DSGE: An Introduction

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Background

What is Macroeconomics?

- Macroeconomics studies aggregate growth and fluctuations.
- The ultimate goal of macroeconomic research is to evaluate policy and improve social welfare.
- Macroeconomic research needs both qualitative (theory) and quantitative analysis.

Convergency in Methodology

1. **From small to larger models**
   thanks to the technological progress: Computational & Computer

2. **From equation-by-equation to system estimation**
   e.g., SVAR, structural estimation of DSGE (MLE, SMM)

What is the Fashion in Macroeconomics

Kocherlakota (2009): "Some Thoughts on the State of Macro".

1. Macroeconomists don't ignore heterogeneity.
2. Macroeconomists don't ignore frictions.
3. Macroeconomic modeling doesn't ignore bounded rationality.
4. Macroeconomic models do incorporate a role for government interventions.
5. Macroeconomists use both calibration and econometrics.

What is the Fashion in Macroeconomics

6. There is no freshwater/saltwater divide – now.

7. These researchers have been much more interested in the consequences of shocks than in their sources.

8. The modeling of financial markets and banks in macroeconomic models is stark.

9. Macroeconomics is mostly math and little talk.

10. The macro-principles textbooks don't represent our field well.

Criticism on Modern Macroeconomic Research

1. Main Criticism: Facts with unknown truth value (FWUTV).
" *Their models attribute fluctuations in aggregate variables to imaginary causal forces that are not influenced by the action that any person takes*."
—Paul Romer: *The Trouble With Macroeconomics*, 2016.

2. Does DSGE have a Future? YES, but conditionally.
—Blanchard, *Does DSGE have a Future*, PIIE Policy Brief, 2016.

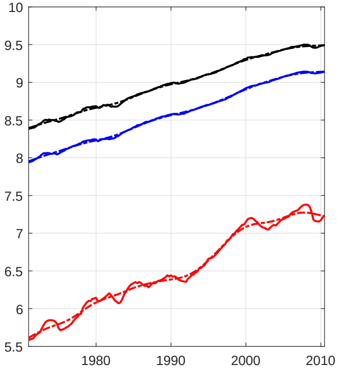
Why Do Macroeconomists Fail to Explain the Reality?

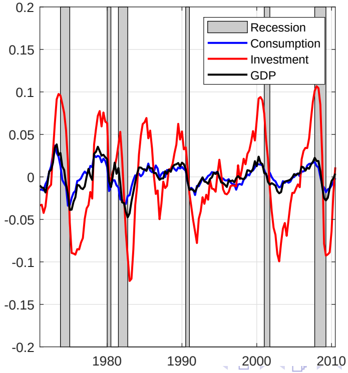
- Because the reality is too complicated...
- It is a big challenge to incorporate followings into a macro-model
  1. the micro foundation: heterogeneous individual behaviors;
  2. dynamics and uncertainties;
  3. interactions among individuals, markets/sectors, and policymakers.

Dynamic Stochastic General Equilibrium

Business Cycle Theory

- aims to explain the fluctuations in aggregate economy
  - what are the sources (shocks) of business cycles
  - what is the propagation mechanism of these shocks
- business cycles in US





Business Cycle Theory

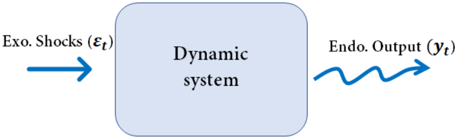
- two major schools
  - Classical
    - supply shock matters $\rightsquigarrow$shocks to tech., cost of prod., ...
    - propagation: real rigidities
  - Keynesian
    - demand shock matters $\rightsquigarrow$shocks to consumption, investment, money, ...
    - propagation: nominal rigidities

Introduction: DSGE

- mainstream macro structural model since 1980s (Kydland and Prescott, 1982)
- key features:
  - dynamics: inter-temporal optimization $\rightarrow$ micro-foundation
  - stochastic: uncertainty $\rightarrow$ expectation matters
  - GE: aggregate price feedback $\rightarrow$ concept of macro equilibrium (endo. price)
- Rational Expectation Equilibrium (REE): RE+E

Dynamic system in DSGE

- DSGE model $\rightarrow$ dynamic system of aggregate economy
- mapping external shocks to endogenous economic variables



- optimal paths (e.g., saddle)

$$\mathbf{y}_t = \mathcal{G}(\mathbf{y}_{t-1}, \varepsilon_t; \Theta)$$

  - $\mathbf{y}_{t-1}$ contains state variables
  - Θ structural parameters

Dynamic system in DSGE: an example

- real business cycle (RBC) model:

$$
\begin{aligned}
(1-\alpha)\frac{y_t}{n_t}u'(c_t) &= v'(n_t) \rightsquigarrow \text{labor} \\
u'(c_t) &= \beta \mathbf{E}_t\left[u'(c_{t+1})\left(\alpha y_{t+1}/k_{t+1} + (1-\delta)\right)\right] \rightsquigarrow \text{capital} \\
c_t &= y_t - \left[k_{t+1} - (1-\delta)k_t\right] \rightsquigarrow \text{consumption \& resource constraint} \\
y_t &= a_t k_t^{\alpha} n_t^{1-\alpha} \rightsquigarrow \text{production} \\
\log a_t &= \rho \log a_{t-1} + \varepsilon_t,\ \ \varepsilon_t \sim \mathbf{N}(0,\sigma^2) \rightsquigarrow \text{exogenous shocks}
\end{aligned}
$$

  - $\mathbf{y_t} = [c_t, n_t, y_t, k_t, a_t]$, k_t: state variable
  - $\Theta = \{\alpha, \beta, \delta, \rho, \sigma, u(.), v(.), ...\}$

- linearization$\rightarrow$ forward iteration $\rightarrow$ REE saddle path $\mathbf{y}_t = \mathcal{G}(\mathbf{y}_{t-1}, \varepsilon_t; \Theta)$

Real Business Cycle (RBC) Theory

Basic environment

- discrete-time Ramsey model + endo. labor decision + stochastic shocks
- key assumptions:
  - flexible prices
  - rational expectations
  - no other frictions:
    - perfect competition, perfect risk sharing, no asymmetric information, no externalities
- competitive equilibrium $\leftrightarrow$ social planner's problem (1st fundamental welfare theorm)

Setup

- Social planner chooses $\{C_t, K_{t+1}, n_t\}$ to solve

$$\max_{\{C_t, K_{t+1}, n_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \psi \log\left(1 - n_t\right)\right]$$

subject to

$$C_t + K_{t+1} - \left(1 - \delta\right) K_t = A_t K_t^{\alpha} n_t^{1-\alpha} \rightsquigarrow \text{resource constraint}$$

- technology process A_t:

$$\begin{aligned} A_t &= a_t X_t^{1-\alpha} \\ \log(a_t / a) &= \rho \log(a_{t-1} / a) + \varepsilon_t, \quad \varepsilon_t \sim N\left(0, \sigma^2\right) \rightsquigarrow \text{stochastic shock} \quad (1) \\ X_t &= \gamma X_{t-1}, \quad \gamma > 1 \rightsquigarrow \text{deterministic trend} \end{aligned}$$

A stationary (detrended) model

- Define the detrended variables as $c_t \equiv C_t / X_t$, $k_{t+1} \equiv K_{t+1} / X_{t+1}$
- Social planner's problem

$$\max_{\{c_t, k_{t+1}, n_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \psi \log (1 - n_t)\right]$$

subject to

$$c_t + k_{t+1}\gamma - (1 - \delta) k_t = a_t k_t^{\alpha} n_t^{1-\alpha} \quad \rightarrow \lambda_t \tag{2}$$

  - a_t follows AR(1) process of Eq. (1)
  - investment: $i_t = k_{t+1}\gamma - (1 - \delta) k_t$
  - λ_t: Lagrangian multiplier

Optimal decisions

- First order conditions for $\{c_t, n_t, k_{t+1}\}$

$$\frac{1}{c_t} = \lambda_t \rightsquigarrow \text{consumption} \tag{3}$$

$$\frac{\psi}{1-n_t} = \lambda_t \left[(1-\alpha) \, a_t k_t^{\alpha} n_t^{-\alpha} \right] \rightsquigarrow \text{labor} \tag{4}$$

$$\lambda_t = \frac{\beta}{\gamma} \mathbb{E}_t \left[\lambda_{t+1} \left(\alpha a_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha} + 1 - \delta \right) \right] \rightsquigarrow \text{capital} \tag{5}$$

- Full dynamic system: (2), (3), (4), (5), and (1) for $\{c_t, k_{t+1}, n_t, \lambda_t, a_t\}$
  - highly nonlinear! $\longrightarrow$ linearize the system around the steady state
  - solving the steady state & log-linearization: perturbation approach

Solving steady state

- (5) implies $1 = \frac{\beta}{\gamma}\left(\alpha \frac{y}{k} + 1 - \delta\right) \rightarrow$ the capital-output ratio $\frac{k}{y}$

$$\frac{k}{y} = \frac{\alpha\beta}{\gamma - (1-\delta)\,\beta}$$

- investment-output ratio:

$$\frac{i}{y} = (\gamma - 1 + \delta)\,\frac{k}{y}$$

- resource constraint (2) implies

$$\frac{c}{y} = 1 - (\gamma - 1 + \delta)\,\frac{k}{y}$$

Solving steady state

- (4) and (3) imply

$$\frac{\psi}{1-n} = \frac{1}{c}(1-\alpha)\frac{y}{n}$$

- steady-state labor:

$$n^{ss} = \frac{1}{\frac{c}{y}\frac{\psi}{1-\alpha} + 1} < 1$$

- production function $y = ak^{\alpha}n^{1-\alpha}$ implies

$$\frac{y}{k} = a\left(\frac{n}{k}\right)^{1-\alpha}$$

Solving steady state

- solve the level of each variable:

$$
\begin{aligned}
k^{ss} &= \left(\frac{y/k}{a}\right)^{\frac{1}{\alpha-1}} n^{ss} \\
y^{ss} &= a\left(k^{ss}\right)^{\alpha}\left(n^{ss}\right)^{1-\alpha} \\
c^{ss} &= y^{ss}\frac{c}{y} \\
i^{ss} &= y^{ss}\frac{i}{y}
\end{aligned}
$$

Log-linearization

- consider a general equation

$$h_t = f\left(x_t, y_t\right) \Rightarrow e^{\tilde{h}_t} = f\left(e^{\tilde{x}_t}, e^{\tilde{y}_t}\right)$$

  - define $\tilde{x}_t \equiv \log x_t, \tilde{y}_t \equiv \log y_t, \tilde{h}_t \equiv \log h_t$

- do Taylor expansion w.r.t. $\{\tilde{x}_t, \tilde{y}_t\}$ around S.S. $\{\tilde{x}^{ss}, \tilde{y}^{ss}\}$

$$e^{h^{ss}}\left(\tilde{h}_t - \tilde{h}^{ss}\right) \approx f_x\left(e^{\tilde{x}^{ss}}, e^{\tilde{y}^{ss}}\right) e^{\tilde{x}^{ss}}\left(\tilde{x}_t - \tilde{x}^{ss}\right) + f_y\left(e^{\tilde{x}^{ss}}, e^{\tilde{y}^{ss}}\right) e^{\tilde{y}^{ss}}\left(\tilde{y}_t - \tilde{y}^{ss}\right)$$

- define $\hat{x}_t = \left(\tilde{x}_t - \tilde{x}^{ss}\right), \hat{y}_t = \left(\tilde{y}_t - \tilde{y}^{ss}\right), \hat{h}_t = \left(\tilde{h}_t - \tilde{h}^{ss}\right)$ $\rightsquigarrow$ (% deviation from SS)

$$\Rightarrow \hat{h}_t = \frac{f_x\left(x^{ss}, y^{ss}\right) x^{ss}}{f\left(x^{ss}, y^{ss}\right)}\hat{x}_t + \frac{f_y\left(x^{ss}, y^{ss}\right) y^{ss}}{f\left(x^{ss}, y^{ss}\right)}\hat{y}_t$$

Log-linearization: examples

- $f(x,y) = ax + by \longrightarrow \hat{h}_t = a\frac{x^{ss}}{f^{ss}}\hat{x}_t + b\frac{y^{ss}}{f^{ss}}\hat{y}_t.$
- $f(x,y) = axy \longrightarrow \hat{h}_t = \hat{x}_t + \hat{y}_t.$
- $f(x,y) = g(x_t) - g(x_{t-1}) \longrightarrow \hat{h}_t = g_x(\hat{x}_t - \hat{x}_{t-1})$
  - where $\hat{h}_t = h_t - h^{ss}, \hat{x}_t = x_t - x^{ss}$
- $f(x,y) = \left[\phi_2\left(\frac{y_t}{x_{t-1}} - \frac{y^{ss}}{x^{ss}}\right)^{\phi_1} + 1\right] y_t \longrightarrow \hat{h}_t = \hat{y}_t$

Log-linearized system for RBC

- full system:

$$\frac{c}{y}\hat{c}_t + \frac{\gamma k}{y}\hat{k}_{t+1} - (1-\delta)\frac{k}{y}\hat{k}_t = \hat{a}_t + \alpha\hat{k}_t + (1-\alpha)\hat{n}_t \tag{6}$$

$$-\hat{c}_t = \hat{\lambda}_t \tag{7}$$

$$\frac{n^{ss}}{1-n^{ss}}\hat{n}_t = \hat{\lambda}_t + \hat{a}_t + \alpha\left(\hat{k}_t - \hat{n}_t\right) \tag{8}$$

$$\hat{\lambda}_t = \mathbb{E}_t\hat{\lambda}_{t+1} + \frac{y/k}{y/k+1-\delta}\mathbb{E}_t\left[\hat{a}_{t+1} + (\alpha-1)\left(\hat{k}_{t+1} - \hat{n}_{t+1}\right)\right] \tag{9}$$

$$\hat{a}_t = \rho\hat{a}_{t-1} + \varepsilon_t \tag{10}$$

- state: k_t, controls: $\{c_t, n_t\}$, co-state: λ_t

Dynamic system in matrices

- state & costate $\{k_t, \lambda_t\}$:

$$\begin{bmatrix} \frac{\gamma k}{y} & 0 \\ \frac{(1-\alpha)y/k}{y/k+1-\delta} & -1 \end{bmatrix} \mathbb{E}_t \begin{bmatrix} \hat{k}_{t+1} \\ \hat{\lambda}_{t+1} \end{bmatrix} = \begin{bmatrix} (1-\delta)\frac{k}{y} + \alpha & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{bmatrix}$$

$$+ \begin{bmatrix} -\frac{c}{y} & 1-\alpha \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{n}_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{(1-\alpha)y/k}{y/k+1-\delta} \end{bmatrix} \mathbb{E}_t \begin{bmatrix} \hat{c}_{t+1} \\ \hat{n}_{t+1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \hat{a}_t + \begin{bmatrix} 0 \\ \frac{y/k}{y/k+1-\delta} \end{bmatrix} \mathbb{E}_t \hat{a}_{t+1}$$

- controls $\{c_t, n_t\}$:

$$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n^{ss}}{1-n^{ss}} + \alpha \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{n}_t \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ \alpha & 1 \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \hat{a}_t$$

Dynamic system in matrices

- rewrite more compactly as

$$\begin{bmatrix} \hat{c}_t \\ \hat{n}_t \end{bmatrix} = A_1 \begin{bmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{bmatrix} + A_2 \hat{a}_t \tag{11}$$

$$\mathbb{E}_t \begin{bmatrix} \hat{k}_{t+1} \\ \hat{\lambda}_{t+1} \end{bmatrix} = B_1 \begin{bmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{bmatrix} + B_2 \hat{a}_t + B_3 \mathbb{E}_t \hat{a}_{t+1} \tag{12}$$

- solving the above system by forward iteration
- decompose $B_1 = P \Lambda P^{-1}$

$$P^{-1} \mathbb{E}_t \begin{bmatrix} \hat{k}_{t+1} \\ \hat{\lambda}_{t+1} \end{bmatrix} = \Lambda P^{-1} \begin{bmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{bmatrix} + P^{-1} B_2 \hat{a}_t + P^{-1} B_3 \mathbb{E}_t \hat{a}_{t+1} \tag{13}$$

Solving optimal/saddle path by forward iteration

- redefine $\begin{bmatrix} \hat{x}_{1t} \\ \hat{x}_{2t} \end{bmatrix} = P^{-1} \begin{bmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{bmatrix} = \begin{bmatrix} p_{11}\hat{k}_t + p_{12}\hat{\lambda}_t \\ p_{21}\hat{k}_t + p_{22}\hat{\lambda}_t \end{bmatrix}$
- (13) $\Longrightarrow$

$$\mathbb{E}_t \hat{x}_{t+1} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \hat{x}_t + \tilde{B}_2 \hat{a}_t + \tilde{B}_3 \mathbb{E}_t \hat{a}_{t+1}, \tag{14}$$

  - where $\tilde{B}_2 \equiv P^{-1} B_2$, $\tilde{B}_3 \equiv P^{-1} B_3$.

Existence of saddle path (BK condition)

- \# of eigenvalues in Λ greater than 1 = \# of co-state variables
- in the above RBC model, Λ must have one eigenvalue greater than 1
- indeterminancy (multiple equilibria):
  - \# of eigenvalues in Λ greater than 1 $<$ \# of co-state variables
- intuition: consider a simple AR(1) process

$$\mathbb{E}_t x_{t+1} = \phi x_t + \varepsilon_t$$

  - $\phi > 1 \Rightarrow$ unique saddle by forward iteration
  - $\phi < 1 \Rightarrow$ multiple equilibria:

$$\mathbb{E}_t(x_{t+1} + u_{t+1}) = \phi(x_t + u_t) + \varepsilon_t, \text{ for any } u_t = \phi u_{t-1}$$

Solving optimal/saddle path by forward iteration

- WLOG, suppose $\lambda_2 > 1$, then (14) $\Rightarrow$

$$\hat{x}_{2t} = \frac{1}{\lambda_2}\mathbb{E}_t\hat{x}_{2t+1} - \frac{1}{\lambda_2}\left[\tilde{B}_2(2,1)\,\hat{a}_t + \tilde{B}_3(2,1)\,\mathbb{E}_t\hat{a}_{t+1}\right]$$

- do forward iteration $\Rightarrow$

$$\begin{aligned}
\hat{x}_{2t} &= \sum_{j=0}^{\infty} -\left[\frac{1}{\lambda_2}\right]^j \frac{1}{\lambda_2}\mathbb{E}_t\left[\tilde{B}_2(2,1)\,\hat{a}_{t+j} + \tilde{B}_3(2,1)\,\hat{a}_{t+1+j}\right] \\
&= -\frac{1}{\lambda_2}\tilde{B}_2(2,1)\,\hat{a}_t - \sum_{j=1}^{\infty}\left(\frac{1}{\lambda_2}\right)^j\left(\frac{1}{\lambda_2}\tilde{B}_2(2,1) + \tilde{B}_3(2,1)\right)\mathbb{E}_t\hat{a}_{t+j} \\
&= \sum_{j=0}^{\infty}\phi_j\mathbb{E}_t\hat{a}_{t+j},
\end{aligned}$$

  - where $\phi_0 = -\frac{1}{\lambda_2}\tilde{B}_2(2,1)$, $\phi_j = -\left(\frac{1}{\lambda_2}\right)^j\left[\frac{1}{\lambda_2}\tilde{B}_2(2,1) + \tilde{B}_3(2,1)\right]$

Solving optimal/saddle path by forward iteration

- definition of $\hat{x}_{2t}$ implies

$$\hat{x}_{2t} \equiv p_{21}\hat{k}_t + p_{22}\hat{\lambda}_t = \sum_{j=0}^{\infty} \phi_j \mathbb{E}_t \hat{a}_{t+j} \Rightarrow \hat{\lambda}_t = \frac{1}{p_{22}} \sum_{j=0}^{\infty} \phi_j E_t \hat{a}_{t+j} - \frac{p_{21}}{p_{22}} \hat{k}_t$$

- combining with $\hat{k}_{t+1} \equiv p_{11}\hat{k}_t + p_{12}\hat{\lambda}_t \Rightarrow$ the policy function of k

$$\hat{k}_{t+1} = \left[B_1(1,1) - B_1(1,2) \frac{p_{21}}{p_{22}} \right] \hat{k}_t + \sum_{j=0}^{\infty} \tilde{\phi}_j \mathbb{E}_t (\hat{a}_{t+j})$$

  - where $\tilde{\phi}_0 = B_2(1,1) + B_1(1,2) \frac{1}{p_{22}} \phi_0$, $\tilde{\phi}_1 = B_3(1,1) B_1(1,2) \frac{1}{p_{22}} \phi_1$, $\tilde{\phi}_j = B_1(1,2) \frac{1}{p_{22}} \phi_j$ for all $j > 1$.

State-space representation of equilibrium path

- policy function of k + Eq. (11) $\Rightarrow$ optimal paths for controls
- express the equilibrium path as

$$\hat{k}_{t+1} = \mathbf{M}_{sk}\hat{k}_t + \sum_{j=0}^{\infty} \tilde{\phi}_j \mathbb{E}_t \hat{a}_{t+j} \quad (15)$$

$$\begin{bmatrix} \hat{c}_t \\ \hat{n}_t \end{bmatrix} = \mathbf{M}_{ck}\hat{k}_t + \mathbf{M}_{ca} \sum_{j=0}^{\infty} \theta_j \mathbb{E}_t \hat{a}_{t+j} \quad (16)$$

- the solution procedure is rational expectation equilibrium (REE)
  - forward iteration is an idea of fixed point solution

State-space representation of equilibrium path

- a general form of the optimal path

$$\hat{\mathbf{S}}_t \equiv \begin{bmatrix} \hat{k}_{t+1} \\ \hat{a}_t \end{bmatrix} = \mathbf{M}_1 \hat{\mathbf{S}}_{t-1} + \mathbf{M}_2 \varepsilon_t \quad (17)$$

$$\hat{\mathbf{C}}_t \equiv \begin{bmatrix} \hat{c}_t \\ \hat{n}_t \\ \hat{y}_t \\ \cdots \end{bmatrix} = \Pi_1 \hat{\mathbf{S}}_t \quad (18)$$

- the above system is essentially a constrained VAR system with structural shocks and deep parameters

Quantitative Exercises

Calibration

- optimal path (DGP) depends on deep parameters
- set parameter values to target US economy

Table: Calibration for U.S. economy

parameter		value/target
α	capital share	0.4
γ	average growth rate	1
γ/β	real interest rate	0.99
δ	depreciation rate	0.025
ψ	disutility on leisure	$n^{ss} = 0.33$
ρ	AR(1) coefficient	0.979
σ	std of tech. shock ε_t	0.0072

- Chinese economy would be different

Optimal path under calibration

- optimal path is solved in `Dynare`

$$\begin{bmatrix} \hat{c}_t \\ \hat{y}_t \\ \hat{k}_{t+1} \\ \hat{n}_t \\ \hat{i}_t \\ \hat{w}_t \\ \hat{r}_t \\ \hat{a}_t \end{bmatrix} = \begin{bmatrix} 0.6096 & 0.4223 \\ 0.2591 & 1.3382 \\ 0.9595 & 0.0909 \\ -0.2349 & 0.6137 \\ -0.6209 & 3.6372 \\ 0.4940 & 0.7245 \\ -0.7409 & 1.3382 \\ 0 & 0.9700 \end{bmatrix} \times \begin{bmatrix} \hat{k}_t \\ \hat{a}_{t-1} \end{bmatrix} + \begin{bmatrix} 0.4353 \\ 1.3796 \\ 0.0937 \\ 0.6326 \\ 3.7497 \\ 0.7469 \\ 1.3796 \\ 1.0000 \end{bmatrix} \times \varepsilon_t$$

Simulation: impulse response function (IRF)

- the dynamic effects of one-unit of shock on the economy
  - definition of impulse response of ε_t on variable x_t

$$IR(j) = \frac{\partial x_{t+j}}{\partial \varepsilon_t}, \text{for } j \geq 0$$

  - IRF of $\hat{\mathbf{S}}_t$, see (17), is

$$IR_s(j) = \mathbf{M}_2 \mathbf{M}_1^j$$

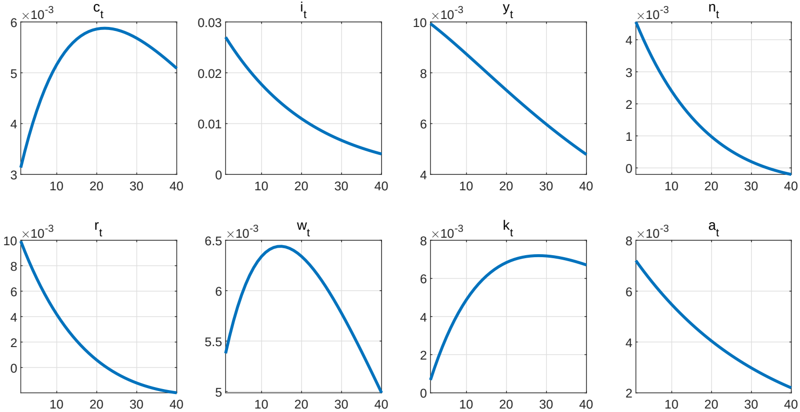
  - IRF of $\hat{\mathbf{C}}_t$, see (18), is

$$IR_c(j) = \Pi \mathbf{M}_2 \mathbf{M}_1^j$$

- IRF characterizes transmission mechanism of external shocks to real economy

IRF under a positive technology shock

- one std increase in a_t leads to



Propagation mechanism in RBC?

- the RBC model is lack of propagation channel to amplify fluctuation
- Cogley and Nason (1995, AER)
- why?

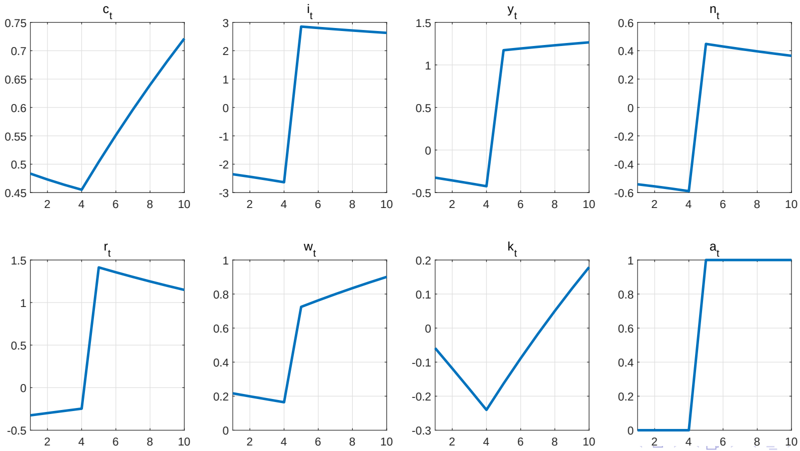
$$\hat{y}_t = 0.2591 \times \hat{k}_t + 1.3796 \times \hat{a}_t = 0.38173 \times \frac{1 - 0.8985\mathbf{L}}{1 - 0.9595\mathbf{L}} \hat{a}_t$$

Business cycle moments: model vs data

	y	c	i	n
	Standard deviations relative to Y			
U.S. Data	1.0000	0.5452	2.4672	1.0539
RBC	1.0000	0.3490	2.7151	0.4608
	First-order autocorrelations			
U.S. Data	0.9003	0.9023	0.8671	0.9255
RBC	0.7366	0.7995	0.7282	0.7262
	Correlation with Y			
U.S. Data	1.0000	0.9296	0.9705	0.8208
RBC	1.0000	0.9295	0.9923	0.9823

Expectation driven business cycles

- data shows news shock about future TFP generates comovement among $\{y, c, i\}$ (Beaudry & Portier, 2006, AER)
- the standard RBC model cannot explain EDBCs



Expectation driven business cycles

- why standard RBC cannot explain?
- intuition:
  - a permanent increase in future TFP $\Rightarrow$ consumption & leisure $\uparrow$(PIH) $\Rightarrow$ labor $\downarrow$
  - k_t is a pre-determined state variable and a_t unchanged at t
  - prod. fun. $y_t = a_t k_t^{\alpha} n_t^{1-\alpha} \Rightarrow y_t \downarrow \Rightarrow$ investment $\downarrow$

Expectation driven business cycles

- add more real frictions into RBC for EDBCs (Jaimovich & Rebelo, 2009, AER)
  - capacity utilization $\rightarrow y_t = a_t(u_t k_t)^\alpha n_t^{1-\alpha}$
  - special utility function to remove income effects on labor
    - Greenwood-Hercowitz-Huffman (GHH) preferences: $u\left(C_t, N_t\right) = \frac{\left(C_t - \psi N_t\right)^{1-\xi} - 1}{1-\xi}$
  - investment adjustment cost to mitigate temporary drop in investment
    - convex IAC: $K_{t+1} = \left(1 - \delta_t\right) K_t + \left[1 - \varphi\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$

Expectation driven business cycles

- EDBCs in Jaimovich & Rebelo (2009)