcal{T}| < 1 + \mathcal{D}.$$

Stability property

Local Uniqueness/Multiplicity

Definition

Predetermined variable: the variable whose initial value is given, as k, h, and b;

Definition

Jump variable: the variable whose initial value is not given, as c, l, and p (sometimes).

Theorem

Conditions for local uniqueness/multiplicity:
1. If the number of stable roots $=$ the number of predetermined variables $\Rightarrow$ Saddle path (Determinacy);
2. If the number of stable roots $<$ the number of predetermined variables $\Rightarrow$ Source (Explosive);
3. If the number of stable roots $>$ the number of predetermined variables $\Rightarrow$ Sink (Indeterminacy).

Solve for the recursive law of motion with method of undetermined coefficients.

State variables: $\hat{k}_{t-1}$, $\hat{z}_t$

The dynamics of the model should be described by **recursive laws of motion** in terms of the state variables,

$$\begin{aligned}\hat{k}_t &= v_{kk}\hat{k}_{t-1} + v_{kz}\hat{z}_t,\\ \hat{c}_t &= v_{ck}\hat{k}_{t-1} + v_{cz}\hat{z}_t.\end{aligned}$$

We need to solve for v_{kk}, v_{kz}, v_{ck} and v_{cz}, the "undetermined" coefficients.

Coefficient interpretation: *elasticities*.

Recall: the log-linearized system consists of

$$\hat{k}_t = \frac{1}{\beta}\hat{k}_{t-1} - \frac{\overline{C}}{\bar{K}}\hat{c}_t + \frac{\widetilde{\delta}}{\alpha\beta}\hat{z}_t,$$

$$\sigma \mathrm{E}_t \hat{c}_{t+1} + \widetilde{\delta}\left(1-\alpha\right)\hat{k}_t - \widetilde{\delta}\mathrm{E}_t\hat{z}_{t+1} = \sigma\hat{c}_t,$$

$$\hat{z}_t = \psi\hat{z}_{t-1} + \varepsilon_t.$$

Recursivity

- Substitute the postulated linear recursive law of motion into the dynamic equations until only $\hat{k}_{t-1}$ and $\hat{z}_t$ remain. E.g.

$$\mathrm{E}_t \hat{z}_{t+1} = \psi \hat{z}_t,$$

$$\begin{aligned}
\mathrm{E}_t \hat{c}_{t+1} &= \mathrm{E}_t (v_{ck} \hat{k}_t + v_{cz} \hat{z}_{t+1}) \\
&= v_{ck} \left(v_{kk} \hat{k}_{t-1} + v_{kz} \hat{z}_t \right) + v_{cz} \psi \hat{z}_t \\
&= v_{ck} v_{kk} \hat{k}_{t-1} + \left(v_{ck} v_{kz} + v_{cz} \psi \right) \hat{z}_t .
\end{aligned}$$

- Compare coefficients.

Recursivity

For the first equation (budget constraint)

$$\begin{aligned} \hat{k}_t &= \frac{1}{\beta}\hat{k}_{t-1} - \frac{\bar{C}}{\bar{K}}\hat{c}_t + \frac{\widetilde{\delta}}{\alpha\beta}\hat{z}_t, \\ v_{kk}\hat{k}_{t-1} + v_{kz}\hat{z}_t &= \frac{1}{\beta}\hat{k}_{t-1} - \frac{\bar{C}}{\bar{K}}\left(v_{ck}\hat{k}_{t-1} + v_{cz}\hat{z}_t\right) + \frac{\widetilde{\delta}}{\alpha\beta}\hat{z}_t, \end{aligned}$$

$$\left(\frac{1}{\beta} - \frac{\bar{C}}{\bar{K}}v_{ck} - v_{kk}\right)\hat{k}_{t-1} + \left(\frac{\widetilde{\delta}}{\alpha\beta} - \frac{\bar{C}}{\bar{K}}v_{cz} - v_{kz}\right)\hat{z}_t = 0.$$

Comparing coefficients: since the equation has to be satisfied for any value of $\hat{k}_{t-1}$ and $\hat{z}_t$, we have

$$\begin{aligned} \text{for } \hat{k}_{t-1} \quad &: \quad v_{kk} = \frac{1}{\beta} - \frac{\bar{C}}{\bar{K}}v_{ck} \\ \text{for } \hat{z}_t \quad &: \quad v_{kz} = \frac{\widetilde{\delta}}{\alpha\beta} - \frac{\bar{C}}{\bar{K}}v_{cz} \end{aligned}$$

Recursivity

For the second equation (Euler equation/asset pricing)

$$\sigma \mathrm{E}_t \hat{c}_{t+1} + \widetilde{\delta} (1-\alpha) \mathrm{E}_t \hat{k}_t - \widetilde{\delta} \mathrm{E}_t \hat{z}_{t+1} = \sigma \hat{c}_t ,$$

$$\begin{aligned} & \sigma \left[v_{ck} v_{kk} \hat{k}_{t-1} + (v_{ck} v_{kz} + v_{cz} \psi) \hat{z}_t \right] + \widetilde{\delta} (1-\alpha) \left(v_{kk} \hat{k}_{t-1} + v_{kz} \hat{z}_t \right) - \\ = \; & \sigma \left(v_{ck} \hat{k}_{t-1} + v_{cz} \hat{z}_t \right) , \end{aligned}$$

$$\begin{aligned} & \left[\sigma v_{ck} v_{kk} + \widetilde{\delta} (1-\alpha) v_{kk} - \sigma v_{ck} \right] \hat{k}_{t-1} + \\ & \left[\sigma (v_{ck} v_{kz} + v_{cz} \psi) + \widetilde{\delta} (1-\alpha) v_{kz} - \widetilde{\delta} \psi - \sigma v_{cz} \right] \hat{z}_t \\ = \; & 0. \end{aligned}$$

Comparing coefficients, we have

$$\begin{aligned} \text{for } \hat{k}_{t-1} \quad & : \quad \sigma v_{ck} (1 - v_{kk}) = \widetilde{\delta} (1-\alpha) v_{kk} , \\ \text{for } \hat{z}_t \quad & : \quad \sigma v_{cz} (1-\psi) = \left[\sigma v_{ck} + \widetilde{\delta} (1-\alpha) \right] v_{kz} - \widetilde{\delta} \psi . \end{aligned}$$

Comparing coefficients

Collecting the results, and comparing coefficients on $\hat{k}_{t-1}$,

$$\sigma v_{ck}\left(1 - v_{kk}\right) = \widetilde{\delta}\left(1-\alpha\right) v_{kk}, \tag{1}$$

$$v_{kk} = \frac{1}{\beta} - \frac{\bar{C}}{\bar{K}} v_{ck}. \tag{2}$$

To solve v_{kk}, we substitute v_{ck} and obtain a quadratic equation

$$v_{kk}^2 - \left[1 + \frac{1}{\beta} - \frac{\widetilde{\delta}\left(1-\alpha\right)}{\sigma}\frac{\bar{C}}{\bar{K}}\right] v_{kk} + \frac{1}{\beta} = 0.$$

Solving the quadratic equation

$$0 = v_{kk}^2 - \gamma v_{kk} + \frac{1}{\beta},$$

where

$$\gamma = 1 + \frac{1}{\beta} + \frac{\widetilde{\delta}\,(1-\alpha)}{\sigma}\,\frac{1 - [1 - (1-\alpha)\,\delta]\,\beta}{\alpha\beta}.$$

The solution is a high school math problem:

$$v_{kk} = \frac{\gamma}{2} - \sqrt{\left(\frac{\gamma}{2}\right)^2 - \frac{1}{\beta}}.$$

Why we delete the root

$$\frac{\gamma}{2} + \sqrt{\left(\frac{\gamma}{2}\right)^2 - \frac{1}{\beta}}.\ ?$$

Solving the quadratic equation

$$\begin{aligned} (v_{kk} - \lambda_1)\,(v_{kk} - \lambda_2) &= 0, \\ v_{kk}^2 - (\lambda_1 + \lambda_2) v_{kk} + \lambda_1 \lambda_2 &= 0, \end{aligned}$$

$$\begin{aligned} \lambda_1 \lambda_2 &= \frac{1}{\beta} > 1, \\ \lambda_1 + \lambda_2 &= \gamma > 1 + \frac{1}{\beta}, \\ \text{hence, } \lambda_1 + \lambda_2 &> 1 + \lambda_1 \lambda_2, \\ (1 - \lambda_1)(\lambda_2 - 1) &> 0. \end{aligned}$$

so λ_1 and λ_2 both positive, one root > 1, and the other root < 1.
We need to delete the explosive solution!

Solving the quadratic equation

Once v_{kk} is solved, the others can be solved easily.
Plugging it into equation (2),

$$v_{ck} = \left(\frac{\widetilde{\delta}}{\alpha\beta} - \delta \right) \left(\frac{1}{\beta} - v_{kk} \right),$$

we get v_{ck}.

Solving the quadratic equation

Then for coefficients on $\hat{z}_t$

$$v_{kz} = \left(\delta - \frac{\widetilde{\delta}}{\alpha\beta}\right) v_{cz} + \frac{\widetilde{\delta}}{\alpha\beta}, \tag{3}$$

$$\sigma v_{cz}\left(1-\psi\right) = \left[\sigma v_{ck} + \widetilde{\delta}\left(1-\alpha\right)\right] v_{kz} - \widetilde{\delta}\psi, \tag{4}$$

i.e.

$$v_{cz} = \frac{\sigma v_{ck} + \widetilde{\delta}\left(1-\alpha\right) - \alpha\beta\psi}{\sigma\left(1-\psi\right) - \left[\sigma v_{ck} + \widetilde{\delta}\left(1-\alpha\right)\right]\left(\delta - \frac{\widetilde{\delta}}{\alpha\beta}\right)} \frac{\widetilde{\delta}}{\alpha\beta},$$

where v_{ck} is known. Substituting v_{cz} into (3), we solve v_{kz}.

Solving the quadratic equation

We could obtain

$$\begin{aligned}\hat{k}_t &= v_{kk}\hat{k}_{t-1} + v_{kz}\hat{z}_t,\\ \hat{c}_t &= v_{ck}\hat{k}_{t-1} + v_{cz}\hat{z}_t,\\ \hat{z}_t &= \psi\hat{z}_{t-1} + \varepsilon_t,\end{aligned}$$

where

$$v_{kk} = \frac{\gamma}{2} - \sqrt{\left(\frac{\gamma}{2}\right)^2 - \frac{1}{\beta}},\ v_{ck} = \left(\frac{\widetilde{\delta}}{\alpha\beta} - \delta\right)\left(\frac{1}{\beta} - v_{kk}\right),$$

$$v_{cz} = \frac{\sigma v_{ck} + \widetilde{\delta}\,(1-\alpha) - \alpha\beta\psi}{\sigma\,(1-\psi) - \left[\sigma v_{ck} + \widetilde{\delta}\,(1-\alpha)\right]\left(\delta - \frac{\widetilde{\delta}}{\alpha\beta}\right)}\frac{\widetilde{\delta}}{\alpha\beta},$$

$$v_{kz} = \left(\delta - \frac{\widetilde{\delta}}{\alpha\beta}\right)v_{cz} + \frac{\widetilde{\delta}}{\alpha\beta}.$$

Calibration and simulation

Assuming quarterly data with

$$\begin{array}{ll} \beta = 0.99 & \alpha = 0.36 \\ \sigma = 1.0 & \delta = 0.025 \\ \bar{Z} = 1 & \psi = 0.95 \end{array}$$

Then we get...

Calibration and simulation

- *Impulse response* analysis: trace out all variables for $\varepsilon_1 = 1$, $\varepsilon_t = 0$ for $t > 1$, when starting from the steady state.
- Because

$$\hat{z}_t = \psi \hat{z}_{t-1} + \varepsilon_t,$$

we have

$$\hat{z}_1 = \psi \hat{z}_0 + \varepsilon_1 = \varepsilon_1,$$

$$\hat{z}_2 = \psi \hat{z}_1 + \varepsilon_2 = \psi \varepsilon_1,$$

$$\hat{z}_j = \psi^{j-1} \varepsilon_1.$$

and

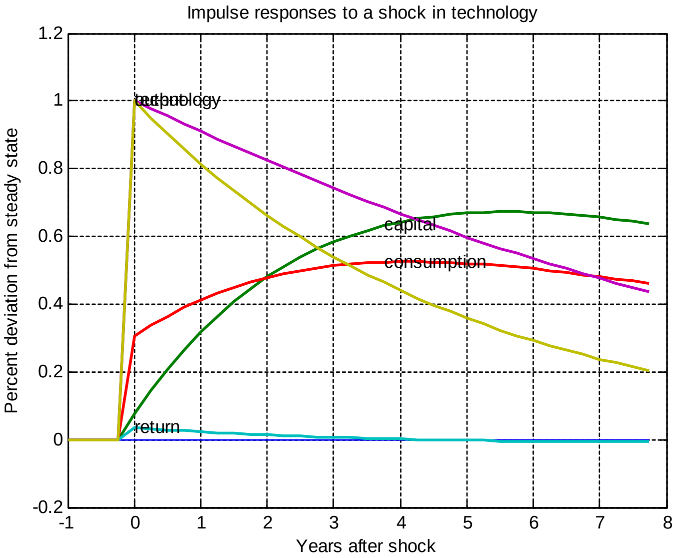
$$\hat{k}_1 = v_{kk} \hat{k}_0 + v_{kz} \hat{z}_1 = v_{kz} \varepsilon_1,$$

$$\hat{k}_2 = v_{kk} \hat{k}_1 + v_{kz} \hat{z}_2 = v_{kk} v_{kz} \varepsilon_1 + v_{kz} \psi \varepsilon_1 = \left(v_{kk} + \psi \right) v_{kz} \varepsilon_1,$$

$$\hat{k}_j = v_{kk} \hat{k}_{j-1} + v_{kz} \psi^{j-1} \varepsilon_1 = \sum_{i=0}^{j-1} \left(v_{kk}^{i} \psi^{j-i-1} \right) v_{kz} \varepsilon_1.$$

Calibration and simulation

Impulse response functions



RBC models with factor-generated externalities

Terminologies

- (local) indeterminacy – infinite number of equilibrium paths converging to the same steady state.
- Sunspot equilibria (Shell, 1977; Cass and Shell, 1983): self-fulfilling expectations; self-fulfilling prophecies
  - agents receive different allocations across states with identical fundamentals
  - equilibrium allocations influenced by purely extrinsic belief shocks in general equilibrium models;
  - agents have identical fundamentals: preferences, endowments and technology;
  - agents have different consumption and/or production;
  - cannot occur under Arrow-Debreu structure.
- indeterminacy implies the existence of stationary sunspot equilibria.
- existence of sunspot equilibria implies economy can fluctuate in the absence of schocks to fundamentals.
- use indeterminacy, sunspots, self-fulfilling expectations, self-fulfilling prophecies interchangeably.

RBC models with factor-generated externalities

Some strands of literature

- *Business cycles*: the role of beliefs in business fluctuations, Benhabib and Farmer (1994), Farmer and Guo (1994), Wen (1998), Benhabib and Farmer (1996), Benhabib, Meng and Nishimura (2000);

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- *Monetary transmission mechanism:* MIU model in Calvo (1979), Money in the production function model in Benhabib and Farmer (1996), CIA model in Cooley and Hansen (1989, 1991);

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- *Policy Feedback*: Fiscal policy (Leeper, 1991; Leith and Thadden, 2008), Monetary policy (Leeper, 1991; Dupor, 2001; Carlstrom and Fuerst, 2005, Huang, Meng and Xue, 2009, 2018, 2019);

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- *Policy Feedback*: Fiscal policy (Leeper, 1991; Leith and Thadden, 2008), Monetary policy (Leeper, 1991; Dupor, 2001; Carlstrom and Fuerst, 2005, Huang, Meng and Xue, 2009, 2018, 2019);
- *Endogenous economic growth*: Benhabib and Perli (1994), Xie (1994), Mino (2001);

With factor-generated externalities

- Benhabib-Farmer-Guo model, the representative agent solves

$$\max \sum_{t=0}^{\infty} E_t \rho^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{L_t^{1+\chi}}{1+\chi} \right), \tag{5}$$

  subject to

$$C_t + K_{t+1} \leq (1-\delta) K_t + Y_t \text{ and } Y_t = A_t K_t^a L_t^b, \tag{6}$$

  where $a + b = 1$, and $A_t = \overline{K}_t^{\alpha - a} \overline{L}_t^{\beta - b}$.
- Wen (1996): capacity utilization;

$$y_t = \overline{e}_t \left(u_t K_t \right)^{\alpha} L_t^{1-\alpha}, \text{ where } \overline{e}_t = \left(\overline{u}_t \overline{K}_t \right)^{\gamma\theta} \overline{L}_t^{(1-\gamma)\theta},$$

  where

$$\dot{K}_t = I_t - \delta_t K_t \text{ and } \delta_t = \tau u_t^{\eta}.$$

- Benhabib and Farmer (1996): two-sector model with sector-specific externalities.

$$y_t^j = \left[\left(K_t^j \right)^{\alpha} \left(L_t^j \right)^{1-\alpha} \right]^{\theta_j} \left[K_t^{\alpha} L_t^{1-\alpha} \right]^{\sigma_j} \left(k_t^j \right)^{\alpha} \left(l_t^j \right)^{1-\alpha}, \text{ where } j = I, C$$

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  - Increasing returns-to-scale: Duffy and Xiao (2008); Huang and Meng (2009).

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- **Policy rule**

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