DSGE Lecture III: Financial Intermediation and Credit Policies

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DSGE is in general referred to as a methodology rather than a theory!
What is the DSGE?

- **Dynamic**: the economy evolves over time, and all decisions made are intertemporal.
- **Stochastic**: random shocks—exogenous shocks follow stochastic processes (i.e. AR(1)).
- **General**: all prices are endogenous; decisions made by each sector in the economy are optimal.
- **Equilibrium**: optimality + market clearing.
What can we do with DSGE Models?

- Qualitative analysis
  - Transmission of exogenous shocks (i.e. IRF V.S. Structural VAR)

- Quantitative analysis
  - Business cycle properties (i.e. volatility, comovement, cyclicity, lead-lag pattern)
  - Variance decomposition
  - Historical decomposition
Structure of Basic DSGE Models

- Sectors: households, firms (i.e. intermediate, final), banks, central banks, government, foreign sectors
- Preference
- Technology
- Constraints/Frictions
- Shocks
- Market clearing
Frictions in DSGE Models

- Heterogeneity (i.e. heterogeneous preferences, heterogeneous circumstances (idiosyncratic risks), heterogeneous productions)
- Incomplete market (i.e. monopolistic competitions)
- Price/wage rigidity
- Financial frictions
- Collateral frictions
- Searching frictions (i.e. random/directed search)
- Adjustment/transaction costs
- Asymmetric info.
- Externality
Examples of DSGE Models

- RBC Models (benchmark models)
- New Keynesian Models
How to deal with DSGE Models?

- Solve for the model: optimality conditions
- Loglinearization if necessary (for analytical solutions only)
- Calibration/Estimation
- Solve for steady state
- Generate IRF
- Simulations
- Sensitivity/Robustness Test
Objectives of This Lecture

- solve for a DSGE model with financial intermediation and credit policies
- Calibration/Estimation
- Steady State
- Introduce how to use Dynare
- Use Dynare to replicate basic results of Gertler and Kiyotaki (2009)
Previous literature did not anticipate all the key empirical phenomena that have played out during the Great Recession.

Much of the earlier macro literature with financial frictions emphasized credit market constraints on non-financial borrowers and treated intermediaries as a veil (i.e. BGG (1999))

Two broad aspects of the crisis have not been fully captured in previous works:

- The current crisis has featured a significant disruption of financial intermediation
- To combat the crisis, various unconventional policy measures have been employed by the monetary and fiscal authorities.
Objectives of Gertler and Kiyotaki (2009)

Two issues being addressed:

- How disruption in financial intermediation can induce a crisis that affects real activities
- How various market interventions by the central bank and the Treasury of the type might work to mitigate the crisis
Model Environment

- In the economy, there is a continuum of islands
- Sectors: Households, Final Goods Sector, Intermediate Goods Sector, Financial Sector
- Households (unit measure), firms (unit measure) and banks (unit measure) are located on a continuum of islands
Each period investment opportunities arrive randomly to a fraction $\pi^i$ of islands.

The arrival of investment opportunities is i.i.d. across time/islands.

Only firms on islands with investment opportunities can acquire new capital.

The structure of this idiosyncratic risk provides a simple way to introduce liquidity needs by firms.
In the economy, the law of motion for capital is given by

\[ K_{t+1} = \psi_{t+1}[l_t + \pi^i(1 - \delta)K_t] + \psi_{t+1}\pi^n(1 - \delta)K_t \]

\[ = \psi_{t+1}[l_t + (1 - \delta)K_t] \]

Capital quality shock serves as an exogenous trigger of asset price dynamics.

\[ \psi_{t+1} \] is best thought of as capturing some form of economic obsolescence, as opposed to physical depreciation.
The capital quality shock follows an AR(1) process:

\[ \ln \psi_{t+1} = \rho \ln \psi_t + e_{t+1} \] (1)

where \( \rho \) is a persistence parameter of the shock equal to 0.66, and \( e_{t+1} \) is the innovation with mean and variance (0, 0.25)
Households

- A continuum of members of measure unity
- Within the household, there are
  - $1 - f$ “workers” who supply labor to firms and return their wages to the household
  - $f$ “bankers” who manage a “bank” and transfer nonnegative dividend back to household subject to its flow of fund constraint
- Households do not hold capital directly
- Rather, households deposit funds in banks
Households’ Problem

A representative HH solves the following problem

\[
\max E_t \sum_{i=0}^{\infty} \beta^i \left[ \ln(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1 + \varepsilon} L^1_{t+i} \right]
\]

Subject to the flow of funds constraints

\[
C_t = W_t L_t + R_t D_{ht-1} - D_{ht} + \Pi_t - T_t
\]
Household’s Optimality Conditions

Household’s optimality conditions for labor supply and consumption/saving are given by

\[ E_t u_{Ct} W_t = \chi L_t^e \]  \hspace{1cm} (2)

\[ E_t \Lambda_{t,t+1} R_{t+1} = 1 \]  \hspace{1cm} (3)

with

\[ u_{Ct} = (C_t - \gamma C_{t-1})^{-1} - \beta \gamma (C_{t+1} - \gamma C_t)^{-1} \]

\[ \Lambda_{t,t+1} = \beta \frac{u_{Ct+1}}{u_{Ct}} \]
Final Goods Firms

- Firms are homogenous
- Firms operate under a CRS technology with capital and labor
- Firms operate under perfect competition
- A goods producer with an opportunity to invest obtains/borrows funds from a bank by issuing new state-contingent securities (equity) at the price $Q^i_t$
- Each unit of equity is a state-contingent claim to the future returns from one unit of investment
- The goods producer buys new capital from intermediate goods firms with the funds borrowed from a bank
Final Goods Firms’ Problem

A representative final goods producer solves the following problem

$$\min W_t L_t + Z_t K_t$$

subject to the production function given by,

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} \quad (4)$$

Note: Do not write it as the firm’s profit maximization problem here because this is actually technically incorrect (although people often write it this way).
Final Goods Firms’ Optimality Conditions

The final goods firms’ optimality conditions are given by

\[ W_t = (1 - \alpha) \frac{Y_t}{L_t} \]

(5)

\[ Z_t = \frac{Y_t - W_t L_t}{K_t} = \alpha \frac{Y_t}{K_t} \]

(6)
Intermediate Goods Firms

- Firms are homogenous
- Firms operate under perfect competitions
- They make new capital using input of final output and subject to convex adjustment costs
Intermediate Goods Firms’ Problem

A representative intermediate goods producer solves the following problem

$$\max_{E_t} \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \{ Q_i^i I_\tau - [1 + f(\frac{l_\tau}{l_{\tau-1}})] l_\tau \}$$

where $f(\frac{l_\tau}{l_{\tau-1}}) l_\tau$ reflects convex adjustment costs, with $f(1) = f'(1) = 0$ and $f''(\frac{l_\tau}{l_{\tau-1}}) > 0$.

⇒ The aggregate production function of capital goods producers is DRS in the short-run and is CRS in the long-run.
The optimality condition for intermediate goods producers are given by

\[ Q_t^i = 1 + f\left(\frac{l_t}{l_{t-1}}\right) + \frac{l_t}{l_{t-1}} f'(\frac{l_t}{l_{t-1}}) - E_t \Lambda_{t,t+1} \left(\frac{l_{t+1}}{l_t}\right)^2 f'(\frac{l_{t+1}}{l_t}) \]  \hspace{1cm} (7)

Profits (which arise only outside of steady state), are redistributed lump sum to households

Note: Due to the implicit adjustment cost function appearing in this optimality condition, we have to loglinearize it in order for the dynamic system of the model to be tractable!
Banks raise funds in a national financial market (closed financial market).

The national financial market includes a retail market and a wholesale market.

- Within the retail market, banks obtain deposits from households, and make loans to nonfinancial firms.
- Within the wholesale market, banks borrow and lend amongst one and another (interbank borrowing and lending).
Banks can only make loans to nonfinancial firms (i.e. final goods firms) located on the same islands.

Banks on islands with new investment opportunities will borrow funds from those on islands with no new investment opportunities.

Assume that the bank is efficient at evaluating and monitoring non-financial firms of the same island, and also at enforcing contractual obligations with these borrowers.

We also assume the costs to a bank of performing these activities are negligible.
Thus, for simplicity, there is no friction for a firm to borrow funds from banks in the same island (no borrowing constraints tied to firms).

In this regard, a bank can lend frictionlessly to non-financial firms of the same island against their future profits.

Firms are able to offer banks perfectly state-contingent equity.
For an individual bank, the flow-of-funds constraint is

$$Q_t^h s_t^h = n_t^h + b_t^h + d_t$$

(8)

where

$$n_t^h = [Z_t + (1 - \delta)Q_t^h]\psi_t s_{t-1} - R_{bt} b_{t-1} - R_t d_{t-1}$$

(9)

Note that asset price $Q_t^h$ is location specific.
To motivate an endogenous constraint on the bank’s ability to borrow/lend, we introduce the following simple agency problem:

- After the bank obtains funds, the banker may transfer a fraction $\theta$ of “divertable” assets to his/her family.
- Divertable assets consist of total gross assets $Q_h^h s_t^h$ net a fraction $\omega$ of interbank borrowing $b_t^h$.
- If a bank divert funds for personal use, it defaults on its debt and is forced to shut down.
- The creditors may re-claim the remaining fraction $1 - \theta$ of funds.
- Because its creditors recognize the bank’s incentive to divert funds, they will restrict the amount they lend.
- In this way, a borrowing constraint may arise.
The bank’s incentive constraint Con’t

Let $V_t(s^h_t, b^h_t, d_t)$ be the maximized value of $V_t$, given an asset and liability $(s^h_t, b^h_t, d_t)$ at the end of period $t$

In order to insure the bank does not divert funds, the following incentive constraint must hold for each bank type:

$$V_t(s^h_t, b^h_t, d_t) \geq \theta(Q_t s^h_t - \omega b^h_t)$$  \hspace{1cm} (10)
Turnover of bankers

- In order to limit banker’s ability to save to overcome financial constraints, we allow for turnover b/w bankers and workers.

- In particular, we assume a banker, with prob. \(1 - \sigma\), exits next period. 
  \[\Rightarrow \text{average survival time} = \frac{1}{1-\sigma}\]

- Upon exiting, a banker transfers retained earning to his/her family and becomes a worker.

- A worker becomes a new banker to take over the business with no costs, and receive a “start up” transfer from his/her family as a small constant fraction of the total assets of ongoing banks/bankers.
The Bank’s Problem

An individual bank maximizes the expected present value of future dividends

\[ V_t = E_t \sum_{i=1}^{\infty} (1 - \sigma) \sigma^{i-1} \Lambda_{t,t+i} n_{t+i}^h \]

subject to the flow-of-funds constraint and the incentive constraint.
The Banks’ Bellman Equation

The value of the bank at the end of period $t - 1$ satisfies the Bellman Equation

$$V_{t-1}(s_{t-1}, b_{t-1}, d_{t-1})$$

$$= E_{t-1} \Lambda_{t-1,t} \sum_{h=i,n} \pi^h \{(1 - \sigma)n_t^h + \sigma \max_d V_t(s_t^h, b_t^h, d_t)\}$$

s.t.

$$Q_t^h s_t^h = n_t^h + b_t^h + d_t$$

$$V_t(s_t^h, b_t^h, d_t) \geq \theta(Q_t^h s_t^h - \omega b_t^h)$$
The Value Function of the Bank

To solve the bellman equation above, we guess the value function is linear:

\[ V_t(s_t, b_t, d_t) = \nu_{st}s_t^h - \nu_{bt}b_t^h - \nu_t d_t \]

The linearity of the value function will be verified later
The Bank’s Optimality Conditions

The optimality conditions for an individual bank are

\[(\nu_{bt} - \nu_t)(1 + \bar{\lambda}_t) = \theta \omega \bar{\lambda}_t \]  
(11)

\[\left(\frac{\nu_{st}^h}{Q_t^h} - \nu_{bt}\right)(1 + \lambda_t^h) = \lambda_t^h \theta (1 - \omega) \]  
(12)

\[[\theta - \left(\frac{\nu_{st}^h}{Q_t^h} - \nu_t\right)] Q_t^h s_t^h - [\theta \omega - (\nu_{bt} - \nu_t)] b_t^h \leq \nu_t n_t^h \]  
(13)

where $\lambda_t^h$ is the Lagrangian multiplier for the incentive constraint faced by bank of type $h$; and $\bar{\lambda}_t$ is the average of multipliers across states, that is

\[\bar{\lambda}_t = \sum_{h=i,n} \pi^h \lambda_t^h \]
Case 1: Frictionless Wholesale Financial Market \((\omega = 1)\)

The perfect interbank market implies that the marginal value of assets in terms of goods must equal to the marginal cost of borrowing on the interbank market, that is

\[
\frac{\nu_{st}}{Q_t} = \nu_{bt}
\]  

and

\[
\mu_t = \frac{\nu_{st}}{Q_t} - \nu_t > 0
\]

where \(\mu_t\) denotes the excess value of a unit of assets relative to deposits.

Note: we can drop the \(h\) superscript in this case since asset prices are equal across island types.
Case 1: Frictionless Wholesale Financial Market ($\omega = 1$)

Cont’d

It then follows that the incentive constraint in this case can be written as

$$Q_t s_t - b_t = \phi_t n_t$$  \hspace{1cm} (16)

with

$$\phi_t = \frac{\nu_t}{\theta - \mu_t}$$  \hspace{1cm} (17)
Case 1: Frictionless Wholesale Financial Market \((\omega = 1)\)

Cont’d

After combining the conjectured value function and the Bellman equation, we can verify the value function is linear in \((s_t, b_t, d_t)\) if \(\mu_t\) and \(\nu_t\) satisfy

\[
\nu_t = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \tag{18}
\]

\[
\mu_t = E_t \Lambda_{t,t+1} \Omega_{t+1} (R_{kt+1} - R_{t+1}) \tag{19}
\]

with

\[
\Omega_{t+1} = 1 - \sigma + \sigma (\nu_{t+1} + \phi_{t+1} \mu_{t+1}) \tag{20}
\]

\[
R_{kt+1} = \psi_{t+1} \frac{Z_{t+1} + (1 - \delta) Q_{t+1}}{Q_t} \tag{21}
\]
Case 1: Frictionless Wholesale Financial Market ($\omega = 1$)

Cont’d

Since the leverage ratio net of interbank borrowing, $\phi_t$, is independent of both bank-specific factors and island-specific factors, summing acrosss individual banks to obtain the following expression:

$$Q_t S_t = \phi_t N_t$$  \hspace{1cm} (22)

⇒ In the case of perfect interbank borrowing, banks do not face idiosyncratic liquidity risks. Aggregate bank lending is simply constrained by aggregate bank capital
Case 1: Frictionless Wholesale Financial Market ($\omega = 1$)

Cont’d

- A perfect interbank market leads to arbitrage in returns to assets across market as follows:

$$E_{t+1} \Omega_{t+1} R_{kt+1} = E_{t+1} \Omega_{t+1} R_{bt+1} > E_{t+1} \Omega_{t+1} R_{t+1}$$ (23)

- A crisis in such economy is associated with an increase in the excess return on assets for banks of all types.
Case 2: Frictional Wholesale Market \((\omega = 0)\)

- In this case, banks can divert the funds obtained in either retail or wholesale financial markets.
- As a result, interbank loans and deposits become perfect substitutes as sources of finance.
- The imperfect interbank market implies that the marginal cost of interbank borrowing is equal to the marginal cost of deposits, that is

\[
\nu_{bt} = \nu_t
\]  

(24)
Case 2: Frictional Wholesale Market ($\omega = 0$)

- Even if banks on investing islands are financially constrained, banks on non-investing islands may or may not be.
- This raises the likelihood that banks on non-investing islands may earn zero excess returns on their assets.
- Because asset supply per unit of bank net worth is larger on investing islands than on non-investing islands, the asset price is lower,

$$Q^i_t < Q^n_t$$

- Hence, frictions in the interbank market limit the degree of arbitrage.
Case 2: Frictional Wholesale Market ($\omega = 0$)

- A lower asset price on the investing island implies a higher expected return, then we have

$$\mu^i_t > \mu^n_t \geq 0$$

- Thus, the leverage ratios for banks on each island type are given by

$$\frac{Q^i_t s^i_t}{n^i_t} = \phi^i_t = \frac{\nu_t}{\theta - \mu^i_t}$$ (25)

$$\frac{Q^n_t s^n_t}{n^n_t} \leq \phi^n_t = \frac{\nu_t}{\theta - \mu^n_t}, \text{ and } \left(\frac{Q^n_t s^n_t}{n^n_t} - \phi^n_t\right)\mu^n_t = 0$$ (26)
Case 2: Frictional Wholesale Market ($\omega = 0$)

Combining the conjectured value function and the Bellman equation, we have

$$
\nu_t = E_t \Lambda_{t,t+1} \Omega_{t+1}^{h'} R_{t+1} = E_t \Lambda_{t,t+1} \sum_{h'=i,n} \pi^{h'} \Omega_{t+1}^{h'} R_{t+1} \quad (27)
$$

$$
\mu_t^h = E_t \Lambda_{t,t+1} \Omega_{t+1}^{h'} (R_{kt+1}^{hh'} - R_{t+1}) \quad (28)
$$

with

$$
\Omega_{t+1}^{h'} = 1 - \sigma + \sigma (\nu_{t+1} + \phi_{t+1}^{h'} \mu_{t+1}^{h'}) \quad (29)
$$

$$
R_{kt+1}^{hh'} = \psi_{t+1} \frac{Z_{t+1} + (1 - \delta) Q_{t+1}^{h'}}{Q_{t}^{h}} \quad (30)
$$
Case 2: Frictional Wholesale Market ($\omega = 0$)

- Because leverage ratios differ across islands, we aggregate separately across bank-type to obtain the aggregate relations:

\[ Q_t^i S_t^i = \phi_t^i N_t^i \]  

\[ Q_t^n S_t^n \leq \phi_t^n N_t^n, \text{ and } (Q_t^n S_t^n - \phi_t^n N_t^n)\mu_t^n = 0 \]
Case 2: Frictional Wholesale Market ($\omega = 0$)

- An imperfect interbank market leads to arbitrage in returns to assets across market as follows:

$$
E_t \Lambda_{t,t+1}^h \Omega_{t+1}^{h'} R_{kt+1}^{i_{h'}} > E_t \Lambda_{t,t+1}^h \Omega_{t+1}^{h'} R_{kt+1}^{n_{h'}}
$$

$$
\geq E_t \Lambda_{t,t+1}^h \Omega_{t+1}^{h'} R_{bt+1} = E_t \Lambda_{t,t+1}^h \Omega_{t+1}^{h'} R_{t+1}
$$

with $\geq$ holds with strict inequality iff $\mu_t^n > 0$ and holds with equality iff $\mu_t^n = 0$
Case 2: Frictional Wholesale Market ($\omega = 0$)

- With an imperfect interbank market, a crisis is associated with
  - a rise in the excess return for banks on investing islands
  - and an increase in the dispersion of returns between island types
- For the case where $0 < \omega < 1$ (general case), the interbank rate will lie between the return on loans and the deposit rates
Recall that a new bank will enter and take over the business of the exiting bank’s business if a bank exit.

The total net worth for type $h$ banks is

$$N_t^h = N_{ot}^h + N_{yt}^h \quad (34)$$

where

$$N_{ot}^h = \sigma \pi^h \{[Z_t + (1 - \delta) Q_t^h] \psi_t S_{t-1} - R_tD_{t-1}\} \quad (35)$$

$$N_{yt}^h = \pi^h \xi[Z_t + (1 - \delta) Q_t^h] \psi_t S_{t-1} \quad (36)$$
Market Clearing

The market clearing conditions are given as follows

\[ Y_t = C_t + \left[1 + f\left(\frac{l_t}{l_{t-1}}\right)\right]l_t \]  
\[ (37) \]

\[ D_t = \sum_{h=i,n} (Q_t^h S_t^h - N_t^h) \]  
\[ (38) \]

\[ (1 - \alpha) \frac{Y_t}{L_t} E_t u_C t = \chi L_t^\varepsilon \]  
\[ (39) \]

\[ S_t = S_t^i + S_t^n \]  
\[ (40) \]

\[ S_t^i = l_t + (1 - \delta) \pi^i K_t \]  
\[ (41) \]

\[ S_t^n = (1 - \delta) \pi^n K_t \]  
\[ (42) \]
Calibration

- We set the values to parameters via calibration and estimation.
- Calibration: Pin down the values of parameters via targets obtained from the data (i.e. $K/Y$, $I/Y$, $C/Y$); the number of parameters being calibrated should equal to the number of targets.
- Bayesian Estimation: Use priors (obtained from the literature) and observables (obtained from the data) to estimate the values of parameters.
### Table 1: Parameter Values for Baseline Model

#### Households

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.990</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.500</td>
<td>Habit parameter</td>
</tr>
<tr>
<td>$\chi$</td>
<td>5.584</td>
<td>Relative utility weight of labor</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.333</td>
<td>Inverse Frisch elasticity of labor supply</td>
</tr>
</tbody>
</table>

#### Financial Intermediaries

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^i$</td>
<td>0.250</td>
<td>Probability of new investment opportunities</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.383</td>
<td>Fraction of assets divertable: Perfect interbank market</td>
</tr>
<tr>
<td></td>
<td>0.129</td>
<td>Fraction of assets divertable: Imperfect interbank market</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.003</td>
<td>Transfer to entering bankers: Perfect interbank market</td>
</tr>
<tr>
<td></td>
<td>0.002</td>
<td>Transfer to entering bankers: Imperfect interbank market</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.972</td>
<td>Survival rate of the bankers</td>
</tr>
</tbody>
</table>

#### Intermediate good firms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.330</td>
<td>Effective capital share</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Steady state depreciation rate</td>
</tr>
</tbody>
</table>

#### Capital Producing Firms

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$If''/f'$</td>
<td>1.500</td>
<td>Inverse elasticity of net investment to the price of capital</td>
</tr>
</tbody>
</table>

#### Government

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G/Y$</td>
<td>0.200</td>
<td>Steady state proportion of government expenditures</td>
</tr>
</tbody>
</table>
Steady State

- We need to solve for the steady states before solving for the dynamic system of the model.
- Conventional way to solve for steady states: ratio $\rightarrow$ level
- We use the steady state values in the Dynare as initial values.
- Dynare calculates steady state values of the endogenous variables of the model for you, based on your guess of the steady state.
- If you give it bad guess, Dynare may not converge and you will get an error.
- In practice, I strongly suggest you to solve the steady states by yourself rather than obtaining them via Dynare (i.e. multiple SS).
In our baseline model, we need to solve for eleven prices

\((R_b, Q^n, Z, \lambda^i, \lambda^n, \nu_b, \nu, \nu_s, \frac{N^i}{K}, \frac{N^n}{K}, D)\)

and quantity variables \((K, I, C, L)\)

Details for determining SS are given in Appendix 7.1
Crisis Experiment: Transmission of Financial Shocks

- An exogenous capital quality shock triggers a decline in the value of banks’ assets.
- The initial exogenous decline is then magnified in two ways:
  - Because banks are leveraged, the effect of decline in asset values on bank net worth is enhanced by a factor equal to the leverage ratio.
  - The drop in net worth tightens the banks’ borrowing constraint, inducing effectively a fire sale of assets that further depresses asset value.
- The crisis then feeds into real activity as the decline in asset values leads to a fall in investment.
Figure 1. Crisis Experiment: Perfect Interbank Market

- \( \psi \)
- \( r \)
- \( E(r_k) - r \)
- \( y \)
- \( c \)
- \( E(r_k) - r \)
- \( k \)
- \( labor \)
- \( q \)
- \( net\ worth \)

Graphs depict changes in various economic variables under the Perfect Interbank Market and RBC models.
Figure 2. Crisis Experiment: Imperfect Interbank Market

Imperfect Interbank Market ($\pi_i = 0.25$)

RBC

Perfect Interbank Market
A negative financial shock produces only a modest downturn in the frictionless model. This is due to the fact that high returns to capital induce an increase in investment and employment.
Crisis Experiment: The Frictional Model

- With financial frictions, a negative financial shock has a magnified effect on the economy
  - The magnified effect is due to bank leverage and to the decline in the market price of capital
  - The contraction in asset prices induces a decline in investment
  - The enhanced decline in investment is responsible for the magnified drop in output
- Financial factors are at work during the crisis is reflected in the behavior of the spread b/w the expected return to capital and the riskless interest rate
- Financial factors also contribute to the slow recovery back to trend
Credit Policies

- **Direct lending**: the central banks sets up for direct acquisition of high quality private securities (expansion of equity demand)

- **Discount Window Lending**: the central bank uses the discount window to lend funds to banks that in turn lend them out to nonfinancial borrowers (expansion of banks’ funds)

- **Equity Injections**: the fiscal authority coordinates with the monetary authority to acquire ownership positions in banks (expansion of banks’ net worth)
Credit Policies

- These credit policies will ultimately depress the spread, leading to a decline in the costs of capital financed by nonfinancial firms, and hence, mitigate the effects of an exogenous financial shock on real activities.
Direct Lending

- Direct lending is meant to broadly characterize the facilities the Fed set up for direct acquisition of high quality private securities.
- The central bank can buy non-financial firms’ securities by issuing government debt (borrowed from the Treasury).

Advantages of the central bank (Fed):
- The central bank is not balance constrained and households do not have to worry about defaulting.
- The central bank can lend in many markets (any islands).

Disadvantages of the central banks:
- The central bank is less efficient at intermediating funds due to the shortage of specific knowledge of a particular market.
Direct Lending

- Total intermediation of type $h$ assets is

$$Q_t^h S_t^h = Q_t^h (S_{pt}^h + S_{gt}^h) \quad (43)$$

- Let's assume that the central bank chooses to intermediate the fraction $\varphi_t^h$ of total credit in market $h$:

$$S_{gt}^h = \varphi_t^h S_t^h \quad (44)$$

where $\varphi_t^h$ denotes an instrument of central bank credit policy.
In the case with imperfect interbank borrowing ($\omega = 0$), the expressions above implies that

$$Q^i_t S^i_t = \frac{1}{1 - \varphi^i_t N^i_t}$$  \hspace{1cm} (45)$$

and

$$Q^n_t S^n_t = 1 - \varphi^n_t N^n_t$$ \hspace{1cm} \text{iff} \ \mu^n_t > 0 \hspace{1cm} (47)$$

$$Q^n_t S^n_t = Q^n_t S^n_{pt} + \varphi^n_t Q^n_t S^n_t$$ \hspace{1cm} \text{iff} \ \mu^n_t = 0 \hspace{1cm} (46)$$
The central bank (Fed) adjusts the fraction of private credit it intermediates to the difference b/w spread on investing islands and its steady state value as:

\[ \varphi_t = \nu_g \left[ (E_t R_{kt+1}^{ih'} - R_{t+1}) - (ER_k^{ih'} - R) \right] \] (48)

where \( \nu_g \) is the policy parameter equal to 100

Note that this rule applies only during a crisis (i.e. “unusual and exigent” circumstances)
Figure 3. Lending Facilities: Perfect Interbank Market

\[ E(r_k) - r \]

\[ \nu_g = 0 \]
\[ \nu_g = 100 \]
Figure 4. Lending Facilities: Imperfect Interbank Market

- $\psi$
- $r$
- spread
- $\psi$
- $c$
- investment
- $k$
- labor
- $q$
- net worth
- fraction of government assets

$v_g = 0$
RBC
$v_g = 100$
Effects of the Credit Policy Response

- The increase in central bank credit significantly reduces the rise in the spread.
- It in turn reduces the overall drop in investment.
- The policy intervention dampen the output decline by mitigating the increase in the spread.
- By directly facilitating credit flows in investing regions, a given level of central bank intermediation can be more effective in relaxing financial constraints in the model with imperfect interbank market, relative to that with perfect interbank market.
Steps of writing the Dynare Code:

- Declare the variables of the model: use command “var”
- Specify the exogenous variables (white noises of the shocks): use command “varexo”
- Declare the parameters of the model: use command “parameters”
- Declare the model: write optimality conditions + market clearing + shock processes in the “model” block; you begin this part of the code with the command “model;” and capped off with “end;”
- Specify the initial values (steady states) of all the endogenous variables in the “initial” block; you begin this part of the code with the command “initial;” and capped off with “end;”
An Introduction to the Dynare

Steps of writing the Dynare Code Con’t:

- Specify the variance of the shocks: this part of the code starts with “shock;”, followed by a specification of the variance, followed by “end;”

- Tell Dynare to calculate the steady state values of the endogenous variables of the model: use command “steady;”

- The last step is to solve the model, produce the policy functions, and generate IRF and unconditional second moments: use command “stoch_simul(option);”
Output of the Dynare

The default output concludes

- (1) steady state values of endogenous variables
- (2) a model summary, which counts variables by type
- (3) covariance matrix of shocks
- (4) the policy and transition functions
- (5) theoretical first and second moments
- (6) a theoretical correlation matrix
- (7) theoretical autocovariances up to order 5
- (8) impulse responses (by default, 2nd-order approximation about the steady state)
Where is the Output Stored

- It is a bit challenging to find where your output is stored
- Most of outputs are stored in “oo.” file
- “$x_e$” stores the IRF of variable $x$ to shock $e$
- “$oo_dr.ghx$” stores the coefficients of state variables; the matrix rows correspond to all endogenous variables in DR-order and columns correspond to state variables in DR-order
- “$oo_dr.ys$” stores the steady state values; the vector rows correspond to all endogenous variables in declaration order
- “$oo_dr.ghu$” stores the coefficients of shocks; the matrix rows correspond to all endogenous variables in DR-order and columns correspond to exogenous variables in declaration order
Where is the Output Stored

- DR ordering is based on the following ordering of variables by type:
  - static variables (only dated $t$)
  - backward-looking variables (dated $t$ and $t-1$)
  - mixed variables (dated $t-1$, $t$, and $t+1$)
  - forward-looking variable (dated $t$ and $t+1$)

- use command 
  
  \[ \text{oo\_dr\_inv\_order\_var}(k) \] 

  to map the declared order into the DR order

- use command 
  
  \[ \text{oo\_dr\_order\_var}(k) \] 

  to map the DR order into the declared order