HANK Made Easy

–Macro Fluctuations and Policies in **THANK**–

(*Tractable Heterogeneous-Agent New Keynesian Models*)

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Lecture Notes on THANK, this version January 2020[1]

[1]Notes used to teach advanced mini-courses over the last years at European Central Bank Research Department, Paris School of Economics Summer School and APE program, HEC Lausanne, etc.

Inspiring Quotes

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4. *"If you can't solve a problem, then there is an easier problem you can solve: find it."* **George Polya**

Inspiring (Funny) Quotes

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2. *"If I have not seen as far as others, it is because there were giants standing on my shoulders."* **Hal Abelson**

Disclaimers

1. Heterogeneity in ... many things, here **only households** and **very limited**
   - Precisely why **tractable**: imagine non-tractable even with such a limited scope
2. NOT an exhaustive review of HANK
   - Centered on *my own work* within this paradigm (15+ yrs & what I *happen to* know best—no doubt many know it better)
   - + related contributions by others

- Overlap with *the slides I use to present my research*

Motivation

- **2008 Great Expansion**—*stabilization* policies (mon&fisc)
  - + inequality-redistribution, i.a. Bernanke, Yellen, Draghi
- Micro data & solving HA models Krusell Smith, Den Haan, Reiter (...)
- *Aggregate* Euler? Hall; Cambell Mankiw (...) zero net worth: Wolff (...)
- Consumption—Income: Johnson, Parker, Souleles; Surico et al; etc.
- *Liquidity* constr. & MPC: Kaplan Violante; Cloyne Ferreira Surico; Gorea Midrigan

HA

- Heterogeneity and constraints: many (parallel and often deeply similar) ways throughout the decades
  - HANK is the culmination–synthesis
- Ex: Bewley-Aiyagari-Huggett (Imrohoroglu, Krusell Smith, Rios-Rull, Heathcote, etc.)
- vs. Deaton-Caroll-Zeldes (Kimball, Mankiw, Campbell, etc.)
- Looks like a *divide* and we did not even mention prices yet … (up to the historians of thought).

HA

NK

$$\frac{\text{HA}}{{}_{\text{HA}}\text{NK}}$$

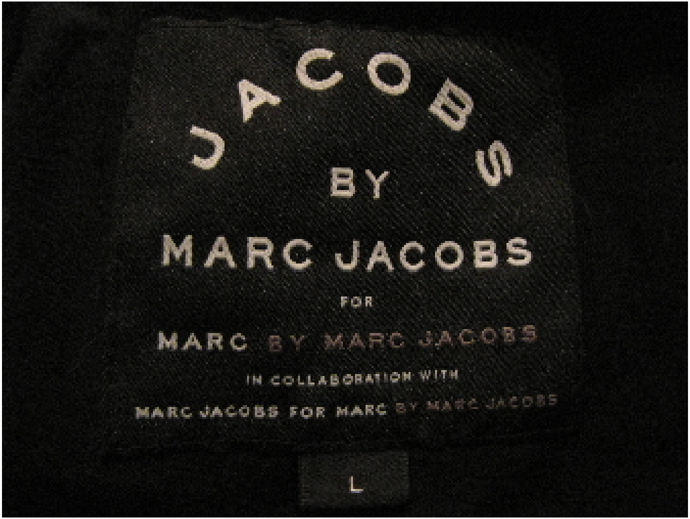
2000s: TANK, Macro to Micro

2010s: Micro to Macro

HANK

HANK

2000s: TANK, Macro to Micro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Based on (15+ years)

- Limited Asset Market Participation, Monetary Policy, and (Inverted) Aggregate Demand Logic, **2008** *Journal of Economic Theory* (Ch. 1, **2004** PhD Thesis)
- The New Keynesian Cross, 2017a *Journal of Monetary Economics*
- Monetary Policy and Heterogeneity: An Analytical Framework, 2017b *Mimeo*

  Please cite & acknowledge the above 3 if using these slides

- Joint work w/ **R. Straub** (2004 Mimeo, 2012 JEDC, 2013 REStat); **Meier and Mueller** (2008 JMCB); **Monacelli and Perotti** (2011 Mimeo; 2013 EJ); **Ragot** (2016 Mimeo)
- Ongoing work with **Primiceri and Tambalotti, Känzig and Surico, Monacelli and Perotti, etc.**

	Quantitative	Tractable
TANK 2000s		Bilbiie **2008** *JET* (2004) LAMP, Mon. Pol. & (Inv.) Aggregate Demand
HANK 2010s		χ

Key channel: $\chi \gtrless 1 \sim$ Cyclical Inequality

Elasticity of individual to aggregate income: F(**profits'** (re)distribution) link

Some ECB history (disclaimer applies ...)

EUROPEAN CENTRAL BANK

WORKING PAPER SERIES

NO. 408 / NOVEMBER 2004

THE GREAT INFLATION, LIMITED ASSET MARKETS PARTICIPATION AND AGGREGATE DEMAND: FED POLICY WAS BETTER THAN YOU THINK [1]

by Florin O. Bilbiie [2]

In 2004 all publications will carry a motif taken from the €100 banknote.

This paper can be downloaded without charge from http://www.ecb.int or from the Social Science Research Network electronic library at http://ssrn.com/abstract_id=601028.

1 I am particularly highly indebted to Roberto Perotti and Giancarlo Corsetti for numerous discussions and close advice and to Jordi Gali and Andrew Scott for detailed comments on my thesis. I am grateful for comments at various stages to an anonymous referee, Kosuke Aoki, Mike Artis, Paul Bergin, Giuseppe Bertola, Fabrice Collard, Roger Farmer, Mike Haliassos, Stephanie Schmitt-Grohe, Fabio Ghironi, David Lopez-Salido, John Muellbauer, Gernot Mueller, Kris Nimark, Fabrizio Perri, Jon Skinner, Mike Woodford, participants at the European Summer Symposium in Macroeconomics in Taragona, and seminars at Nuffield College, Oxford and EUI. I thank Jeff Fuhrer and Liz Walat for data and Bergljot Barkbu for help with codes for the empirical part. Special thanks to Roland Straub, my co-author in related work. I am grateful to the Directorate General Research of the European Central Bank for hospitality during part of writing this paper. All errors are mine.

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EUROPEAN CENTRAL BANK

EUROSYSTEM

WORKING PAPER SERIES

NO 1438 / MAY 2012

ASSET MARKET PARTICIPATION, MONETARY POLICY RULES AND THE GREAT INFLATION

by Florin O. Bilbiie and Roland Straub

In 2012 all ECB publications feature a motif taken from the €50 banknote.

NOTE: This Working Paper should not be reported as representing the views of the European Central Bank (ECB). The views expressed are those of the authors and do not necessarily reflect those of the ECB.

Literature

- **TANK 2000s** Bilbiie 08; Galí Lopez-Salido Vallés 07 (Mankiw 00); Bilbiie Straub; Bilbiie Meier Muller; Colciago; Ascari, Colciago and Rossi; Eser, etc.; **different:** Iacoviello 05; Eggertsson Krugman; Curdia Woodford; Nistico; Bilbiie Monacelli Perotti
- **HANK 2010s** Oh Reis, Guerrieri Lorenzoni, Gornemann Kuester Nakajima; **Kaplan Moll Violante; McKay Nakamura Steinsson; Auclert**; Auclert Rognlie; Bayer Luetticke Pham-Dao Tjaden; Luetticke; Ravn Sterk; Den Haan Rendahl Riegler; McKay Reis; Challe Matheron Ragot Rubio; Debortoli Galí; Hagedorn Manovskii Mitman (Luo); Ferrière Navarro; Auclert Rognlie Straub; **Analytical:** Acharya Dogra; Bilbiie; Broer, Hansen, Krusell, Oberg; Holm; Ravn Sterk; Werning
- **Determinacy in RANK**: Leeper; Woodford; Cochrane; Lubik Schorfheide; **Forward Guidance puzzle** (Del Negro, Giannoni, Patterson): ~~perfect information/rational expectations~~ Kiley; Carlstrom Fuerst Paustian; Garcia-Schmidt Woodford; Farhi Werning; Wiederholt; Andrade et al; Gabaix; Angeletos Lian; **G balance sheet**: Cochrane; Diba Loisel; Michaillat Saez; Hagedorn
- **Optimal policy TANKs** Bilbiie 08, Ascari et al; Nistico; Curdia Woodford); **HANKs**: Bhandari Evans Golosov Sargent; Nuno Thomas; Challe; Bilbiie Ragot; Cui Sterk

Core Model: **THANK**

Max(Micro in Macro)

s.t. Tractable

Core Model: **THANK**

1. Projection of **several** quantitative-HANK channels

2. Tractable affords *closed-form analytical, full-blown NK*

   — policymakers, central banks
   — public communication
   — students
   — colleague economists
   — discipline empirical work

Plan (all topics about Monetary and Fiscal Policy)

1. What is RANK not enough for?
   - Aggregate Demand and Keynesian Cross
2. TANK (RANK-isomorphic)
   - The New Keynesian Cross
3. THANK:
   - idiosyncratic risk & precautionary saving
   - liquidity
   - cyclical risk
   - (the Catch-22)
4. *Optimal Monetary(-Fiscal) Policies* in TANK & THANK
5. Application: *Liquidity Traps*

Further developments/extensions: THANK &

6. *Money, i.e. Liquidity* (w/ Ragot)
7. *Capital K, i.e. illiquid wealth* (w/ Känzig and Surico)
8. *DSGE, estimated*, w/ Primiceri and Tambalotti
9. Ways forward

Preview: The 3-Equation THANK Model

$$
\begin{aligned}
c_t &= \delta E_t c_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi}\left(i_t - E_t \pi_{t+1} - \rho_t\right) \\
&: \quad (\text{with } \delta \equiv 1 + (\chi - 1)\frac{1-s}{1-\lambda\chi}) \\
\pi_t &= \kappa c_t + \beta E_t \pi_{t+1} + u_t \\
i_t &= \phi \pi_t \text{ (or LQ-}\textit{optimal policy}\text{)}
\end{aligned}
$$

- Heterogeneity $\sim$ colors

RANK: A Keynesian-Cross Representation

- Complete markets

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^j, N_t^j\right) \text{ s.t.}$$

$$Z_{t+1}^j + \Theta_{t+1}^j V_t \quad \leq \quad B_t^j + \Theta_t^j \left(V_t + P_t D_t\right) + W_t N_t^j - P_t C_t^j.$$

  - Z_{t+1}^j nominal *end of period* t portfolio of all state-contingent assets (except shares)
  - B_t^j beginning of period wealth. Θ_t^j shares

- **No-arbitrage** $\rightarrow \exists$ pricing kernels/stoch. disc. factors:

$$\frac{Z_{t+1}^j}{P_t} = E_t\left[Q_{t,t+1}^j \frac{B_{t+1}^j}{P_{t+1}}\right] \text{ and } \frac{V_t}{P_t} = E_t\left[Q_{t,t+1}^j\left(\frac{V_{t+1}}{P_{t+1}} + D_{t+1}\right)\right],$$

- Gross **real** rate (definition)

$$\frac{1}{R_t} = E_t Q_{t,t+1}^j$$

RANK: A Keynesian-Cross Representation

- No-arbitrage + wealth → flow BC, + 'natural' borrowing limit by state, anticipate equil. all agents hold constant fraction of shares (no trade) Θ^j:

$$\text{IBC:} \qquad E_t \sum_{i=0}^{\infty} Q^j_{t,t+i} C^j_{t+i} \leq E_t \sum_{i=0}^{\infty} Q^j_{t,t+i} Y^j_{t+i},$$

$$\text{income} \quad : \quad Y^j_{t+i} \equiv \Theta^j D_{t+i} + \frac{W_{t+i}}{P_{t+i}} N^j_{t+i}$$

Max. U s.t. this, each date and state:

$$\beta \frac{U_C\left(C^j_{t+1}\right)}{U_C\left(C^j_t\right)} = Q^j_{t,t+1}$$

+ IBC with equality (or flow BC w/ equality + transversality $\lim_{i\to\infty} E_t\left[Q^j_{t,t+i} Z^j_{t+i}\right] = \lim_{i\to\infty} E_t\left[Q^j_{t,t+i} V_{t+i}\right] = 0$).

RANK: A Keynesian-Cross Representation

- Substitute in no-arbitrage

$$\frac{1}{R_t} = \beta E_t \left[\frac{U_C\left(C^j_{t+1}\right)}{U_C\left(C^j_t\right)} \right].$$

- Loglinearize IBC $E_t \sum_{i=0}^{\infty} Q^j_{t,t+i} C^j_{t+i} \leq E_t \sum_{i=0}^{\infty} Q^j_{t,t+i} Y^j_{t+i}$, use Euler and stochastic disc. factor

$$c^j_t = -\sigma\beta \sum_{i=0}^{\infty} \beta^i E_t r_{t+i} + (1-\beta) \sum_{i=0}^{\infty} \beta^i E_t y^j_{t+i},$$

write in recursive form ...

RANK "not enough" 1: (lack of) amplification

- "consumption function" of agent j;

$$c_t^j = (1-\beta)\,\hat{y}_t^j - \sigma\beta r_t + \beta E_t c_{t+1}^j.$$

- Euler-IS by market clearing $c_t^j = \hat{y}_t^j \equiv y_t^j - t_t^j$:

$$c_t^j = E_t c_{t+1}^j - \sigma r_t$$

- MP shock with persistence p and FP $g_t = t_t$

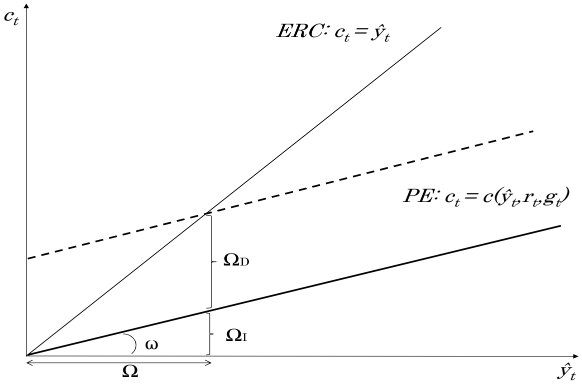
$$\begin{aligned}
\Omega &\equiv \frac{dc_t^j}{d\left(-r_t\right)} = \frac{\sigma}{1-p} \\
\omega &\equiv \frac{\Omega_I}{\Omega} = \frac{1-\beta}{1-\beta p} \\
\mathcal{M} &\equiv \frac{dy_t}{dg_t} = 1 \left(\frac{dc_t}{dg_t} = 0\right)
\end{aligned}$$

where $\Omega_I \equiv \frac{dc_t^j}{d(-r_t)}|_{r_t=\bar{r}} = \Omega - \Omega_D$; $\Omega_D \equiv \frac{dc_t^j}{d(-r_t)}\Big|_{y_t^j=\bar{y}} = \frac{\sigma\beta}{1-\beta p}$

The (New?) Kenesian Cross

$$c_t = \omega \hat{y}_t - (1-\omega)\,\Omega r_t + (1-\omega)\,(M-1)\,g_t$$

Old Keynesian Cross: Samuelson (1948, pp 256-279)



ω ~aggreg. MPC Ω, M ~ Multipliers

RANK: all shift, no slope

- $\omega \sim 0$ Planned-Expenditure curve is flat
- – virtually no General-Equilibrium
- All partial-equilibrium, direct (shift): $\Omega \sim \Omega_D$
- New ... *what*?

RANK "not enough" 2: FG Puzzle

- add AS $\pi_t = \kappa c_t \, (+\beta E_t \pi_{t+1})$

$$c_t = E_t c_{t+1} - \sigma \left(i_t - E_t \pi_{t+1} \right) = \nu_0 E_t c_{t+1} - \sigma i_t$$

N*ews* on AD under i <u>peg</u>:

$$\nu_0 \equiv 1 + \kappa\sigma \geq 1.$$

- **FG puzzle** (Del Negro, Giannoni, Patterson, ...):

$$c_t = \nu_0 E_t c_{t+1} - \sigma i_t = \nu_0^T E_t c_{t+T} - \sigma \sum_{j=0}^{T-1} \nu_0^j E_t i_{t+j}$$

  - "indeterminacy", rationale for Taylor $i_t = \phi \pi_t, \phi > 1 \longrightarrow$ $\nu = \frac{1+\kappa\sigma}{1+\kappa\sigma\phi} < 1$, solve $\Big\uparrow$ **forward**

TANK

TANK, Bilbiie 2008 version

- (Revisited in light of HANK: *The New Keynesian Cross*)
- A*ssets* or not chi; λ fraction consume all **<u>their</u>** income

$$C_t^H = W_t N_t^H + \mathcal{T}_t^H = Y_t^H$$

- S: complete markets

$$C_t^S + Z_{t+1}^S + v_t \Theta_{t+1} = W_t N_t^S + B_t^S + \Theta_t \left(v_t + D_t \right) + \mathcal{T}_t^S,$$

- Assets held/priced, not traded! ($\neq$ Mankiw 2000, Gali et al 2007)

  $\longrightarrow$isolate role of **income inequality** and **<u>profits</u> (#60)**
- Asset mkt clearing$\rightarrow$

$$C_t^S = W_t N_t^S + \frac{1}{1-\lambda} D_t + \mathcal{T}_t^S = Y_t^S$$

- Redistribution/transfer (**Section 4.3**), tax profits τ^D rebate to $H \sim$ "automatic stabilizer"

$$\lambda \mathcal{T}_t^H = \tau^D D_t = -\left(1-\lambda\right) \mathcal{T}_t^S$$

- Heterogeneity in earnings and income

TANK

- separable $U^j\left(C^j, N^j\right)$; $\sigma^{-1} \equiv -U^j_{CC}C^j/U^j_C$; $\varphi \equiv U^j_{NN}N^j/U^j_N$
- H **work**; S work and trade/price shares (get profits)+all securities. No risk or insurance (later)
- Labor $\varphi n^j_t = w_t - \sigma^{-1}c^j_t$ all j (also aggregate!) together with $c_t = y_t = n_t \rightarrow$

$$w_t = \left(\varphi + \sigma^{-1}\right) c_t$$

- H loglin BC $c^H_t = w_t + n^H_t + \frac{\tau^D}{\lambda} d_t - t^H_t$.
- **government policies:** 1. τ^D; 2. D^{SS}: steady-state subsidy+tax firms (S)$\rightarrow$ $D^{SS} = 0$

$$d_t = -w_t$$

(now $\left(g_t, t^H_t\right) = 0$, add later)

TANK: Model Summary

Equil. condition	Loglinearized
$U_C\left(C_t^S\right) = \beta R_t E_t\left[U_C\left(C_{t+1}^S\right)\right]$	$c_t^S = E_t c_{t+1}^S - \sigma r_t$
$\frac{W_t}{P_t} U_C\left(C_t^S\right) = -U_N\left(N_t^S\right)$	$\varphi n_t^S = w_t - \sigma^{-1} c_t^S$
$\frac{W_t}{P_t} U_C\left(C_t^H\right) = -U_N\left(N_t^H\right)$	$\varphi n_t^H = w_t - \sigma^{-1} c_t^H$
$C_t^H = \frac{W_t}{P_t} N_t^H + \frac{\tau^D}{\lambda} D_t$	$c_t^H = w_t + n_t^H + \frac{\tau^D}{\lambda} d_t$
$D_t = \left(1+\tau^S\right) Y_t - \frac{W_t}{P_t} N_t - T_t^F$	$d_t = -w_t$
$Y_t = C_t \equiv \lambda C_t^H + (1-\lambda) C_t^S$	$y_t = c_t \equiv \lambda c_t^H + (1-\lambda) c_t^S$
$N_t = \lambda N_t^H + (1-\lambda) N_t^S$	$n_t = \lambda n_t^H + (1-\lambda) n_t^S$
$Y_t = N_t$	$y_t = n_t$

- loglin. around SS w/ opt. τ^S, $D^{SS} = 0$, $T_t^F = \tau^S Y_t$

TANK: Reminder

- if prices sticky, add Phillips curve + Taylor rule (π and i)
- above still goes through with Phillips curve if Taylor rule is $i_t = E_t \pi_{t+1} + r_t$
- (just as in RANK)

TANK: Deriving Aggregate Demand

- idea: express individual variables $c^j (= y^j)$ as function of aggregate $c (= y)$

$$
\begin{aligned}
c_t^H &= \underbrace{\chi}_{\text{[KEY]}} y_t = \left[1 + \underbrace{\varphi}_{\text{labor mkt.}} \times \underbrace{\left(1 - \frac{\tau^D}{\lambda} \right)}_{\text{fiscal redistrib.}} \right] y_t \\
c_t^S &= \frac{1 - \lambda \chi}{1 - \lambda} y_t
\end{aligned}
$$

Extra income effect $w \uparrow \rightarrow d \downarrow$ *keystone*: <u>**profits**</u>[2]

- **Gen-Eq.**: demand↑, w↑, $y^H \uparrow$, demand↑↑–amplification spiral *until?*
- S work more=*equilibrium* because $y^S \downarrow$ as $d \downarrow$

Cyclical **Income Inequality:** $\gamma_t = y_t^S - y_t^H = \dfrac{1 - \chi}{1 - \lambda} y_t$

Several income-distribution (χ) models: fiscal incidence, *sticky wages*

TANK: Colciago; Ascari Colciago Rossi, Furlanetto, HANK: Broer et al, Auclert et al.

[2] Profits' cyclicality? 1. off-equilibrium, 2. conditional; 3. *data!?*

TANK: Cyclical (Income) Inequality

- Aggregate Euler-IS-AD: replace c_t^S in Euler S: $c_t^S = E_t c_{t+1}^S - \sigma r_t$:

$$c_t = E_t c_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi} r_t$$

1. **TANK** *Amplification* iff χ >1: **Inequality** *Countercyclical*

Generalizes to rich-HANK: cov(MPC,χ), Auclert JMP 2015; Direct test: Patterson 2019 JMP

aggreg. MPC $\equiv \lambda \times 1 \times \chi + (1-\lambda) \times (1-\beta) \times \frac{1-\lambda\chi}{1-\lambda}$

- $\chi > 1$: AEIS—dc/dr—*increasing* with λ $\left(< \chi^{-1}\right)$; Reason $\uparrow$
- **dampening** with $\chi < 1$ but
  - indirect share ω increasing with λ *regardless* of χ;

The New Keynesian Cross

- *Aggreg.* C, **PE curve** (novel$\neq$Campbell-Mankiw!):

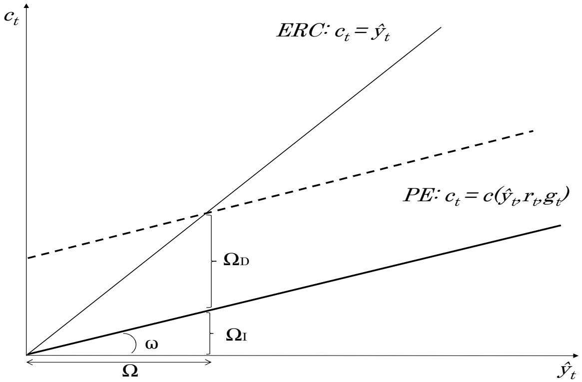
$$c_t = \left[1 - \beta \left(1 - \lambda\chi\right)\right] \hat{y}_t - \left(1 - \lambda\right) \beta\sigma r_t + \beta \left(1 - \lambda\chi\right) E_t c_{t+1}$$

- Partial equilibrium, indirect effect ... **MPC!** keep y fixed
- General equilibrium, total effect ... **Multiplier**: add $c_t = \hat{y}_t \rightarrow$ Aggregate Euler

	Total effect Ω ("multiplier")	Indirect-effect share ω ("aggregate MPC")
TANK	$\frac{\sigma}{1-p}\frac{1-\lambda}{1-\lambda\chi}$	$\frac{1-\beta(1-\lambda\chi)}{1-\beta p(1-\lambda\chi)}$

The New Kenesian Cross

$$c_t = \omega \hat{y}_t - (1-\omega)\,\Omega r_t + (1-\omega)\,(M-1)\,g_t$$



$$\text{aggreg. MPC } \omega \equiv \lambda \times 1 \times \chi + (1-\lambda) \times (1-\beta) \times \frac{1-\lambda\chi}{1-\lambda}$$

TANK Neutrality Special case: A-cyclical Inequality

- Campbell-Mankiw knife-edge $\chi = 1$, intertemporal substitution only difference

188 · CAMPBELL & MANKIW

accrues to individuals to consume their current income, while the remainder $(1-\lambda)$ accrues to individuals who consume their permanent income. If the incomes of the two groups are Y_{1t} and Y_{2t} respectively, then total income is $Y_t = Y_{1t} + Y_{2t}$. Since the first group receives λ of total income, $Y_{1t} = \lambda Y_t$ and $Y_{2t} = (1-\lambda)Y_t$. Agents in the first group consume their current income, so $C_{1t} = Y_{1t}$, implying $\Delta C_{1t} = \Delta Y_{1t} = \lambda \Delta Y_t$. By contrast, agents in the second group obey the permanent income hypothesis, implying $\Delta C_{2t} = (1-\lambda)\epsilon_t$.

The change in aggregate consumption can now be written as

$$\Delta C_t = \Delta C_{1t} + \Delta C_{2t} = \lambda \Delta Y_t + (1-\lambda)\epsilon_t. \quad (1.4)$$

- History of thought: footnote 26 in CM's 3rd and last paper on this, EER 1991

[26] Of course, utility costs would be much larger again if some agents were consuming their *own* current income.

- **neutrality** (RANK); but *indirect effect* (one-to-one);
- Bilbiie 2008 footnote 14; Bilbiie-Straub 2012;
  - Werning 2015: generalization in a complicated model but focusing on "income risk". Here, no risk (yet)

Detour: Aggregate Demand, Inverted

- IS-AD **swivels** when:

$$\lambda > \chi^{-1}$$

- "fallacy of composition" $\rightarrow$ Inverted AD & Taylor principle
- Bilbiie Straub 2013 REStat explain Great Inflation: no sunspots, passive Fed policy OK (Bayesian TANK-DSGE estimation)
- Bilbiie Straub 2012 JEDC (1-eq. GMM): IS slope inverted in the 70s, changed sign post-Volcker. Solves zero-slope puzzle (time-agregate of + and -)
- Key: tremendous financial liberalization and innovation $\rightarrow$ increased participation in the early '80s
- **Takeaway:** to publish "Keynesian" papers with amplification in the 2000s (Great Moderation ...) had to focus on bifurcations, inversions ... non-Keynesian

TANK: Fiscal Multipliers

- Add **Government policy 3:** spend G_t, balanced-budget $T_t = G_t$
- + *exogenous* redistribution (∼progressivity) α

$$\lambda T_t^H = \alpha T_t$$

- loglinearized (around $G = 0$)

$$t_t^H = \frac{\alpha}{\lambda} t_t = \frac{\alpha}{\lambda} g_t = \underbrace{g_t}_{\text{bal.-budg.}} - \underbrace{\left(1 - \frac{\alpha}{\lambda}\right) t_t}_{\text{exog. redist.}}$$

- Implies (income effect $\zeta_H \equiv \left(1 + \varphi^{-1}\sigma^{-1}\right)^{-1}$)

$$c_t^H = \chi \hat{y}_t + \zeta_H \left(\chi - \frac{\alpha}{\lambda}\right) g_t$$

TANK: Fiscal Multipliers

- Aggregate Euler-IS-AD:

$$c_t = E_t c_{t+1} - \frac{1-\lambda}{1-\lambda\chi}\sigma r_t + \frac{\lambda\zeta_H}{1-\lambda\chi}\left(\chi - \frac{\alpha}{\lambda}\right)\left(g_t - E_t g_{t+1}\right).$$

- *Aggreg. C*, **PE curve**:

$$\begin{aligned} c_t &= \left[1-\beta\left(1-\lambda\chi\right)\right]\hat{y}_t - \left(1-\lambda\right)\beta\sigma r_t + \beta\left(1-\lambda\chi\right)E_t c_{t+1} \\ &\quad + \beta\lambda\zeta_H\left(\chi - \frac{\alpha}{\lambda}\right)\left(g_t - E_t g_{t+1}\right) \end{aligned}$$

- Multiplier (fixed-*r* trick from Bilbiie, 2008, 2011)

	Total Ω	Ind. share ω	**Fisc. Mult.** $\mathcal{M}$
TANK	$\frac{\sigma}{1-p}\frac{1-\lambda}{1-\lambda\chi}$	$\frac{1-\beta(1-\lambda\chi)}{1-\beta p(1-\lambda\chi)}$	$1+\frac{\lambda\zeta_H}{1-\lambda\chi}\left(\chi-\frac{\alpha}{\lambda}\right)$

TANK: Fiscal Multipliers

$$\mathcal{M} = 1 + \frac{\lambda \zeta_H}{1 - \lambda \chi} \left[\underbrace{(\chi - 1)}_{\text{NK cross}} + \underbrace{\left(1 - \frac{\alpha}{\lambda}\right)}_{\text{exog. redist.}} \right] > 1$$

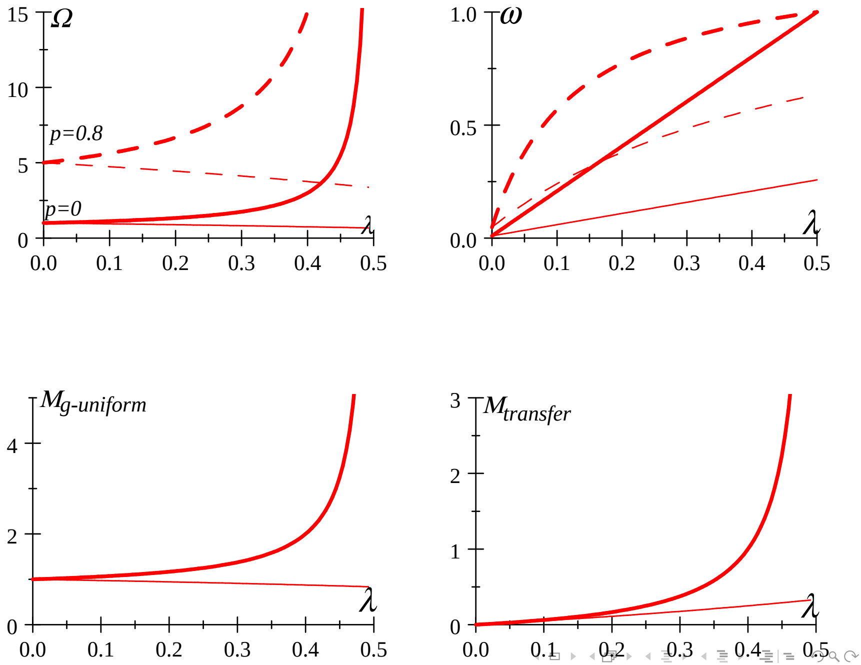
+amplification $\frac{\partial \mathcal{M}}{\partial \lambda} > 0$ w/ uniform t^j, $\alpha = \lambda$ IFF

$$\chi > 1$$

- $\mathcal{M}$ "total effect": Same decomposition, persistence irrelevant.
  - $\chi \uparrow$ increases PE slope; also PE shift but *only if* $\chi > 1$
  - $\alpha \downarrow$ increases PE shift *only if transfer* (progressive shock, $\alpha < \lambda$)
  - λ increases $\mathcal{M}$; but iff $\chi > 1$ when taxation uniform $\alpha = \lambda$

TANK: Fiscal Multipliers, Previous Work

- G **spending** multiplier:
  - numerical, with K (Gali Lopez-Salido Valles JEEA 2007); analytical, no K (dist. taxes) Bilbiie Straub 2004 WP, Bilbiie Meier Mueller 2008 JMCB; Monacelli Perotti IMF EcRev 2012
- Redistribution (**transfer**) multiplier:
  - Bilbiie Monacelli Perotti EJ 2013; Mehrotra IJCB; Giambattista and Pennings EER
- At the **ZLB** with borrower-saver and deleveraging:
  - Eggertsson Krugman QJE 2012



Ω
15
10
5
0
p=0.8
p=0
λ
0.0
0.1
0.2
0.3
0.4
0.5
ω
1.0
0.5
0.0
λ
0.0
0.1
0.2
0.3
0.4
0.5
M_g-uniform
4
2
0
λ
0.0
0.1
0.2
0.3
0.4
0.5
M_transfer
3
2
1
0
λ
0.0
0.1
0.2
0.3
0.4
0.5

What (else) determines χ?

- *Fiscal redistribution:*
  - more general progressive taxation (Heathcote Storesletten Violante; Ferrière Navarro; Auclert Rognlie Straub);
  - crucial ingredient in **all** HANK: here spelled out transparently
- *Sticky wages*
  - TANK: Colciago, w/ Ascari & Rossi; Furlanetto;
  - HANK: Broer et al, Walsh, Auclert Rognlie Straub, Alves Kaplan Moll Violante, Bilbiie Känzig Surico

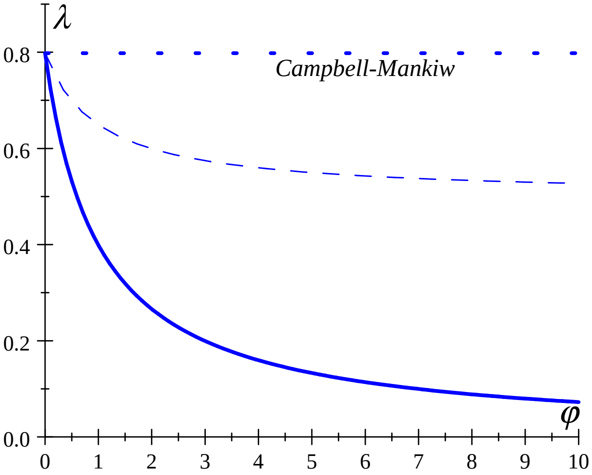
"χ" as in ... Key(nes)!

$$\chi \equiv 1 + \varphi \left(1 - \frac{\tau^D}{\lambda}\right) \gtrless 1.$$

"The amount that the community spends on ***consumption*** *obviously depends on [...] the principles on which* ***income is divided*** *between the individuals composing it (which may suffer* ***modification*** *as* ***output is increased****)."*
"[...] we may have to make an allowance for the possible ***reactions*** *of* ***aggregate consumption*** *to the* ***change in the distribution*** *of a given real* ***income*** *...* ***resulting*** *from a* ***change in the wage****-unit"*.
"If ***fiscal policy*** *is used as a deliberate instrument for the more equal* ***distribution of incomes****, its effect in increasing the* ***propensity to consume*** *is, of course, all the greater."* Keynes [1936] , Ch. 8, Books I and III

"HANK Surface": Indirect Effect = 0.8

(φ, λ) s.t. $\omega = 0.8$, $\tau^D = 0.5$ (dash); iid shock



Indirect Amplification

- Suppose $\mathcal{A}$ times amplification relative to $\lambda = 0$ (RANK):

$$\Omega(\lambda) = \mathcal{A} * \Omega(0)$$

- *Proposition*: the indirect share is **at least** (for iid shocks):

$$\omega \geq 1 - \frac{1}{\mathcal{A}}$$

- twice as much effect, at least half of it is indirect; four times, three quarters is indirect, etc.
- NOTE: invariant to λ and χ

Homework

- Assume a different fiscal redistribution scheme (your choice) and derive the χ
- Is the FG puzzle still a puzzle in TANK? More so, less so, or exactly the same as in RANK?

THANK

THANK Model (Ingredients)

- **Two states**: constrained hand-to-mouth H and unconstrained "savers" S
  - switch *exogenously* (idiosyncratic uncertainty).
- Insurance:
  - *full within* type (after idiosyncratic uncertainty revealed)
  - *limited across* types.
- **Two assets** and *liquidity*:
  - bonds are **liquid** (*can* be used to self-insure, before idiosyncratic uncertainty is revealed)
  - stocks are **illiquid** (cannot ———„———).
- Bond trading
  - equilibrium liquidity
  - or **not (***most analytical* HANK): "**Bondless** limit"

Two-state-, Two-asset, Tractable-HANK

- shocks S$\leftrightarrows$H
- $\mathrm{p}(S|S) = s; \mathrm{p}(H|S) = 1 - s;$
- $\mathrm{p}(H|H) = h; \mathrm{p}(S|H) = 1 - h$
- H mass (<u>unconditional</u> H probability, stationary distribution):
$$\lambda = \frac{1-s}{2-s-h}$$
solution of
$$\begin{pmatrix} \lambda & 1-\lambda \end{pmatrix} \begin{pmatrix} h & 1-h \\ 1-s & s \end{pmatrix} = \begin{pmatrix} \lambda & 1-\lambda \end{pmatrix}$$

Assets

- S: illiquid shares (profits); liquid nominal public debt
  - Adjust portfolio *before* knowing if S or H next
  - If $\rightarrow H$ can only take bonds
- Bond flows (per capita)
  - B^S_{t+1} *beginning-of-period-$t+1$*, after consumption-saving choice, also after changing state and pooling
  - Z^S_{t+1} *end-of-period-t* after the consumption-saving choice but before moving

$$(1-\lambda)\,B^S_{t+1} = (1-\lambda)\,sZ^S_{t+1} + (1-\lambda)\,(1-s)\,Z^H_{t+1}$$
$$\lambda B^H_{t+1} = (1-\lambda)\,(1-s)\,Z^S_{t+1} + \lambda h Z^H_{t+1}.$$

rescaling and using $\lambda = \frac{1-s}{1-s+1-h}$:

$$B^S_{t+1} = sZ^S_{t+1} + (1-s)\,Z^H_{t+1}$$
$$B^H_{t+1} = (1-h)\,Z^S_{t+1} + hZ^H_{t+1}.$$

Family (Head) Optimization

$$
\begin{aligned}
W\left(B_t^S, B_t^H, \Theta_t\right) \quad = & \max_{\{C_t^S, Z_{t+1}^S Z_{t+1}^H, C_t^H, \Theta_{t+1}\}} (1-\lambda)\, U\left(C_t^S\right) + \lambda U\left(C_t^H\right) \\
& + \beta E_t W\left(B_{t+1}^S, B_{t+1}^H, \Theta_{t+1}\right)
\end{aligned}
$$

subject to:

$$
\begin{aligned}
C_t^S + Z_{t+1}^S + v_t \Theta_{t+1} &= Y_t^S + R_t B_t^S + \Theta_t\left(v_t + D_t\right), \\
C_t^H + Z_{t+1}^H &= Y_t^H + R_t B_t^H \\
Z_{t+1}^S, Z_{t+1}^H &\geq 0
\end{aligned}
$$

and the laws of motion for bond flows relating the Zs to the Bs

Euler Equations

Look like Bewley-Aiyagari-Huggett-...

$$U'\left(C_t^S\right) \geq \beta E_t \left\{ \frac{v_{t+1} + D_{t+1}}{v_t} U'\left(C_{t+1}^S\right) \right\} \text{ and } \Theta_{t+1} = \Theta_t = (1-\lambda)^{-1};$$

$$U'\left(C_t^S\right) \geq \beta E_t \left\{ R_{t+1} \left[sU'\left(C_{t+1}^S\right) + (1-s)\, U'\left(C_{t+1}^H\right) \right] \right\} \text{ and}$$

$$0 = Z_{t+1}^S \left[U'\left(C_t^S\right) - \beta E_t \left\{ R_{t+1} \left[sU'\left(C_{t+1}^S\right) + (1-s)\, U'\left(C_{t+1}^H\right) \right] \right\} \right]$$

$$U'\left(C_t^H\right) \geq \beta E_t \left\{ R_{t+1} \left[(1-h)\, U'\left(C_{t+1}^S\right) + hU'\left(C_{t+1}^H\right) \right] \right\} \text{ and}$$

$$0 = Z_{t+1}^H \left[U'\left(C_t^H\right) - \beta E_t \left\{ R_{t+1} \left[(1-h)\, U'\left(C_{t+1}^S\right) + hU'\left(C_{t+1}^H\right) \right] \right\} \right]$$

Two-state-, Two-asset, Tractable-HANK

- *Liquidity* (Kaplan et al, ...): S$\xrightarrow{1-s}$H take **bonds** (liquid), not stock
  - **self-insurance** (bonds priced even when not traded):

$$\left(C_t^S\right)^{-\frac{1}{\sigma}} = \beta E_t \left\{ (1+r_t) \left[s \left(C_{t+1}^S\right)^{-\frac{1}{\sigma}} + (1-s) \left(C_{t+1}^H\right)^{-\frac{1}{\sigma}} \right] \right\}$$

- "wealthy" H: Euler with inequality (constrained):

$$C_t^H = Y_t^H$$

  - no-liquidity, **bondless** limit: *most analytical* HANK
  - w/ *liquidity*: $C_t^H + Z_{t+1}^H = Y_t^H + R_t B_t^H$

Step back

- Designed to capture how ... *which* asset markets work?

Step back

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- Obviously: ...

Step back

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- Obviously: ...
- **None!**

Step back

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- Obviously: ...
- **None!**
- Designed to mimick equilibrium of (be observationally-equivalent to) micro-consistent model that <u>does</u> describe how certain asset markets work

Step back

- Designed to capture how ... *which* asset markets work?
- Obviously: ...
- **None!**
- Designed to mimick equilibrium of (be observationally-equivalent to) micro-consistent model that <u>does</u> describe how certain asset markets work
- Friedman as philosopher of science: approx. *"judge model by implications not (a fortiori unrealistic) assumptions"* (the **F-twist**, cf Samuelson)

Income Inequality and Risk

- **Income inequality (**$\sim$ Gini, generalized entropy**)**

$$\Gamma_t\left(Y_t\right) \equiv \frac{Y_t^S}{Y_t^H}$$

- **Conditional variance** ($\sim$ *income risk*)**:**

$$var\left(\ln Y_{t+1}^S \middle| \ln Y_t^S\right) = s\left(1-s\right)\left(\ln \Gamma_{t+1}\right)^2 .$$

- **Conditional skewness** and **kurtosis**:

$$\begin{aligned} skew\left(\ln Y_{t+1}^S \middle| \ln Y_t^S\right) &= \frac{1-2s}{\sqrt{s\left(1-s\right)}}; \\ kurt\left(\ln Y_{t+1}^S \middle| \ln Y_t^S\right) &= \frac{1}{s\left(1-s\right)} - 3 \end{aligned}$$

- Autocorrelation (> 0 if $s \geq 1-h$)

$$corr\left(\ln Y_{t+1}^j, \ln Y_t^j\right) = s + h - 1 = 1 - \frac{1-s}{\lambda};$$

(Basically Rouwenhorst with 2 states.)

Income Inequality and Risk: Cyclicality

- "**Risk"** (Ravn Sterk; Challe Matheron Ragot Rubio; Werning; Acharya Dogra)

$$1 - s\left(Y_{t+1}\right),\ -s_Y \gtrless 0 \ \rightarrow\ \text{pro-(counter-)cyclical risk (NB: } \lambda \text{ invariant)}$$

- **Skewness** (Guvenen Ozkan Song)

$$\frac{d\left(skew\right)}{dY} = -\frac{s_Y}{2\left[s\left(1-s\right)\right]^{\frac{3}{2}}}$$

- **Variance** (Storesletten Telmer Yaron)

$$\frac{d\left(var\right)}{dY} = \frac{1-s}{Y}\left(\underbrace{\frac{-s_Y Y}{1-s}\left(2s-1\right)\left(\ln\Gamma\right)^2}_{\text{pure risk}} + \underbrace{\frac{\Gamma_Y Y}{\Gamma} s \ln\left(\Gamma\right)^2}_{\text{inequality}}\right)$$

  - $s = 1$: TANK – no risk
  - $s = 0$: oscillating, $\lambda = 1/2$ – no risk (Woodford '90)
  - approximate$\sim$ SS $Y^S = Y^H \rightarrow \Gamma = 1 \rightarrow$ risk 1st-order acyclical

Rest: TANK

- Recall loglin (around $D = 0$) equilibrium:

$$c_t^H = \chi y_t = \left[1 + \underbrace{\varphi}_{\text{labor mkt.}} \times \underbrace{\left(1 - \frac{\tau^D}{\lambda}\right)}_{\text{fiscal redistrib.}} \right] y_t$$

$$c_t^S = \frac{1 - \lambda\chi}{1 - \lambda} y_t$$

Extra income effect $w \uparrow \rightarrow d \downarrow$ *keystone*: **<u>profits</u>**

Cyclical **Income Inequality:** $\gamma_t = y_t^S - y_t^H = \frac{1 - \chi}{1 - \lambda} y_t$

- (χ): **Any** <u>income-distribution model</u>

Aggregate Euler in THANK

- Aggregate, replace $c_t^j \longrightarrow c_t^S = sE_t c_{t+1}^S + (1-s)\, E_t c_{t+1}^H - \sigma r_t$

$$c_t = \underbrace{\left[1 + (\chi - 1)\,\frac{1-s}{1-\lambda\chi}\right]}_{\equiv\delta} E_t c_{t+1} - \sigma \underbrace{\frac{1-\lambda}{1-\lambda\chi}}_{\text{TANK}} r_t$$

Aggregate Euler in THANK

$$c_t = \underbrace{\left[1 + (\chi - 1)\,\frac{1-s}{1-\lambda\chi}\right]}_{\equiv \delta} E_t c_{t+1} - \sigma \underbrace{\frac{1-\lambda}{1-\lambda\chi}}_{\text{TANK}} r_t$$

1. **TANK** *Amplification* iff χ >1: **Inequality** *Countercyclical*

   Generalizes to rich-HANK: cov(MPC,χ), Auclert JMP 2015; Micro evidence: Patterson JMP 2019

aggreg. MPC $\equiv \lambda \times 1 \times \chi + (1-\lambda) \times (1-\beta) \times \frac{1-\lambda\chi}{1-\lambda}$

Aggregate Euler in THANK

$$c_t = \underbrace{\left[1 + (\chi - 1)\frac{1-s}{1-\lambda\chi}\right]}_{\equiv \delta} E_t c_{t+1} - \sigma \underbrace{\frac{1-\lambda}{1-\lambda\chi}}_{\text{TANK}} r_t$$

1. **TANK** *Amplification* iff χ >1: **Inequality** *Countercyclical*

   Generalizes to rich-HANK: cov(MPC,χ), Auclert JMP 2015; Micro evidence: Patterson JMP 2019

   aggreg. MPC $\equiv \lambda \times 1 \times \chi + (1-\lambda) \times (1-\beta) \times \frac{1-\lambda\chi}{1-\lambda}$

2. **THANK Compounding**/Discounting $\delta \gtrless 1$ iff $\chi \gtrless 1$
   - same as in **TANK** but intertemporal! (amplification to news)
   - *Not necesarily* **cyclical** risk

THANK Amplification: Acyclical Risk

$$c_t = \underbrace{\left[1 + (\chi - 1)\frac{1-s}{1-\lambda\chi}\right]}_{\equiv\delta} E_t c_{t+1} - \sigma \underbrace{\frac{1-\lambda}{1-\lambda\chi}}_{\text{TANK}} r_t$$

- *Not necesarily* **cyclical** risk:
- (i) $\sim Y^H = Y^S \to \Gamma = 1 \to$ variance is zero to first order
- (ii) $s = 0$ (Woodford 1990) $\lambda = 1/2$ agents oscilate, Aggregate Euler:

$$c_t = \frac{\chi}{2-\chi} E_t c_{t+1} - \sigma \frac{1}{2-\chi} r_t$$

$$\delta|_{s=0} = \frac{\chi}{2-\chi} \lessgtr 1 \text{ iff } \chi \lessgtr 1.$$

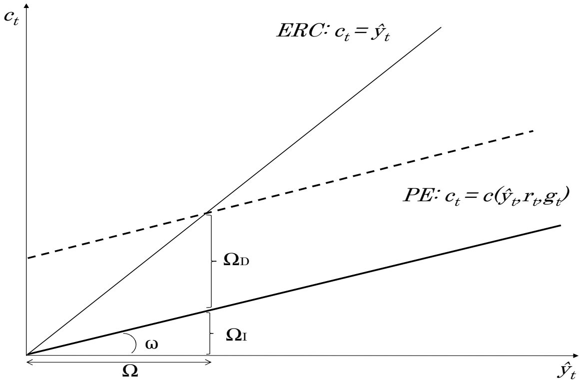
The New Keynesian Cross in THANK

$$\text{PE: } c_t = \left[1 - \beta\left(1 - \lambda\chi\right)\right] y_t - \left(1 - \lambda\right)\beta\sigma r_t + \beta\delta\left(1 - \lambda\chi\right) E_t c_{t+1}$$

	Total effect Ω ("multiplier")	Indirect-effect share ω ("aggregate MPC")
TANK	$\frac{\sigma}{1-p}\frac{1-\lambda}{1-\lambda\chi}$	$\frac{1-\beta(1-\lambda\chi)}{1-\beta p(1-\lambda\chi)}$
THANK	$\frac{\sigma}{1-\delta p}\frac{1-\lambda}{1-\lambda\chi}$	$\frac{1-\beta(1-\lambda\chi)}{1-\delta\beta p(1-\lambda\chi)}$

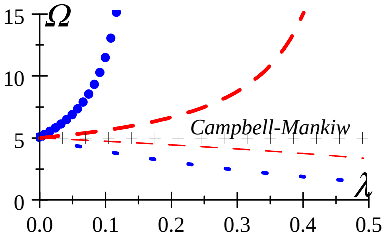
The New Kenesian Cross

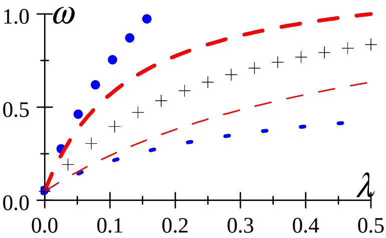
$$c_t = \omega \hat{y}_t - (1-\omega)\,\Omega r_t + (1-\omega)\,(M-1)\,g_t$$



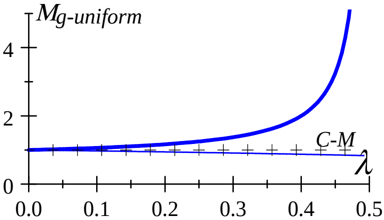
$$\text{aggreg. MPC } \omega \equiv \lambda \times 1 \times \chi + (1-\lambda) \times (1-\beta) \times \frac{1-\lambda\chi}{1-\lambda}$$

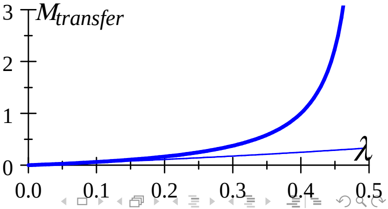
The New Kenesian Cross (in THANK)





Calibrate **KMV** $\Omega/\Omega_{rank} = 1.5, \omega = .8:$ $\chi = 1.42; \lambda = .37; 1 - s = .04$





Calibrating Simple to Match Complicated

Table 1: Approximating HANK

HANK: Equilibrium objects					Implied parameters		
	$\frac{\Omega}{\Omega^*}$	ω	$\frac{\Omega_1^F}{\Omega^*}$	$\frac{\Omega_{20}^F}{\Omega^*}$	χ	λ	$1-s$
Kaplan et al	1.5	.8	—	—	1.48	.41	0 (TANK)
					1.48	**.37**	**.04**
McKay et al	—	—	.8	.4	—	—	0 (TANK)
					.3	.21	.04

Paper: other HANKs (Goremann et al, Debortoli Galí, Hagedorn et al, Auclert et al)

A Common Misconception: "Constant" Euler Wedges?

- No, **not constant:** they are **cyclical** (that is the whole point)
- their *elasticities* (to some *endogenous* variables) are constant around the long-run ergodic steady-state

$$c_t = \underbrace{E_t c_{t+1} - \sigma r_t}_{\text{RANK}} - \underbrace{\sigma \frac{\lambda (\chi - 1)}{1 - \lambda \chi} r_t}_{\text{cyc.-ineq. TANK}} + \underbrace{(\delta - 1) E_t c_{t+1}}_{\text{cyc.-ineq.+risk THANK}} + \underbrace{\eta E_t c_{t+1}}_{\text{(pure) cyc.-risk THANK}}$$

- analytical version of Debortoli Galí decomposition: "between" (TANK) vs "within" (second line)
- last term: later (no cyclical risk yet)

iMPCs in THANK w/ liquidity

- **Auclert Rognlie Straub**; Hagedorn Manovskii Mitman
  - *fiscal policy*
- most compelling critique of **TANK** ... **not of THANK!**
- better still: χ helps match data (Fagereng Holm Natvik)

iMPCs in THANK (w/ liquidity)

- Individual BC w/ liquidity

$$C_t^H + Z_{t+1}^H = \hat{Y}_t^H + R_t B_t^H$$

asset-market equilibrium:

$$Z_{t+1}^H = 0(impatient) \longrightarrow B_{t+1} = (1-\lambda)\, Z_{t+1}^S;$$

$$\begin{aligned} B_{t+1}^H &= \underbrace{(1-h)}_{\left(=\frac{(1-\lambda)(1-s)}{\lambda}\right)} Z_{t+1}^S = \frac{1-h}{1-\lambda} B_{t+1} = \frac{1-s}{\lambda} B_{t+1} \\ B_{t+1}^S &= s Z_{t+1}^S = \frac{s}{1-\lambda} B_{t+1} \end{aligned}$$

- Replace in indiv BCs

$$C_t^S + \frac{1}{1-\lambda} B_{t+1} = \hat{Y}_t^S + \frac{s}{1-\lambda} R_t B_t$$

$$C_t^H = \hat{Y}_t^H + R_t \frac{1-s}{\lambda} B_t$$

iMPCs in THANK (w/ liquidity)

- loglin. indiv. BCs, replace in self-insurance Euler $\rightarrow$ demand for liquidity
- at given income (no govt BC): take partial derivative wrt aggregate income shock, keeping fixed <u>everything</u>
- Special oscillating case: $s = 0$ and $\lambda = \frac{1}{2}$. Asset accumulation eq.

$$b_{t+1} = \frac{\hat{y}_t^S - E_t \hat{y}_{t+1}^H}{2\left(1 + \beta^{-1}\right)} = \frac{1}{2\left(1 + \beta^{-1}\right)} \left[(2 - \chi)\,\hat{y}_t - \chi E_t \hat{y}_{t+1}\right]$$

(dis-)save when expect (higher) lower income tomorrow.

- Consumption function

$$c_t = \frac{2 - \chi + \beta\chi}{2\,(1 + \beta)} \hat{y}_t + \frac{2 - \chi}{2\,(1 + \beta)} \hat{y}_{t-1} + \frac{\beta\chi}{2\,(1 + \beta)} \hat{y}_{t+1}$$

iMPCs in THANK (w/ liquidity)

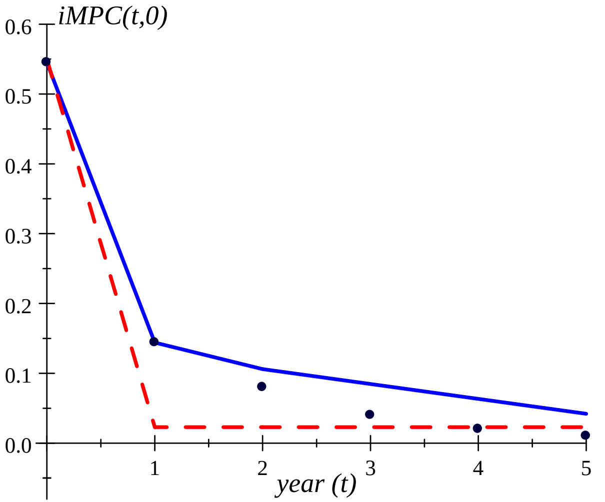
- Proposition: iMPCs in oscillating model $s = 0$

$$\frac{dc_T}{d\hat{y}_T} = \frac{2-\chi+\beta\chi}{2\,(1+\beta)};\ \frac{dc_{T+1}}{d\hat{y}_T} = \frac{2-\chi}{2\,(1+\beta)};\ \frac{dc_{T-1}}{d\hat{y}_T} = \frac{\beta\chi}{2\,(1+\beta)}$$

$\frac{dc_t}{d\hat{y}_T} = 0$ o/w

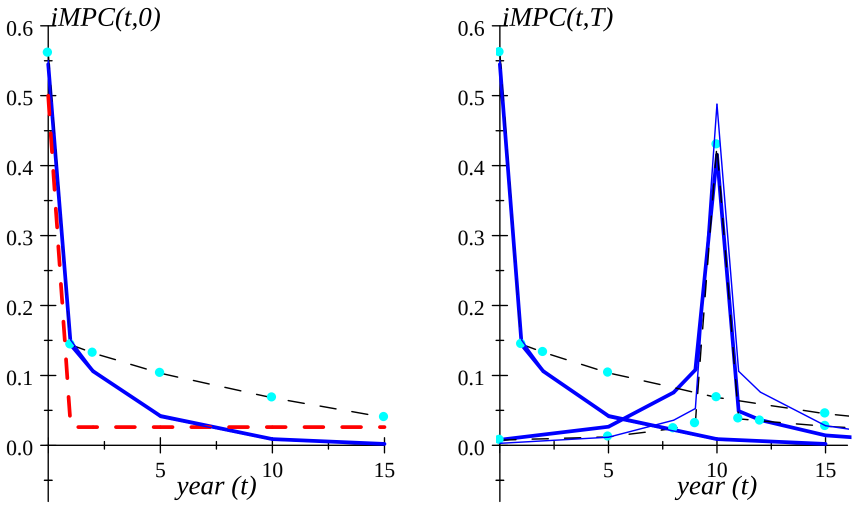
General proposition: paper (closed-form but unwidely)

iMPCs in THANK



iMPCs in THANK (blue solid); TANK (red dash); Data (dots)

iMPCs in THANK



$\chi = 1$ (dot-dash); TANK (red dash); $\chi > 1$ thick and < 1 thin

Recap: THANK as HANK projection

1. Idiosyncratic income uncertainty (variance, skewness, kurtosis)
2. NK Cross, *cyclical inequality* χ
3. Self-insurance, precautionary saving from constraints (extend to prudence later)
4. iK Cross: iMPCs (with liquidity)

Remainder: zero-liq. limit (*passive-Ricardian* FP, zero SS debt)

The 3-Equation THANK Model

$$
\begin{aligned}
c_t &= \delta E_t c_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi}\left(i_t - E_t \pi_{t+1}\right) \\
&: \quad (\text{with } \delta \equiv 1 + (\chi - 1)\frac{1-s}{1-\lambda\chi}) \\
\pi_t &= \kappa c_t + \beta E_t \pi_{t+1} \\
i_t &= \phi \pi_t
\end{aligned}
$$

- (*here* $\pi_t = \kappa c_t$ simple closed forms, paper NKPC)

The 1-Equation THANK Model

$$
\begin{aligned}
c_t &= \delta E_t c_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi}\left(i_t - E_t \pi_{t+1}\right) \\
&: \quad (\text{with } \delta \equiv 1 + (\chi - 1)\frac{1-s}{1-\lambda\chi}) \\
\pi_t &= \kappa c_t \\
i_t &= \phi \pi_t
\end{aligned}
$$

- (*here* $\pi_t = \kappa c_t$ simple closed forms, paper NKPC)

The HANK Taylor Priciple

$$c_t = \frac{\delta + \kappa\sigma\frac{1-\lambda}{1-\lambda\chi}}{1+\phi\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}} E_t c_{t+1} + shocks$$

- $\exists!$ REE (local determinacy) with $\lambda < \chi^{-1}$:

$$\phi > 1 + \frac{\delta - 1}{\kappa\sigma\frac{1-\lambda}{1-\lambda\chi}}.$$

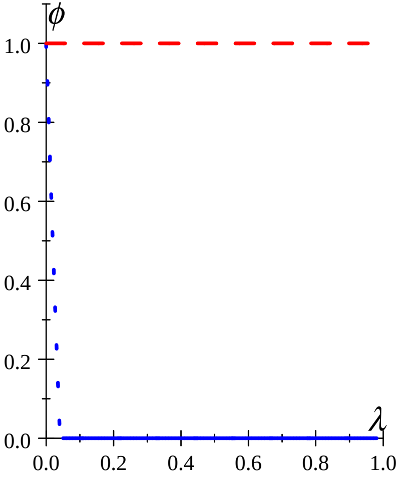
  - **Taylor principle** $\phi > 1$ *sufficient* if:

    $\delta \leq 1 \longrightarrow \chi \leq 1$ ($\Leftrightarrow \Sigma$ (iMPCs) $\gtrless 1$, Auclert Rognlie Straub)

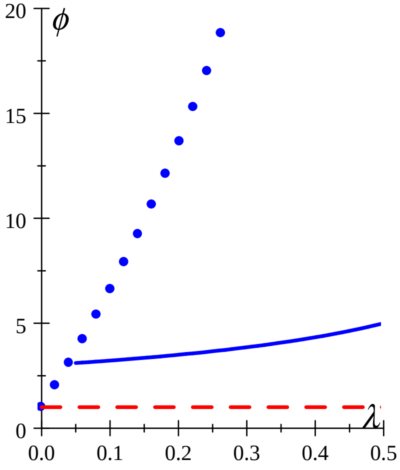
  - subsequently: Acharya Dogra w/ **cyclical (pure) risk**

The HANK Taylor Priciple and Sargent-Wallace

Threshold ϕ: TANK (dash); $s = .96$ (solid); iid: $1 - s = \lambda$ (dots)



PI: $\chi < 1$



CI: $\chi > 1$

Virtues of a Wicksellian PLT Rule in HANK

- Indeterminacy under Taylor pervasive with countercyclical inequality, even more so with countercyclical risk
- **Wicksellian** price-level-targeting: $\exists !$ REE w/

$$i_t = \phi_p p_t \text{ with } \phi_p > 0 \text{ (Woodford \& Giannoni in RANK)}$$

Model:

$$\begin{aligned} c_t &= \nu_0 E_t c_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi}\phi_p p_t; \nu_0 \equiv \delta + \kappa\sigma\frac{1-\lambda}{1-\lambda\chi} \\ \text{PC} &: p_t - p_{t-1} = \kappa c_t. \end{aligned}$$

$$\rightarrow E_t p_{t+1} - \left[1 + \nu_0^{-1}\left(1 + \sigma\frac{1-\lambda}{1-\lambda\chi}\phi_p\kappa\right)\right] p_t + \nu_0^{-1} p_{t-1} = 0$$

  - Intuition: **PID control**–bygones *not* bygones;
- Alternative: Fiscal Quantity-rule policies Hagedorn way

Catch-22: No Puzzle, No Amplification?

1. **HANK Amplification-Multiplier** iff:

$$\chi > 1$$

intuition: *NK Cross*; **paper**: liquidity traps, fiscal multipliers

2. **No-puzzle** iff *HANK-Disc.* > *RANK-Comp.*

$$\nu_0 = \delta + \kappa\sigma \frac{1-\lambda}{1-\lambda\chi} < 1 \longrightarrow \chi << 1$$

Proof: $c_t = \nu_0 E_t c_{t+1} - \sigma \dfrac{1-\lambda}{1-\lambda\chi} i_t^* = \nu_0^{\bar{T}} E_t c_{t+\bar{T}} - \sigma \dfrac{1-\lambda}{1-\lambda\chi} E_t \sum_{j=0}^{\bar{T}-1} \nu_0^j i_{t+j}^*$

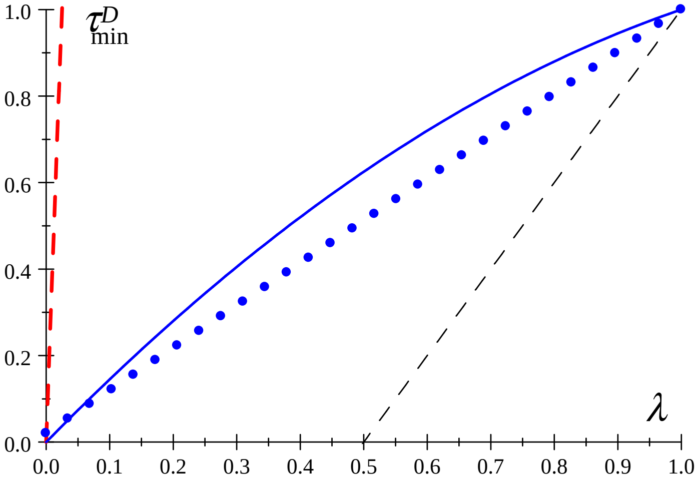
FG Puzzle: Resolved or Aggravated

- **Aggravated** with countercyclical inequality $\chi > 1$
- Also: discounting $\delta < 1$ not sufficient; sufficiency:

$$1 - s > 0 \text{ and } \chi < 1 - \sigma\kappa\frac{1-\lambda}{1-s} < 1$$

- **McKay Nakamura Steinsson**: sufficient conditions for resolving FG puzzle, two special cases
  - analytical, $\chi = 0$, $\delta = s$, iid $s = 1 - \lambda$
  - quantitative: rebate profits uniformly, i.e. disproportionately more to bottom ("poor"), isomorphic to $\tau^D > \lambda$ so $\chi < 1$
- Hagedorn Luo Manovskii Mitman: more quantitative examples of both cases (sticky wages, redistribution, etc.)

No-Puzzle Threshold Redistribution



Redistribution threshold $\tau^D_{\min}$ in TANK $1 - s \to 0$ (dash); $1 - s = 0.04$ (solid); iid HANK $1 - s = \lambda$ (dots).

A Different Cyclical-Risk Channel

- Aggreg. Euler w/ $-s'(Y_{t+1}) \gtrless 0$ ($\Gamma = Y^S/Y^H > 1$):
  Ravn Sterk; Werning; **Acharya Dogra**

$$c_t = (\delta + \eta)\, E_t c_{t+1} - \sigma \frac{1-\lambda}{1-\lambda\chi} r_t$$

$$\eta \equiv \frac{s_Y Y}{1-s}\left(1 - \Gamma^{-1/\sigma}\right)(1-\tilde{s})\,\sigma \frac{1-\lambda}{1-\lambda\chi}$$

- **Similar** equilibrium Euler-discounting/*compounding*
- *Different* "precautionary saving": prudence $\sigma > 0$

Solution to Catch-22? Cyclical Inequality vs Risk

- Yes and No
- *No-Catch-22*: **Amplification without Puzzles** iff

$$\begin{aligned} \text{Countercyclical Inequality} &: \quad \chi \geq 1 \\ \text{Procyclical (enough) Risk} &: \quad \eta < 1 - \delta < 0 \end{aligned}$$

- Flip side: *everything worse* if (pure) **risk countercyclical too**

- **Wicksellian Price-Level Targeting** $i_t = \phi_p p_t$:
  - Amplification, Determinacy & No puzzle[3]

[3]Corrolary: *also* in **RANK**!

Empirical Evidence for Cyclicality of Inequality? Micro

- **Bottomline:** seems countercyclical $\chi > 1$
- Heathcote Storesletten Violante 2010 RED in Y distribution
- Guvenen et al 2004 JPE, etc: U-shaped, skewness, worker betas, etc.– income **risk** (Storesletten Telmer Yaron 2004, Mankiw 1986)
- Cloyne Ferreira Surico 2018 REStud: larger effect for mortgagors, through <u>income</u>
- Lenza Slacalek 2018 – response to MP by Y quantile (key: unemployment)
- Patterson 2019 JMP: first **direct** test, matching MPCs and individual Y cyclicalities
- Alves Kaplan Moll Violante 2019 (fct of permanent Y, Mincerian regressions)
- Slacalek Tristani Violante 2019: constrained (low **liquid** wealth) vs unconstrained

Heathcote, Perri, Violante RED 2010

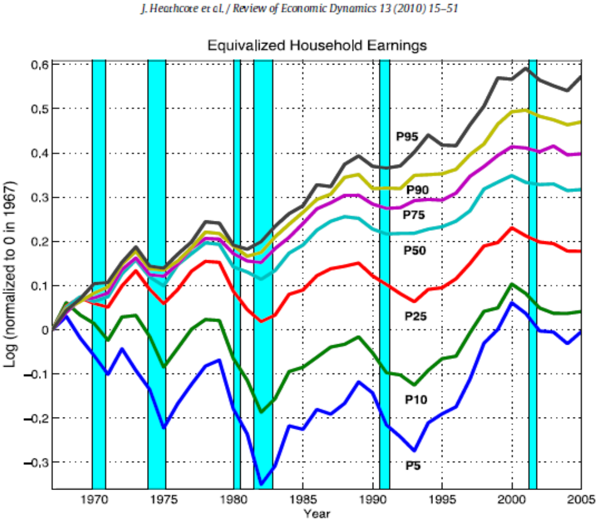


Fig. 9. Percentiles of the household earnings distribution (CPS). Shaded areas are NBER recessions.

Guvenen Ozkan Song 2014 JPE

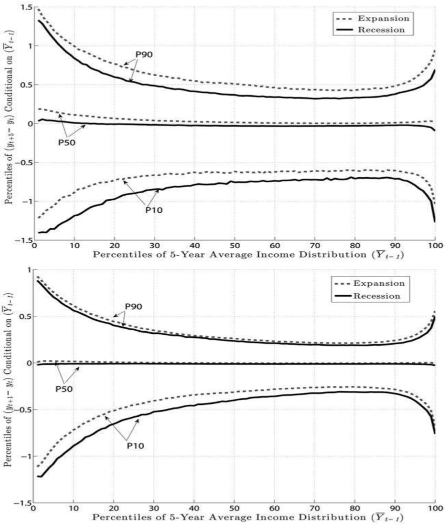
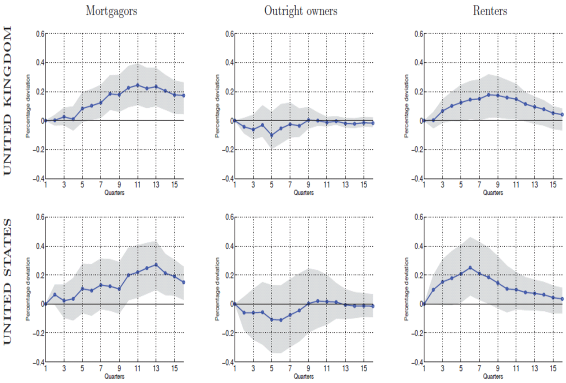


FIG. 7.—Percentiles of the earnings growth distribution: recession versus expansion. *Top*, persistent change. *Bottom*, transitory change.

Cloyne Ferreira Surico 2018 REStud

1. What drives the heterogeneity in consumption?



response to a 25 bps interest rate cut

Source: "Monetary Policy When Households have Debt" (Cloyne, Ferreira, Surico, 2018, ReStud).

Source: P. Surico slides

Cloyne Ferreira Surico 2018 REStud

2A. The indirect effects of MP through income

Table 1: Cumulative changes over four years in US$

Panel A: United Kingdom

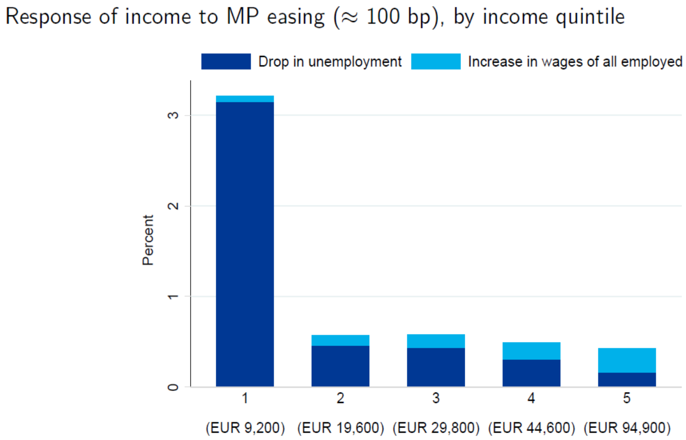
	Non-durable consumption	*Durable expenditure*	*Mortgage or rental payments*	*After-tax income*
Mortgagors	308.3	292.3	-166.4	695.9
	[112.8, 516.1]	[189.2 , 369.0]	[**-272.2** , **-41.7**]	[186.5 , 1105.1]
Outright owners	-62.6	46.5		451.7
	[-148.2 , 77.4]	[-24.6 , 107.6]		[122.5 , 797.2]
Renters	155.3	19.0	64.7	397.3
	[17.9 , 261.8]	[-36.5 , 62.9]	[4.4 , 118.7]	[94.2 , 596.1]

The unequal incidence channel of Bilbiie (2008, 2019): **AMPLIFICATION**

Source: "Monetary Policy When Households have Debt" (Cloyne, Ferreira, Surico, 2018, ReStud).

Source: P. Surico slides

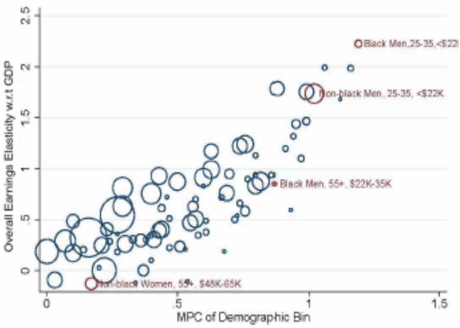
Lenza Slacalek 2018



Patterson 2019

First to do MPCs and "χs"

Figure 1: Recession Exposure and MPC by Demographic Group



Notes: Sample includes the set of all workers employed in a sample state in year $t-1$ from 1995 to 2011. The dependent variable in the regression producing the y-axis estimates is the total change in log earnings for the demographic group. The size of each bubble represents the earnings share of that demographic group. The coefficient on the fitted line for this plot is 1.33. Appendix Figure A14 shows the corresponding figure separately for the intensive and extensive margin of earnings.

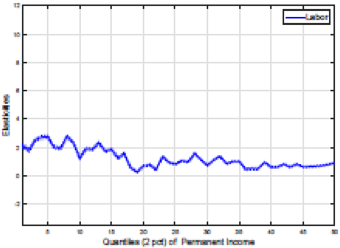
Slacalek Tristani Violante 2020

Table 2: Consumption–Income Ratios, Aggregate Responses to Monetary Policy Shock and Unequal Income Incidence

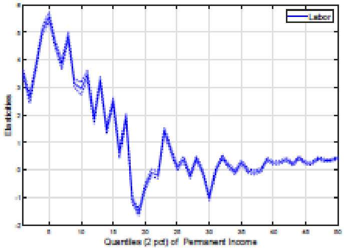
Variable	Germany	Spain	France	Italy
	Country			
Sensitivity of Household Employment to Aggregate Employment by HtM Status				
Poor Hand-to-Mouth	1.7	2.7	1.6	2.1
Wealthy Hand-to-Mouth	0.3	1.6	1.3	1.6
Non Hand-to-Mouth	1.1	0.8	0.9	0.8

See Fig. 15

Alves Kaplan Moll Violante 2019



(a) Elasticity using logarithm



(b) Elasticity using inverse hyperbolic sine

Figure 2: Estimated elasticities of individual earnings to aggregate earnings as a function of permanent income quantile. Dotted lines are the 95% confidence bands. Source: ASEC 1967-2017.

Optimal Monetary (Fiscal?) Policy in THANK

Optimal Policy in THANK

- First-best: perfect insurance. Trivial.
- Ramsey problem

$$\max_{\{C_t^H, C_t^S, N_t^H, N_t^S, \pi_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \{\lambda U\left(C_t^H, N_t^H\right) + (1-\lambda) U\left(C_t^S, N_t^S\right)$$
$$+ \varsigma_{j,t} \Xi_{j,t}$$

$\Xi_{j,t}$: Ramsey constraints (private equilibrium conditions)
$\varsigma_{j,t}$ co-state Lagrange multipliers (with arbitrary initial values: time-0 vs timeless).

Optimal Policy in THANK: Ramsey Constraints

-

$$
\begin{aligned}
\Xi_{1,t} &= \frac{U_N\left(N_t^S\right)}{U_C\left(C_t^S\right)} - \frac{U_N\left(N_t^H\right)}{U_C\left(C_t^H\right)} \\
\Xi_{2,t} &= C_t^H + \frac{U_N(N_t^H)}{U_C(C_t^H)} N_t^H - \frac{\tau^D}{\lambda}\left(1 - \frac{\psi}{2}\pi_t^2 + \frac{U_N(N_t^H)}{U_C(C_t^H)}\right)(\lambda N_t^H + (1-\lambda) N_t^S) \\
\Xi_{3,t} &= \lambda C_t^H + (1-\lambda) C_t^S - (1 - \frac{\psi}{2}\pi_t^2)(\lambda N_t^H + (1-\lambda) N_t^S) \\
\Xi_{4,t} &= \begin{array}{c} \pi_t(1+\pi_t) - \beta E_t\left[\frac{U_C(C_{t+1}^S)}{U_C(C_t^S)} \frac{\lambda N_{t+1}^H + (1-\lambda) N_{t+1}^S}{\lambda N_t^H + (1-\lambda) N_t^S} \pi_{t+1}(1+\pi_{t+1})\right] \\ + \frac{\varepsilon - 1}{\psi}\left[\frac{\varepsilon}{\varepsilon - 1} \frac{U_N(N_t^H)}{U_C(C_t^H)} + 1 + \tau^S\right] \end{array}
\end{aligned}
$$

- Substituted $C_t = \left(1 - \frac{\psi}{2}\pi_t^2\right) Y_t = \left(1 - \frac{\psi}{2}\pi_t^2\right) N_t \ldots$, $W_t = -U_N\left(N_t^S\right)/U_C\left(C_t^S\right) = -U_N\left(N_t^H\right)/U_C\left(C_t^H\right)$, eliminated $D_t = \left(1 - \frac{\psi}{2}\Pi_t^2 - W_t\right)\left(\lambda N_t^H + (1-\lambda) N_t^S\right)$

Optimal Policy in THANK: Ramsey Constraints

- $$\begin{aligned}
\Xi_{1,t} &= \frac{U_N\left(N_t^S\right)}{U_C\left(C_t^S\right)} - \frac{U_N\left(N_t^H\right)}{U_C\left(C_t^H\right)} \\
\Xi_{2,t} &= C_t^H + \frac{U_N(N_t^H)}{U_C(C_t^H)} N_t^H - \frac{\tau^D}{\lambda}\left(1 - \frac{\psi}{2}\pi_t^2 + \frac{U_N(N_t^H)}{U_C(C_t^H)}\right)(\lambda N_t^H + (1-\lambda) N_t^S) \\
\Xi_{3,t} &= \lambda C_t^H + (1-\lambda) C_t^S - (1 - \frac{\psi}{2}\pi_t^2)(\lambda N_t^H + (1-\lambda) N_t^S) \\
\Xi_{4,t} &= \begin{array}{c} \pi_t(1+\pi_t) - \beta E_t\left[\frac{U_C(C_{t+1}^S)}{U_C(C_t^S)} \frac{\lambda N_{t+1}^H + (1-\lambda) N_{t+1}^S}{\lambda N_t^H + (1-\lambda) N_t^S} \pi_{t+1}(1+\pi_{t+1})\right] \\ + \frac{\varepsilon - 1}{\psi}\left[\frac{\varepsilon}{\varepsilon - 1} \frac{U_N(N_t^H)}{U_C(C_t^H)} + 1 + \tau^S\right] \end{array}
\end{aligned}$$
- **Important**: Self-insurance NOT a constraint! ($\sim$ RANK)

$$U_C(C_t^S) = \beta E_t\left[\frac{1+i_t}{1+\pi_{t+1}}\left(s(C_{t+1}) U_C(C_{t+1}^S) + (1 - s(C_{t+1})) U_C(C_{t+1}^H\right.\right.$$

determines i_t residually once we found the allocation

- no longer true in other contexts, i$\longrightarrow$MPC$\longrightarrow$pass-through Y-C

Optimal Policy in THANK

- Optimal long-run inflation rate:

$$\pi^* = 0$$

  - "easy" to show as SS of Ramsey problem (like RANK)
- Approx aggreg. welfare around **first-best, perfect-insurance** y^*

(Woodford 2003 RANK, Bilbiie 2008 TANK)

$$\min_{\{c_t,\pi_t\}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \underbrace{\pi_t^2 + \alpha_y y_t^2}_{\text{RANK}} + \underbrace{\alpha_\gamma \gamma_t^2}_{\text{ineq.-THANK}} \right\},$$

$$\alpha_y \equiv \left(\sigma^{-1} + \varphi\right) / \psi; \quad \alpha_\gamma \equiv \lambda \left(1 - \lambda\right) \sigma^{-1} \varphi^{-1} \alpha_y$$

- note: more general, around target efficient y^*, change constraint – cost-push shocks

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t,$$

Optimal Policy in THANK

- key features: 1. no linear term; 2. recall γ prop. to y

$$\gamma_t = y_t^S - y_t^H = \frac{1-\chi}{1-\lambda} y_t$$

- **result:** risk *irrelevant* (around perf-insurance equil.)
- heterogeneity $\longrightarrow$ *less π stabilization* (key: **profits**)$\rightarrow$*more π volatility* under optimal policy

$$\underline{\text{discretion}}: \pi_t = -\frac{\alpha_y}{\kappa}\left(1 + \frac{\lambda}{1-\lambda}\sigma^{-1}\varphi^{-1}\left(\chi - 1\right)^2\right) y_t$$

  - cyclicality of Γ irrelevant (note *square*) jump
  - survives in quant-HANK: Bhandari Evans Golosov Sargent
- commitment: similar, but price-level targeting eventually
  - side remark: determinacy

Application: Liquidity Traps in THANK

Liquidity Traps with THANK

- use Bilbiie 2016 (*Optimal Forward Guidance*, AEJ-Macro) simple closed-form, FG "state"
  - **first** *closed-form* **optimal policy** in RANK-LT (∼Ramsey) + "simple rule FG" (how long should CB $i = 0$)
- Extend Eggertsson-Woodford to *three states*: P(L→F)= $(1-z)\,q$, P(F→F)= q; E(FG duration)=1/ $(1-q)$

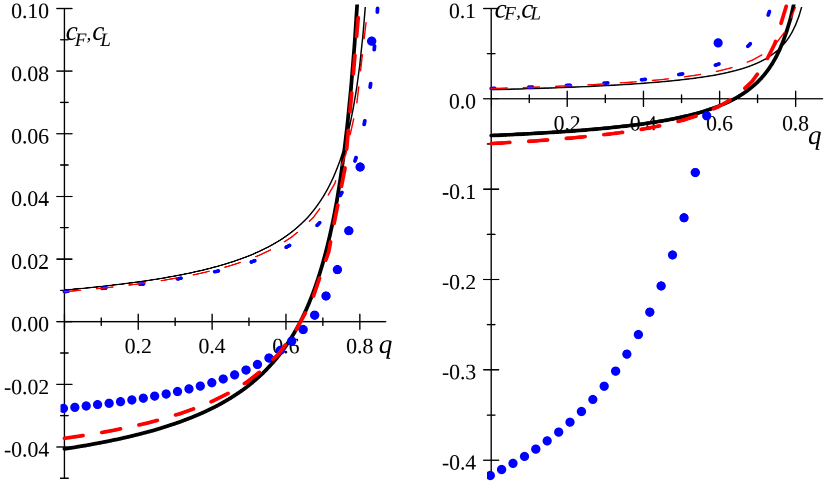
$$E_t c_{t+1} = z c_L + (1-z)\,q c_F + (1-z)\,(1-q)\,0$$

- Equilibrium $dc_F/dq > 0; dc_L/dq > 0$

$$\begin{aligned} c_F &= \frac{1}{1-q\nu_0}\sigma\frac{1-\lambda}{1-\lambda\chi}\rho \\ c_L &= \frac{1-z}{1-z\nu_0}\frac{q\nu_0}{1-q\nu_0}\sigma\frac{1-\lambda}{1-\lambda\chi}\rho + \frac{\sigma}{1-z\nu_0}\frac{1-\lambda}{1-\lambda\chi}\rho_L \end{aligned}$$

FG: Dampening and Amplification

c_L (thick) and c_F (thin): RANK, TANK and iid-HANK



PI: $\chi < 1$

CI: $\chi > 1$

FG Power and Puzzle

- **FG power**:

$$\mathcal{P}_{FG} \equiv \frac{dc_L}{dq} = \left(\frac{1}{1-q\nu_0}\right)^2 \frac{(1-z)\,\nu_0\sigma\frac{1-\lambda}{1-\lambda\chi}}{1-z\nu_0}\rho.$$

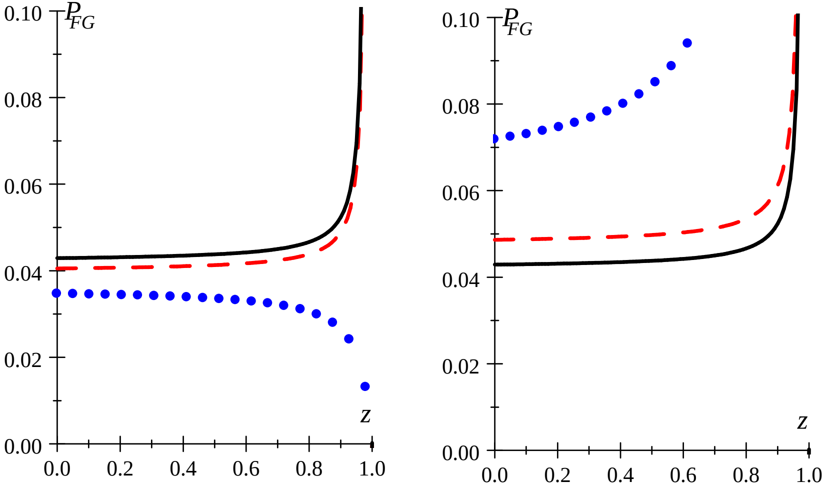
  - $\chi > 1$: $\partial\mathcal{P}_{FG}/\partial\lambda > 0$; $\partial\mathcal{P}_{FG}/\partial\,(1-s) > 0$
- *Corollary*:

$$\textbf{FG puzzle: } \frac{\partial\mathcal{P}_{FG}}{\partial z} \geq 0 \text{ ruled out } \textit{iff}$$

$$\nu_0 < 1.$$

FG Puzzle: Resolution or Aggravation?

RANK, TANK and iid-HANK; $q = 0.5$ $\lambda = 0.1$



PI: $\chi < 1$ CI: $\chi > 1$

Optimal Policy in LT (and the Dark Side of FG Power)

- RANK: Eggertsson Woodford 2003, Jung Teranishi Watanabe 2005, Nakov 2008, Adam Billi 2008, Nakata, Schmidt, Nakata Schmidt, Bilbiie 2016 (analytical)
- E(PDV(Welfare)) w/ Markov chain:

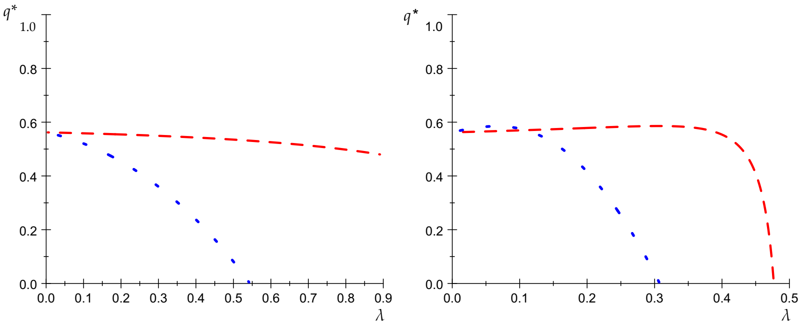
$$W = \frac{1}{1-\beta z}\frac{1}{2}\left[c_L^2 + \omega\left(q\right)c_F^2\right],$$

  - $\omega\left(q\right) = \frac{1-\beta z+\beta(1-z)q}{1-\beta q}$, $\omega'\left(q\right) > 0$: the longer in F, the larger the total welfare cost.
- $\min_q W$ s.t. equilibrium c_F and c_L

$$c_L\frac{dc_L}{dq} + \omega\left(q\right)c_F\frac{dc_F}{dq} + \frac{1}{2}\frac{d\omega\left(q\right)}{dq}c_F^2 = 0$$

- simple case: closed-form q^* (paper)

Optimal FG duration



TANK (red dashed); THANK iid (blue dots)

$q^*(\lambda)$: $\chi < 1$ (left) and $\chi > 1$ (right)

Further Developments and Extensions

1. Money - Liquidity including Optimal Mon. Pol. (w/ Ragot)
2. Capital - (Illiquid) Wealth Inequality (w/ Känzig and Surico)
3. Macro Estimation (w/ Primiceri and Tambalotti)
4. Current work

Convergence

HANK

$\downarrow\uparrow$

$\boxed{\text{THANK}}$

Max(Micro in Macro)

s.t. Tractable

(Duality? probably not yet)

Current Work

- Fiscal Theory and Policy w/ ...
- Optimal Monetary-Fiscal Policy w/ Redistribution (w/ Monacelli and Perotti)
- Secular Stagnation w/ ...
- A Model of χ (cyclical inequality) w/ entry and variety (draws on previous work with Ghironi and Melitz)
- Much more yet to be done

Other ways out

- other ~~puzzles~~ with $\chi > 1$
  - add orthogonal ingredients: deviations from RE, PI, PCC; wealth in U, interest on reserves, etc.
  - **HA**: Hagedorn (2018): a. positive B demand (BIU, HA), b. *choose nominal B*, c. and d. commit to nominal T and to i$\longrightarrow P$
- "Amplification" with $\chi < 1$
  - note: "indirect effect" always there; multipliers with transfers (progressivity increase)
  - wicksellian

Bilbiie (2008 JET) 4 contributions in nutshell

1. TANK analytical⟶income (profits') distribution key for AD&MP

Interest rate changes modify the intertemporal consumption and labor supply profile of *asset holders*, agents who smooth consumption by trading in asset markets. This affects the real wage and hence the demand of agents who have no asset holdings but merely consume their wage income. Variations in the real wage (marginal cost) lead to variations in profits and hence in the dividend income of asset holders. These variations can either reinforce (if participation is not 'too' limited) or *overturn* the initial impact of interest rates on aggregate demand. The latter case occurs if the share of non-asset holders is high enough and/or and the elasticity of labor supply is low enough, for the potential variations in profit income offset the interest rate effects on the demand of asset holders. This is the main mechanism identified by this paper to change dramatically the effects of monetary policy as compared to a standard full-participation case whereby aggregate demand is completely driven by asset holders.

If participation is restricted below a certain threshold, the predictions are **strengthened**: as the share of non-asset holders increases, the link between interest rates and aggregate demand becomes stronger, and monetary policy is more effective; we label this case '*standard aggregate demand logic*', (SADL). However, when participation is restricted beyond a given threshold,

2. aggregate Euler-IS; key $c_j = \varepsilon(\lambda) * y$ (intuition Section 3.1) TANK Intro

Straightforward algebraic manipulation of the equilibrium conditions in Table 1 allows the derivation of aggregate dynamics, similar to the standard, full-participation New Keynesian benchmark. We start by deriving the aggregate Euler equation, or 'IS' curve. To that end, we need to express consumption of asset holders (the only agents whose consumption obeys an Euler equation) in terms of aggregate consumption/output. Since hours of non-asset holders are constant $n_{H,t} = 0$,[13] their consumption tracks real wage, $c_{H,t} = w_t$. Total labor supply (from the labor market clearing condition) is $n_t = [1-\lambda]\, n_{S,t}$. Using these last two expressions, asset holders' labor supply equation, the production function and the goods market clearing condition into the definition of total consumption we find:

$$c_{S,t} = \delta y_t + (1+\mu)(1-\delta)\, a_t, \quad \text{where } \delta \equiv 1 - \varphi \frac{\lambda}{1-\lambda} \frac{1}{1+\mu}. \tag{7}$$

Substituting (7) into the Euler equation of asset holders we find the *aggregate Euler equation*, or 'IS curve':

$$y_t = E_t y_{t+1} - \delta^{-1} [r_t - E_t \pi_{t+1}] + (1+\mu)\left(1-\delta^{-1}\right)[a_t - E_t a_{t+1}]. \tag{8}$$

Direct inspection of (8) suggests the impact that LAMP has on the dynamics of a standard business cycle model through modifying the elasticity of aggregate demand to real interest rates

$$w_t = \chi y_t - \varphi a_t, \quad \text{where } \chi \equiv 1 + \varphi/(1+\mu) \geqslant 1 \geqslant \delta.$$

3.1. *Intuition and the labor market*

How can an increase in interest rates become expansionary when asset market participation is restricted enough? To answer this question, it is useful to conduct a simple mental experiment whereby the monetary authority pursues a one-time discretionary increase in the interest rate ε_t, otherwise pursuing a policy that fully accommodates inflationary expectations, namely $r_t = E_t \pi_{t+1} + \varepsilon_t$. In the standard, full-participation economy, an increase in interest rates leads to a fall in aggregate demand today. Asset holders are also willing to work more at a given real wage (labor supply shifts rightward), but labor demand shifts left because of sticky prices (not all the fall in demand can be accommodated via cutting prices). The new equilibrium is one with lower output, consumption, hours and real wage. Suppose now that we are in an economy with limited participation, but $\lambda < \lambda^*$ either because participation is not restricted 'enough' or labor supply is not inelastic enough. The fall in real wage brought about by the intertemporal substitution of asset holders now means a further fall in demand, since non-asset holders merely consume their wage income. This generates a further shift in labor demand, so the new equilibrium is one with even lower (compared to the full-participation one) output, consumption, hours and real wage.

This effect could at first sight seem monotonic over the whole domain of λ: the more restricted asset market participation, the stronger the contractionary effect on demand and hence on labor demand, and hence the more effective monetary policy. In order to understand why this is not the case, it is helpful to consider the additional distributional dimension introduced by limited asset market participation. The further demand effect that occurs because of non-asset holders has an effect on profits: both marginal cost (wage) and sales (output and hours) fall. The relative

size of these reductions (and the final effect on profits) depends on the relative mass of non-asset holders and on labor supply elasticity. In particular, if labor supply is inelastic enough

Note that such a wage–hours locus implies that the model generates a higher partial elasticity of hours to the real wage, and more so more negative δ is. Importantly, despite the potential decrease, in general equilibrium *actual* profits may not fall, precisely due to the negative income effect making asset holders willing to work more; for as a result of this effect hours will increase by more and marginal cost by less, preventing actual profits from falling. In fact, for certain combinations of parameters, shocks or policies our model would *not imply countercyclical profits* in equilibrium (or at least implies more procyclical profits than a standard full-participation model with countercyclical markups). This is an important point, since it is widely believed that profits are procyclical.[23] It is also important to note that the negative income effect does not mean that

[23] See Section 7 of the working paper version [7] for a detailed discussion

Bilbiie (2008 JET) 4 contributions in nutshell - cont'd

3. fiscal redistribution key for AD amplification of MP; AD elasticity:

$$c_{S,t} = \delta_\tau y_t, \quad \text{where } \delta_\tau = \frac{1}{1-\tau^D}\left[1 - \tau^D \frac{\mu}{1+\mu} + \frac{\tau^D - \lambda}{1-\lambda}\frac{\varphi}{1+\mu}\right].$$

4.3. Redistribution restores Keynesian logic

The mechanism of all the previous results relies on the interaction between labor and asset markets, namely income effects on labor supply of asset holders from the return on shares. This hints to an obvious way to restore Keynesian logic relying on a specific fiscal policy rule that shuts off this channel: tax dividend income and redistribute proceedings as transfers to non-asset holders. We focus on the IADL case whereby in the absence of fiscal policy $\delta < 0$. To make this point, consider the following simplified fiscal rule: profits are taxed at rate τ_t^D and the budget is balanced period-by-period, with total tax income $\tau_t^D D_t$ being distributed lump-sum to all non-asset holders. We focus on the case where profits are zero in steady state. The balanced-budget rule then is $\tau_t^D D_t = \lambda L_{H,t}$ which around the steady state (both profits and transfers are shares of

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Proposition 4. *If the steady state of the model in Section 3 is* ***efficient*** *the aggregate welfare function can be approximated by (ignoring terms independent of policy and terms of order higher than 2):*

$$\mathbf{U}_t = -\frac{U_C C}{2}\frac{\varepsilon}{\psi}E_t\sum_{i=t}^{\infty}\left\{\alpha x_{t+i}^2 + \pi_{t+i}^2\right\}, \tag{18}$$

$$\alpha = \frac{\varphi+\gamma}{1-\lambda}\left[1-\lambda(1-\gamma)(1+\varphi)\right]\frac{\psi}{\varepsilon}.$$