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On the Faustian Dynamics of Policy and Political Power

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This paper examines the *Faustian dynamics* of policy and power. We posit a general class of dynamic games in which current policies affect the future distribution of political power, resulting in the following "Faustian trade-off": if the current ruler chooses his preferred policy, he then sacrifices future political power; yet if he wants to preserve his future power, he must sacrifice his present policy objectives. The trade-off comes from the fact that the current political ruler/pivotal voter cannot uncouple the direct effect of his policy from its indirect effect on future power. A *policy-endogenous (PE) equilibrium* describes this endogenous transfer of power and the resulting evolution of policy and political power over time. We show that the Faustian trade-off in a PE equilibrium is decomposed into two basic rationales. The *political preservation effect* induces more tempered policy choices than if one's policy choice did not affect one's political fortunes. However, the *reformation effect* induces "more aggressive" policies in order to exploit the productivity gains from policies chosen by even more aggressive successors. We distinguish between political systems that give rise to *monotone Faustian dynamics*—political power that progressively evolves towards more fiscally liberal types of leaders—and *cyclical Faustian dynamics*—political power that oscillates between liberal and conservative types of leaders. In each case, we show that the Faustian trade-off moderates the choices of each type of leader.

Keywords: Monotone and cyclical Faustian dynamics, policy-endogenous equilibrium, permanent authority, preservation and reformation effects, distortion-adjusted Euler equation.

JEL Codes: C73, C61, D72, H11

1. INTRODUCTION

In Goethe's *Faust*, a well-meaning Faust seeks knowledge, truth, and beauty but cannot achieve them on his own. The devil appears and strikes a bargain with Faust: the devil will serve Faust while Faust remains here on earth; in exchange Faust must serve the devil in hell. But there is a catch. As part of the agreement, if Faust is so happy with the devil's services that he wants to "freeze" the present moment forever, Faust must then die immediately. Hence, the bargain endows Faust with the power to reach his objective but denies him the ability to savour it.[1]

So it is, quite often, with "political bargains". If a political leader chooses a desirable but unpopular policy, he may lose political power and thus the ability to shape policy in the future.

1. While there are many versions of the Faustian bargain, the most well known is rendered by Goethe (1932). See also www.openlibrary.org/details/faustgoethe00goetiala.

By opting to stay in power, he sacrifices his policy objectives and then faces the same trade-off in the future. Hence, the political bargain endows a leader with the power to determine policy only as long as he does not choose the one he prefers.

These types of "Faustian trade-offs" are not uncommon in politics. In one prominent example, after signing the Civil Rights Act in 1964, President Lyndon Johnson remarked to an aide, "We have just lost the South for a generation". It proved to be a fairly accurate forecast of Democratic losses to come.[2]

This paper examines a dynamic game-theoretic model of this "Faustian trade-off" between policy and power. To highlight the "tragic" nature of this trade-off, we do not presume that rulers crave power for its own sake. Instead, political actors are assumed to be purely policy motivated. Their concern about losing power arises only because the new decision makers have different policy objectives than their own.

We model an ongoing society inhabited by a continuum of infinitely lived citizens. At each date t, one of the citizens, whom we refer to as the *leader*, has effective authority to choose a policy that affects all the citizens in society. The "leader" can be viewed as an elected ruler or, alternatively, as a pivotal voter whose preferences are decisive in determining a policy. In either case, the policy choice of the leader in date t can change political power in a way that ultimately changes the identity of the leader in date $t+1$. This can be done through policies that affect the underlying demographic and/or distributional characteristics of the population. For instance, an increase in a country's income tax changes the future distribution of income. In turn, this may bring about electoral changes in future political power.

Because future political power is endogenously driven by current policy change, we refer to this as a case of *policy-endogenous (PE) political power*. Under PE power, the dynamics induce a feedback loop from policy to power and back to policy. We characterize the *PE equilibria*—smooth Markov perfect equilibria in PE regimes.[3] As a benchmark, PE equilibrium paths of policy and power are compared to those resulting from *permanent authority (PA) equilibria*—equilibria under which political power is permanent.

In the PE equilibrium, a political leader faces a *Faustian trade-off* when his most preferred policy strips him of power and then places it in the hands of a less desirable ruler. A central theme to emerge from the study is that Faustian trade-offs tend to turn political decision makers into "Burkean conservatives". That is, when facing a Faustian trade-off, a leader must overcompensate for the potential loss of power by slowing the rate of political evolution as dictated by his current policy choice. This Burkean effect is compounded by the indirect effect one's decision has on the policies of future leaders: a fiscal conservative whose preferred policy might lead to an electoral loss to a moderate should worry that the moderate's choice might lead to an electoral loss to a fiscal liberal in the longer term. The result is that political shifts and policy changes are more gradual than they would be if these sorts of trade-offs went unrecognized.

While an abundant literature in political economy studies the link from political power to policy, less is known about the "reverse causal link", *i.e.*, from policy to power. One reason for this is that much of the "first-generation" political economy literature studied "undistorted" political mechanisms combining standard majority voting with order-preserving policy changes in population distribution.[4] In these models, an individual who, for instance, is richer than another

2. See http://en.wikipedia.org/wiki/Lyndon_B._Johnson. We thank a referee for pointing out the reference. See also Black and Black (2002) for a detailed account.

3. The restriction to Markov perfection is fairly common in dynamic political games (see, for instance, Battaglini and Coate (2007, 2008)). There is some justification for this, which is detailed in Section 2.

4. See, for example, political economy models of growth and taxation such as Bertola (1993), Alesina and Rodrik (1994), Krusell, Quadrini, and Ríos-Rull (1996, 1997), and Krusell and Ríos-Rull (1999). For a detailed review of papers in this tradition, see Krusell, Quadrini, and Ríos-Rull (1997) and Persson and Tabellini (2000).

today is still expected to be richer after the policy change goes into effect. Consequently, a fixed median voter emerges in equilibrium, and so no change in political power occurs.

The traditional emphasis on "undistorted" systems is a natural starting point. Yet, biases that produce PE distortions have, historically, been the rule rather than the exception. Until the late 19th century, most governments explicitly weighted votes by some form of wealth or property value. In democracies today, the bias is less formulaic but no less real. Representation in the U.S. Senate, for example, is biased in favour of less populous states, hence towards characteristics of rural rather than urban voters. Small minority parties in parliamentary systems often have disproportionate influence in the formation of majority governments. In addition, recent studies by Benabou (2000), Campante (2007), and Bartels (2008) all document the wealth bias implicit in the U.S. political system.

By themselves, these biases may not create a Faustian trade-off. However, policy choices typically have distributional effects—changes in, say, income inequality—and we show that these effects can create a Faustian trade-off when coupled with the political bias.

Something akin to a Faustian trade-off arises in some recent studies of endogenous electoral outcomes. These include Besley and Coate (1998), Bourguignon and Verdier (2000), Hassler et al. (2003, 2005), Dolmas and Huffman (2004), Ortega (2005), Azzimonti (2005), Campante (2007), and Acemoglu and Robinson (2008). Many of these focus on ways in which particular political mechanisms such as Downsian competition affect future voter preferences. These and other related papers are discussed in more detail in Section 5. We are not aware of a systemic study that identifies common features of the Faustian trade-off (including the longer-term indirect effects mentioned above) across a large sweep of environments and political systems. This paper builds on the recent literature by working towards that end.

We develop the model in two stages. First, we posit a stylized model of public investment in which the level of investment augments a public capital good such as infrastructure or education. We then extend the results to a general (non-parametric) model. Each model features a "detail-free" mechanism by which the policy–power link is specified in reduced form as a function from population characteristics to the type or identity of the leader. This mechanism is later "endogenized" by showing that any member of the class of reduced-form functions considered here can be generated by a majority voting rule in which votes are weighted by wealth or income.

We analyse both transition dynamics and steady-state properties of the model. In the stylized model, the Faustian trade-off moderates each leader's incentives to invest in public capital. For instance, when public capital is below its equilibrium steady-state level, each leader chooses a lower level of government investment than he would if his authority were permanent. Likewise, starting above the steady-state investment, a leader chooses a larger investment than if his power were permanent. As a consequence, public capital initially changes more slowly than it would in the absence of a Faustian trade-off.

Yet, despite the moderating effect on individual incentives, political power does evolve over time. We distinguish between two cases: monotone and cyclical dynamics. Monotone dynamics arise in polities with a *reinforcing distortion*—a distortion that reinforces the natural pattern of capital accumulation. In this case, it means that an increase in public capital moves political power progressively towards more fiscally liberal types who prefer larger increases in government spending. In turn, this leads to even larger increases in spending in the future. Viewed in this way, the virtually uninterrupted increase in overall U.S. government expenditures after World War II can be interpreted as both a cause and a consequence of a gradual political shift in preferences towards a greater role for government.[5]

5. See Peltzman (1980), Husted and Kenny (1997), and Lott and Kenny (1999).

By contrast, cyclical dynamics arise *only* in polities with *countervailing distortion*—a distortion that runs counter to the pattern of capital accumulation. Increases (decreases) in public capital move political power towards more fiscally conservative (liberal) leaders. This results in oscillations between liberal and conservative types. Each type's policy moves power towards the other side of the political spectrum. A natural example of this is immigration policy. Ortega (2005) presents a calibrated model of skill complementarities with a countervailing distortion. By allowing in immigrants with complementary skills, voters from one skill group endogenously increase the political opposition since the immigrants will have opposing political preferences regarding the composition of immigration in the future.

We also compare two political institutions of differing degrees of the same type of distortion. The result is somewhat surprising. Using the monotone case to illustrate, we show that there exists a cut-off state above which the more distorted polity yields more liberal leaders and below which it yields more conservative leaders. In other words, the more distorted polity yields a more gradual evolution of power in the short run, and a less gradual evolution in the long run. The short run–long run distinction is significant because it indicates that there are critical features in the transition dynamics of the Faustian model that would not be evident by focusing only on its steady-state properties.

The monotone results are generalized in the non-parametric model. When certain supermodularity restrictions hold, the main results characterize PE equilibria in terms of a "distortion-adjusted" Euler equation in which a leader's motives may be decomposed into the following two rationales.

First, the "political preservation effect" turns all leaders into Burkean conservatives regardless of political preference. That is, each leader chooses more moderately/less aggressively than he would if he did not face a Faustian trade-off. Roughly, the political preservation effect represents the distortion in the current leader's incentives due to the effect his policy choice has on the identities of future leaders. Because policy preferences across individuals differ, each distinct leader determines a distinct policy rule. Hence, a change in current policy alters the trajectory of policy *rules* (rather than just the policies). The larger the wedge between current and future leaders' preferences, the greater the distortion. As a result, today's leader slows the rate of political change with his policy choice. We show that the preservation effect on a given individual's incentives increases in magnitude through time.

Yet, the preservation effect is partly offset by a second rationale, the "reformation effect", which represents the distortion in the current leader's incentives due to the effect his policy choice has on policies (rather than policy *rules*) chosen in the subsequent period. The reformation effect isolates the effect of current policy on next period's productivity by ignoring the changes to future policy rules themselves. Looking just at productivity, a more aggressive policy choice in the subsequent period increases the marginal productivity of policies in the present. In short, the reformation effect pushes the current leader towards a more aggressive policy choice.

While the preservation and reformation effects refer to distorted incentives of a *fixed* decision maker, it is important to remember that decision authority changes hands over time. The overall effect is that power evolves towards more progressive leaders, and, in fact, the policy trajectory is more progressive than that under PA in the long run. The steady-state stock is larger and the leader more progressive than in a case of no Faustian trade-offs.

The general model is introduced in Section 2. Section 3 elaborates on the parametric model. The model displays some of the salient features of political systems that give rise to PE power. Section 4 returns to the abstract model and contains the main decomposition result. Section 5 examines the related literature and examples. Finally, Section 6 discusses various extensions. The proofs are contained in an Appendix at the end.

2. THE BASIC SETUP

In this section, we describe a general model in which Faustian trade-offs occur. The level of generality underscores the fact that PE political change is not necessarily an isolated feature of a small set of environments. However, for concreteness a special case of the general model is presented in Section 3 in the form of a stylized model of public sector investment and growth.

2.1. *The Environment*

At each date $t = 0, 1, 2, \ldots,$ a government must undertake a policy decision that affects all members of society. The policy interacts with a state variable that fully summarizes the economy at that date. Let a_t denote the policy choice and ω_t the state. Feasible policies and states are restricted to compact intervals.

The state is assumed to evolve according to a deterministic Markov process determining the future state as a function of current states and actions. The transition function Q is assumed smooth, non-decreasing in ω_t, increasing in a_t, and jointly concave in the pair ω_t and a_t.

Society comprises continuum $I = [0, 1]$ of infinitely lived *citizen types*. Given any sequence of states $\{\omega_t\}$ and policies $\{a_t\}$, the dynamic pay-off to citizen type $i \in I$ is

$$\sum_{t=0}^{\infty} \delta^t u(i, \omega_t, a_t), \tag{1}$$

where δ is a common discount factor and the pay-off function u is smooth, increasing in ω_t, decreasing and strictly concave in a_t, and jointly concave in the pair ω_t and a_t.

The assumptions on u and Q reflect the idea that the policy a_t is a tax or public investment that, while costly in the present, augments the future value of the state. In turn, the state ω_t is a parameter that determines a capital stock or an income distribution. An obvious example is an income tax that generates revenue to fund public infrastructure.

2.2. *The Permanent Authority Benchmark*

Consider a benchmark case of a decision maker who faces no Faustian trade-off. There are a few ways this can happen. For instance, if all individuals have identical preferences over policy, then a purely policy-motivated type is indifferent between retaining and losing political power. Alternatively, an individual can lose power for purely exogenous reasons unrelated to his current policy choice.

The most natural benchmark, however, is that of an individual who maintains political power regardless of his policy action. This "king" or "dictator", whom we label i_0, chooses a policy a_t at each date t, fully anticipating that his authority is perpetual. We refer to this as the *PA* regime. PA regimes are not, almost by definition, common in modern democracies. They were common, however, in many European monarchies prior to the 20th century.

The PA regime can also be interpreted normatively as a time-consistent social planner. In our model, there is no qualitative difference between the PA of a fixed citizen type and the authority of a social planner who aggregates pay-offs across citizen types. Significantly, most political economy models either assume explicitly a PA regime (*e.g.*, a social planner) or derive one in equilibrium under special assumptions on preferences, technology, and political institutions.

Consider the problem of a PA i_0. His policy choices are characterized by a policy function $\psi(\omega_t) = a_t$ (omitting the notational dependence on i_0) that solves the Bellman equation

$$V(i_0,\omega_t;\psi) \;=\; \max_{a_t}\;\big[u(i_0,\omega_t,a_t)\;+\;\delta\,V(i_0,\omega_{t+1};\psi)\big] \tag{2}$$

subject to $\omega_{t+1}=Q(\omega_t,a_t)$. We refer to the function ψ that solves equation (2) as a *PA* equilibrium. The PA equilibrium serves as a baseline for comparison.[6]

2.3. *Policy-Endogenous Political Power*

We compare the PA regime in which there are no Faustian trade-offs to one in which there are. In an environment with *PE political power*, the current policy choices induce changes in future political power. To focus on decision-theoretic aspects, political power is modelled in reduced form by assuming that each period the political system produces an outcome that is rationalized by the preferences of a pivotal individual. This individual (*e.g.*, pivotal voter or political leader) is, in effect, endowed with the exclusive right in period t to choose the policy action a_t. The assumption that the political process admits a pivotal leader clearly involves some loss of generality. There are, however, commonly known conditions on policy preferences, notably the class of intermediate preferences (Grandmont, 1978), and preferences satisfying single crossing properties (Gans and Smart, 1996; Rothstein, 1990) that do admit pivotal voters.

Henceforth, we refer to the pivotal decision maker as the *leader*. Political power is therefore represented by a mapping from states (*e.g.*, capital stocks, income distributions) to citizen types. Formally, the mapping is assumed to be a smooth, weakly monotone function $\mu:\omega\mapsto i$ such that $i_t=\mu(\omega_t)$ is the leader who decides on policy in state ω_t.

Because μ determines “who is in charge” in each state, we refer to it as the *authority function*. The key attribute of an authority function, for our purposes, is that it admits the possibility that political power changes endogenously due to changes in the state. Current policy changes produce changes in the state that, in turn, produce changes in the identity of the leader through μ.

To facilitate a comparison with the PA regime, we fix i_0 as both the PA and the *initial* decision maker under PE power. The change in the identity of the leader, as described by μ, defines a dynamic game with a potentially infinite set of players. The players’ choices result in a stationary Markov process that realizes states $\{\omega_0,\omega_1,\omega_2,\ldots\}$, leaders $\{i_0,i_1,i_2,\ldots\}$, and policies $\{a_0,a_1,a_2,\ldots\}$. We refer to these processes as the *Faustian dynamics* of PE political power.

In much of the paper, μ is treated as exogenous, although we give an explicit micro foundation for it in Section 3. In the simplest case of majority voting, $\mu(\omega_t)$ is the median type whenever the median voter theorem holds. However, in order to obtain a state-dependent authority function μ, votes in some cases may need to be weighted by, say, wealth or income. The idea, roughly, is that changes in the state “distort” the income distribution, and wealth-weighted voting is sensitive to these types of distortions—see Section 3 for an explicit description of such a voting mechanism.

This sensitivity of voting to changes in the wealth distribution could be explicitly built into the polity as in the case of the U.S. Senate or in Germany in the 19th century. In other cases, the sensitivity is implicit such as when a citizen’s political influence depends on his wealth.[7]

6. Implicitly, the PA equilibrium characterizes a time-consistent optimal strategy for i_0 if the private sector’s dynamic response is Markov or is absent altogether. In the case where the private sector is absent, the PA equilibrium amounts to a single-agent dynamic programming problem, in which case it coincides with the full commitment solution.

7. Polities that weight votes by wealth, at least implicitly, are not exotic. Benabou (2000), Bartels (2008), and Campante (2007) all provide evidence of bias towards the affluent in the U.S. electoral politics. They show that campaign contributions (Campante) or differential turnout rates (Benabou, Bartels) produce roughly the same effect as if the votes were weighted by wealth.

Hence, policy changes such as tax cuts may change the identity of the pivotal voter in a heterogeneous economy if campaign contributions affect the outcome of an election. Alternatively, changes in immigration laws, education levels, or fertility policy can have (longer-run) electoral effects even under ordinary median voter rules since the demographic changes themselves may be biased towards the participation of certain groups or social classes. For example, Tichenor (2002) describes 19th-century immigration policy in the U.S., sometimes restrictive but more often expansive, that ultimately brought about large political shifts towards urban regions that came to be reflected in congressional and presidential elections.

Under PE power generally, the leader $i_t = \mu(\omega_t)$ in period t chooses an action a_t anticipating the effect that it has on the future trajectory of states, leaders, and policies. A *PE equilibrium* is defined as a Markov perfect equilibrium in the PE environment. More precisely, a PE equilibrium, denoted by Ψ^*, is a Markov policy function (with $\Psi^*(\omega_t) = a_t$) such that Ψ^* is a best response by citizen type i against any *history-contingent* strategy that differs from Ψ^* only in states ω for which $\mu(\omega) = i$.[8] Under Ψ^*, the continuation pay-off to a citizen type i is given by

$$V(i, \omega_t; \Psi^*) \;=\; \sum_{\tau=t}^{\infty} \delta^{\tau-t}\, u(i, \omega_\tau, \Psi^*(\omega_\tau)) \tag{3}$$

given the sequence $\{\omega_t\}$ induced by transition function Q. A standard argument shows that the so-called *one-shot deviation principle* applies to PE equilibria. Namely, Ψ^* is a PE equilibrium if and only if for all ω_t and for all a_t,

$$V(\mu(\omega_t), \omega_t; \Psi^*) \;\geq\; u(\mu(\omega_t), \omega_t, a_t) \;+\; \delta\, V(\mu(\omega_t), Q(\omega_t, a_t); \Psi^*). \tag{4}$$

Clearly, there are other types of subgame perfect equilibria one might examine in a Faustian model. However, we think that the restriction to Markov perfection is sensible in this context. First, it facilitates a direct comparison with the PA equilibrium in equation (2). Second, the restriction seems natural in large populations. Costs of coordination are presumably higher in larger economies, and so strategies that therefore depend only on the current, pay-off relevant state may reduce these costs. Third, if policy makers have uniformly bounded recall, then it can be shown that *any* subgame perfect equilibrium in our model *must* be Markov.[9]

Given a PE equilibrium Ψ^*, consider what policy *would have been chosen* in state ω_t by an arbitrary citizen type i? This question is hypothetical because an arbitrary i is not the authority in state ω_t unless it happens that $i = i_t = \mu(\omega_t)$. The question is important nevertheless because it allows one to compare the Faustian incentives for different political types in any situation. Let $a_t = \Psi(i, \omega_t)$ denote the hypothetical decision of type i. Call Ψ a *hypothetical equilibrium* if for all ω_t, all i, and all a_t,

$$u(i, \omega_t, \Psi(i, \omega_t)) \;+\; \delta\, V(i,\; Q(\omega_t, \Psi(i, \omega_t)); \Psi^*) \;\geq\; u(i, \omega_t, a_t) \;+\; \delta\, V(i, Q(\omega_t, a_t); \Psi^*). \tag{5}$$

The inequality in (5) coincides with (4) in those states for which $i = \mu(\omega_t)$. The hypothetical equilibrium and the PE equilibrium are therefore related by $\Psi(\mu(\omega_t), \omega_t) \;=\; \Psi^*(\omega_t)$.

8. To be clear, it should be noted that Markov perfect equilibria can be alternatively described as subgame perfect equilibria in Markov strategies. This means that *feasible strategies and pay-offs, fully described, may possibly vary with the entire history*. For brevity, however, we omit the full description of such strategies and pay-offs and limit our description to pay-offs evaluated by equilibrium (Markov) strategies.

9. An economy has uniformly bounded recall if there is a finite bound m such that every decision maker's memory of the past history cannot exceed m periods back. The argument for the stated assertion relies on asynchronous decision making, which applies to the present model. A simple proof is found in Bhaskar and Vega-Redondo (2002).

Hence, starting from an initial state ω_0, the decision maker i_0 chooses $\Psi(i_0,\omega_0) \equiv \Psi^*(\omega_0)$. Type i_0 correctly anticipates that his chosen policy a_0 leads to a possible change in decision authority at date $t = 1$. This change is given by $i_1 = \mu(\omega_1)$, where the next period's state ω_1 is determined by $\omega_1 = Q(\omega_0, \Psi^*(\omega_0))$. It is generally true that $\Psi(i_0,\omega_1) \neq \Psi^*(\omega_1)$ because i_0 no longer makes the decision in state ω_1 in the PE equilibrium, and the decision type i_1 will generally have different preferences over policy. The more interesting comparison, however, is between $\Psi(i_0,\omega_1)$ and $\psi(\omega_1)$. It tells us how an arbitrary citizen type would react to the loss of power in a given state, where the extent of the loss is determined by that state.

In certain instances, the current leader faces no Faustian trade-off even when μ is distortionary. For instance, suppose that period pay-offs are of the form $u(i,\omega,a) = u_1(i) + u_2(\omega,a)$ or the form $u_1(i)u_2(\omega,a)$. In either case, the additive or multiplicative separability implies that individual-specific characteristics do not affect policy preferences. Individuals' policy preferences are therefore the same, and so changes in power have no effect on policy.

3. A STYLIZED MODEL OF PUBLIC SECTOR INVESTMENT

This section examines how Faustian dynamics work in a concrete, special case of the general model. We examine a stylized model of public sector investment where political authority is derived explicitly from a system of weighted voting. We examine two cases: one where the public sector expands monotonically as political power steadily evolves towards more fiscally liberal types, and the other where public sector alternately expands and contracts and political power oscillates between fiscally liberal and conservative types of decision makers. We use the model to illustrate effects of Faustian dynamics on the evolution of political power, public sector growth, and income inequality.

3.1. *The Environment*

Society invests each period in a public capital good (infrastructure) that is produced according to the linear transition

$$\omega_{t+1} = Q(\omega_t, a_t) = (1-d)\omega_t + a_t. \tag{6}$$

Here, ω_t is the current stock of the public capital, a_t is the public investment, and $d \in (0,1]$ is the depreciation rate. Public investment a_t is produced from a lump-sum tax $\mathcal{T}_t$, according to a concave production technology $a_t = (2\mathcal{T}_t)^{1/2}$. This technology can be alternatively expressed as the cost $\mathcal{T}_t = \frac{1}{2}a_t^2$ of providing public investment a_t.

Each citizen is assumed to provide labour inelastically with a time allocation normalized to one. His labour is combined with public capital to produce income

$$y(i,\omega_t) = g(i) + f(i)\omega_t$$

with $g(i)$ and $f(i)$ denoting type i's efficiency utilization of labour and capital, respectively. We assume that $f' \geq 0$, so that higher types have (weakly) higher efficiency utilization of public capital.

A citizen's flow pay-off is assumed to be linear in his consumption, and we assume (without loss of generality, due to the linear pay-off) that there is no private borrowing or saving. A citizen therefore consumes his after-tax income $y(i,\omega_t) - \mathcal{T}_t$. His flow pay-off is then given by

$$u = y(i,\omega_t) - \mathcal{T}_t = g(i) + f(i)\omega_t - \frac{a^2}{2}.$$

Dropping the labour efficiency term $g(i)$ yields an indirect utility function of the form in (1), given by

$$u(i, \omega_t, a_t) = f(i)\omega_t - \frac{a_t^2}{2}. \quad (7)$$

Note that the idiosyncratic labour productivity $g(i)$ drops out of equation (7) as it plays no role in citizen i's policy preference. However, $g(i)$ *will* play an indirect role in the evolution of the economy through its effect on the political process.

3.2. *A Voting Foundation for* μ

The stylized model has a natural political interpretation. The state ω may be regarded as a proxy for the size of government. Hence, the assumption $f' \geq 0$ implies that the dynamic policy preferences are well ordered: higher types are more "fiscally liberal" in the sense that they prefer a larger public sector.[10] Given preferences satisfying equation (7), one can show that the median voter theorem applies. Hence, any voting system, regardless of the way in which votes are weighted, admits a pivotal voter whose preferred policy choice beats any other in a pairwise comparison.

One such voting system is that of wealth-weighted voting discussed in Section 2. Specifically, consider a polity that allocates $y(i, \omega_t)$ votes to each type i in state ω_t.[11] Then, applying the median voter theorem (where "median" is wealth weighted here), the authority function $\mu(\omega)$ is endogenously determined by

$$\frac{\int_0^{\mu(\omega_t)} y(i, \omega_t)di}{\int_0^1 y(i, \omega_t)di} = \frac{\int_0^{\mu(\omega_t)} [g(i) + f(i)\omega]di}{\int_0^1 [g(i) + f(i)\omega]di} = \frac{1}{2}. \quad (8)$$

Hence, while order-preserving shifts in the wealth distribution do not change the median voter, they do change the wealth-weighted pivotal voter. In this case, μ increases (decreases) whenever Lorenz inequality increases (decreases) in the state ω_t.

An authority function μ can therefore be computed from equation (8) given functions f and g. A particularly tractable form of μ, given by

$$i_t = \mu(\omega_t) = \frac{\kappa_0 + \kappa\omega_t}{\kappa_0 + \kappa\omega_t + 1}$$

with parameters κ and $\kappa_0 > 0$, may be obtained from a marginal efficiency f satisfying: $f(i) = \frac{i}{1-i}$ for all $i \leq \bar{i}$ and $f(i) = f(\bar{i})$ for all $i \geq \bar{i}$. The marginal labour efficiency g can easily be backed out using equation (8). The details are contained in a technical appendix.[12]

For the purposes of solving the model, the most relevant property of this construction is that for all ω_t,

$$f(\mu(\omega_t)) = \kappa_0 + \kappa\omega_t. \quad (9)$$

10. To see this, observe that when flow pay-offs satisfy equation (7), the value function is an affine function of $f(i)$ regardless of the policy function Ψ^*:

$$V(i, \omega_t; \Psi^*) = f(i)V^1(\omega_t; \Psi^*) + V^2(\omega_t; \Psi^*).$$

11. In general, we could have considered any weighting system that places positive weight on one's income/wealth. See footnote 7 for real-world examples.

12. See www9.georgetown.edu/faculty/lagunofr/onlineappendix.pdf.

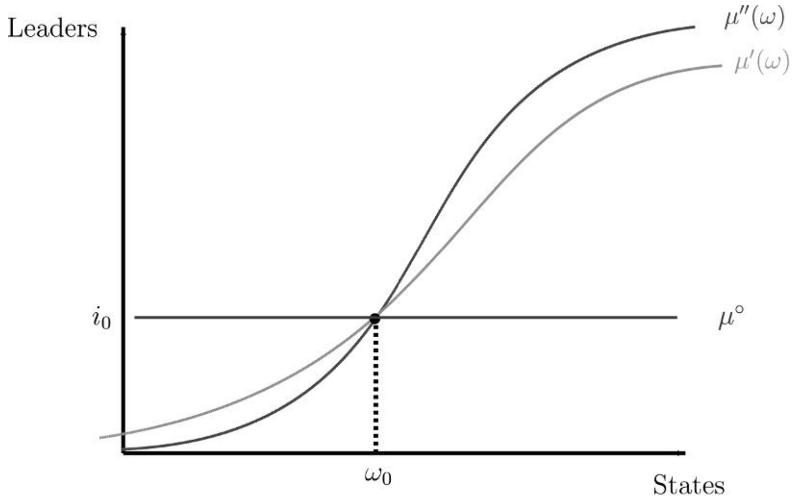

FIGURE 1
Authority functions in the PE and permanent power regimes

In other words, authority is given to the type i_t for whom the marginal value of public sector capital is an affine function of the stock itself. Increases in the stock therefore correspond to more fiscally liberal authorities (for $\kappa > 0$). By varying parameters κ and κ_0 such that all authority functions intersect the initial point (ω_0, i_0), equation (9) maps out a one-dimensional *class* of authority functions, each of which differs by the adjustment speed and direction of political power.[13]

Figure 1 illustrates three examples of authority functions that satisfy equation (9). In the figure, μ°, μ', and μ'' correspond to parameter values for $\kappa = 0, \kappa'$, and κ'', respectively, where $0 < \kappa' < \kappa''$. The special case of $\kappa = 0$ corresponds to the "undistorted" case of PA, with i_0 as the permanent leader. Clearly, there is no Faustian trade-off for i_0 in this case. When $\kappa = 0$, it is not hard to show that the PA equilibrium is state invariant: $\psi(\omega_t) = \overline{\psi}$.[14]

The value $|\kappa|$ measures the degree of institutional distortion. An increase in $|\kappa|$ (while adjusting κ_0 to keep the same initial point) implies faster structural evolution of political authority. In the figure, μ' and μ'' both begin with i_0 as the initial decision maker. The function μ' is "less distorted" than μ'' in the sense that μ'' offers a starker Faustian trade-off for leader i_0. Intuitively, one might call μ' more "conservative" in the "Burkean" sense of offering a more gradual structural evolution of political power, and μ'' is more "progressive" in the sense of inducing accelerated change. A central feature of the Faustian model, as we later show, is that equilibrium

13. This class of authority functions are those that satisfy $\mu(\omega_t) = f^{-1}(\kappa_0 + \kappa\omega_t)$ such that κ and κ_0 satisfy the linear equation $f(i_0) = \kappa_0 + \kappa\omega_0$.

14. Because $f(i)$ is a monotone function, the PA equilibrium can be identified as a solution to a social planner's problem. To see this, suppose that h is a density on $[0,1]$ in which $h(i)$ is the welfare weight assigned to citizen type i. Then there exists i_0 such that

$$f(i_0) = \int h(i) f(i) di.$$

In other words, the social welfare function coincides with the utility function of a specially chosen i_0. Therefore, the social planner's problem is the same as the PA problem with PA vested in type i_0.

responses of individuals may undercut this seemingly straightforward comparison: the more conservative rule does not always produce a more gradual evolution of political authority in equilibrium.

In either case, $\kappa > 0$ implies that μ is increasing. An increase in public sector capital therefore adjusts political power upward towards more fiscally liberal types—those with higher marginal value of government expenditure. But more liberal types choose higher levels of government expenditure that increase the public capital stock. In this sense, μ represents a *reinforcing distortion*. If instead $\kappa < 0$, then an increase in public capital would adjust political power downward towards the more conservative "small government" types. In that case, because the authority function μ moves in opposition to the transition technology, it represents a *countervailing distortion*.

3.3. *Monotone Faustian Dynamics*

Consider first the case of $\kappa > 0$, thus a reinforcing distortion. Since μ slopes upward, increases in public capital put power in the hands of more fiscally liberal leaders. In the following result, ψ refers to the state-invariant PA equilibrium.

Proposition 1. *Consider any authority function μ satisfying equation (9). If $0 < \kappa < d\left(\frac{1}{\delta} - 1 + d\right)$, then the following hold:*

(i) There exist an increasing, affine PE equilibrium policy rule $\Psi^*(\omega_t)$ and a non-increasing, affine hypothetical rule $\Psi(i_0, \omega_t)$, each of which are unique in the corresponding class of affine equilibria.

(ii) The PE equilibrium path of states $\{\omega_t\}$ converges monotonically to a unique steady state $\omega^* = Q(\omega^*, \Psi^*(\omega^*))$. If $\omega_0 < \omega^*$, then $\omega_0 < \omega_t$ implies that $\Psi(i_0, \omega_t) < \psi(\omega_t)$ and $\Psi(i_0, \omega_t) < \Psi^*(\omega_t)$, whereas if $\omega_0 > \omega^*$, then $\omega_t < \omega_0$ implies that $\Psi(i_0, \omega_t) > \psi(\omega_t)$ and $\Psi(i_0, \omega_t) > \Psi^*(\omega_t)$.

(iii) If $\omega_0 < \omega^*$, then there exists a state $\hat{\omega}$ with $\omega_0 < \hat{\omega} < \omega^*$ such that

$$\Psi^*(\omega_t) < \psi(\omega_t), \quad \forall \omega_0 \leq \omega_t < \hat{\omega},$$

$$\Psi^*(\omega_t) > \psi(\omega_t), \quad \forall \omega_t > \hat{\omega},$$

whereas if $\omega_0 > \omega^*$, then there exists a state $\hat{\omega}$ with $\omega^* < \hat{\omega} < \omega_0$ such that

$$\Psi^*(\omega_t) > \psi(\omega_t), \quad \forall \hat{\omega} < \omega_t \leq \omega_0,$$

$$\Psi^*(\omega_t) < \psi(\omega_t), \quad \forall \omega_t < \hat{\omega},$$

The upper bound on κ is required to satisfy a transversality condition that bounds the rate of growth. The proof in the Appendix shows the policy function of the affine form $\Psi^*(\omega_t) = (d - K)\omega^* + K\omega_t$, where K is a constant (in ω_t) with $0 \leq K < d$ and ω^* is the unique steady state. The coefficients implicitly vary in the exogenous parameters κ_0, κ, and δ and the depreciation rate d.

Parts (ii) and (iii) state what will turn out to be quite general properties of Faustian dynamics. Part (ii) compares the hypothetical rule to both the PE and the PA rules. If the initial state is below the steady state, then political power evolves from fiscally conservative towards more fiscally liberal types. By choosing a smaller expansion in government expenditures than he would if his power were permanent, type i_0 slows the rate of political change as it evolves away from his type. In this sense, fiscal conservatism coincides with "Burkean" conservatism.

By contrast, when the initial state is above the steady state, then political power evolves from fiscally liberal towards fiscally conservative types. In that case, type i_0, a fiscal liberal, acts as a Burkean conservative by choosing a larger expansion in government expenditures than he would if his power were permanent. By doing so, he once again slows the rate of political change.

For purposes of intuition, we restrict our discussion and pictures to the case of $\omega_0 < \omega^*$. Then the (hypothetical) policies of leader i_0 become more conservative over time/states because more distant types assume power as the state progresses upward. But, as Part (iii) shows, individual caution is juxtaposed with progressive evolution of policy. The PE equilibrium starts out more conservative than PA but winds up more fiscally liberal in the long run. The intuition is as follows. Consider the incentives of the initial leader i_0. He anticipates a growing economy under Ψ^*. However, i_0 also anticipates the corresponding shift of power to more liberal types in the future. Because the higher tax rates chosen by these liberal types are undesirable from i_0's viewpoint, i_0 slows the process of political evolution towards these types by choosing a lower tax rate than even he himself would want. The fact that $\Psi(i_0, \omega_t)$ is decreasing in the state demonstrates, in fact, that the incentive to "slow things up" intensifies as the public sector grows larger. This is seen in the first diagram in Figure 2. The second diagram displays transition dynamics using the notation $\theta(\omega_t) = Q(\omega_t, \psi(\omega_t))$ for PA, $\Theta^*(\omega_t) = Q(\omega_t, \Psi^*(\omega_t))$ for PE, and $\Theta(i_0, \omega_t) = Q(\omega_t, \Psi(i_0, \omega_t))$ for hypothetical PE.

This "Faustian" motive for gradualism described above can be identified in the leader's Euler equation. Using the parametric assumptions, the value function for the current leader i_t (but not for an arbitrary i) is

$$V\left(i_t, \omega_t;\ \Psi^*\right) = \max_{a_t}\left\{ f(i_t)\omega_t - \frac{a_t^2}{2} + \delta\, V\left(i_t, \omega_{t+1};\ \Psi^*\right)\right\} \text{ subject to } \omega_{t+1} = (1-d)\,\omega_t + a_t. \tag{10}$$

If $\Psi^*(\omega_t)$ lies in the interior of the feasible policy space, then it satisfies the first-order condition[15]

$$0 = -\Psi^*(\omega_t) + \delta D_{\omega_{t+1}} V\left(i_t, (1-d)\,\omega_t + \Psi^*(\omega_t);\ \Psi^*\right). \tag{11}$$

The first term is clearly the marginal cost of an increase in government spending, while the second is the discounted marginal benefit in the future. Consider next period's decision from the point of view of the *current* decision maker i_t. Since next period's decision maker i_{t+1} is different from i_t, the next period's decision induces a marginal distortion away from i_t's optimal policy choice in $t+1$. This distortion is given by

$$\Delta(i_t, \omega_{t+1}; \Psi^*) = -\Psi^*(\omega_{t+1}) + \delta D_{\omega_{t+2}} V\left(i_t, (1-d)\,\omega_{t+1} + \Psi^*(\omega_{t+1});\ \Psi^*\right). \tag{12}$$

Notice that the R.H.S. of equation (12) is of the same form as equation (11), shifted one period ahead. Intuitively, because the government's capital stock ω_t increases, power shifts towards policy makers who prefer higher levels of government spending. Hence, $\Delta(i_t, \omega_{t+1}; \Psi^*) < 0$ since i_{t+1} chooses a higher level of spending than i_t himself would choose in state ω_{t+1}.

Differentiating the value function V in equation (10), substituting in the distortion equation (12), and iterating one period, the discounted marginal benefit $D_{\omega_{t+1}} V(i_t, \omega_{t+1};\ \Psi^*)$ of an increase in current investment can now be expressed as

$$D_{\omega_{t+1}} V\left(i_t, \omega_{t+1};\ \Psi^*\right) = \left[f(i_t) + (1-d)\,\Psi^*(\omega_{t+1})\right] + \left[(K+1-d)\,\Delta(i_t, \omega_{t+1};\ \Psi^*)\right]. \tag{13}$$

15. Throughout the paper, the partial derivative of a function $F(x, y)$ is expressed as $D_x F$.

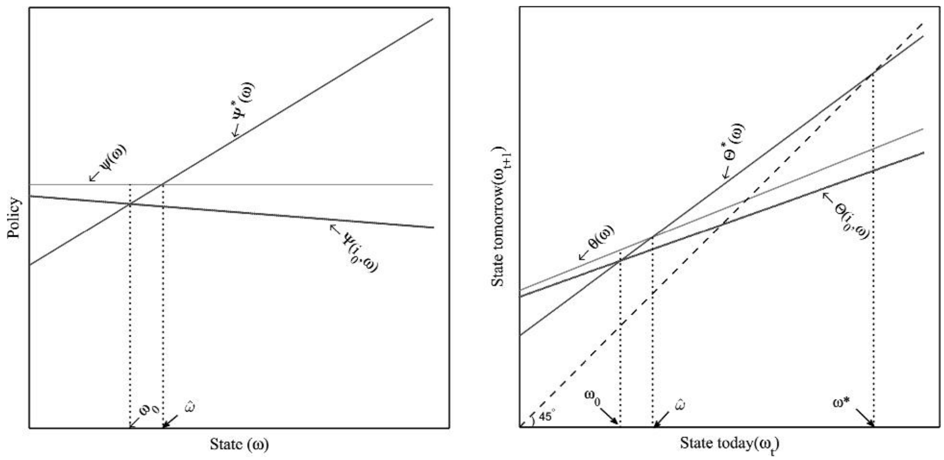

FIGURE 2
Monotone policy functions and monotone transition dynamics in the PE and PA equilibria

The marginal continuation value $D_{\omega_{t+1}}V$ can be decomposed into two effects. The first bracketed term $[\cdot]$ on the R.H.S. describes the direct effect that current policy has on the next period's state. This term also exists (is non-zero) under PA, although the particular values differ between the two regimes. We refer to this difference (between PE and PA) as the *reformation effect* since it describes the net direct incentive to increase public investment via changes in the marginal value of public sector capital. The reformation effect does not include the "Faustian" distortion in incentives due to endogenous change in future leadership. This distortion is captured by the second bracketed term $[\cdot]$. This term exists *only* in PE decision problems. This distortion creates the "Burkean" incentive by all leaders to slow the evolution of political power as it moves away from the current leader. This is seen by the fact $\Delta(i_t,\omega_{t+1};\ \Psi^*)<0$. Hence, we refer to it as the *political preservation effect*. General properties of both these effects are described in the next section.

Consider the decision of i_0 at date 0. His PE policy may be well below his PA policy, and it takes time before the PE path overtakes that of PA. Part (iii) shows that this eventually happens (see Figure 2). For ω_t close to ω_0, the leader type i_t is not so different from i_0, and so the preference for conservative change may place $\Psi^*(\omega_t)$ below $\psi(\omega_t)$ for a time.

Notice finally that the Faustian dynamics of states and leaders also move monotonically. To see this, observe that if ω_0 is close to zero, it lies below the steady state. Hence, the equilibrium state transition $\Theta^*(\omega_t)\equiv Q(\omega_t,\Psi^*(\omega_t))$ is increasing in the state, and, consequently, the equilibrium paths $\{\omega_t\}$ and $\{i_t\}$ are increasing. This is illustrated in the second diagram in Figure 2, which displays the comparison between the PE transition Θ^*, the hypothetical transition Θ, and the PA transition θ.

So far, we have compared the PE to the PA regime. But this comparison can be applied to any two PE political institutions, one yielding more gradual change in political power than the other. This is illustrated in Figure 1.

Proposition 2. *Consider two authority functions μ and $\widetilde{\mu}$, each corresponding to a parameter pair (κ_0,κ) and $(\widetilde{\kappa}_0,\widetilde{\kappa})$ according to equation (9) and both of which satisfy the initial condition. Suppose $\kappa>\widetilde{\kappa}\geq 0$ (and, consequently, $\kappa_0<\widetilde{\kappa}_0$). Let Ψ^* and $\widetilde{\Psi}$ denote PE equilibria under μ and $\widetilde{\mu}$, respectively, and let ω^* and $\widetilde{\omega}$ denote their corresponding (unique) steady states.*

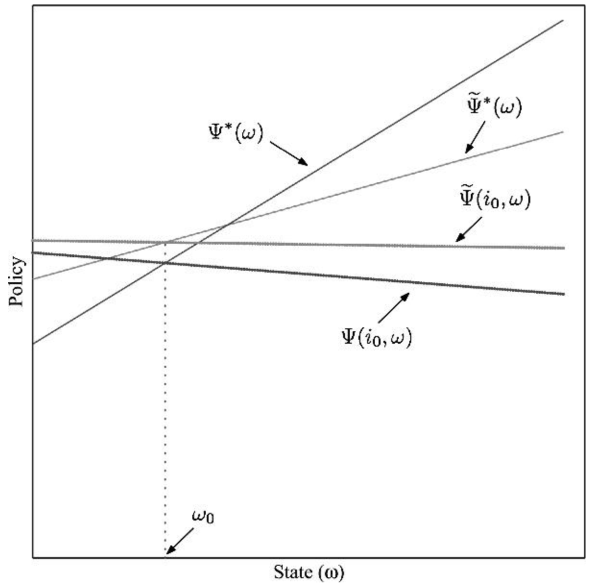

FIGURE 3
Comparison of two political institutions

Then, $\omega^* > \widetilde{\omega}$ *iff* $\omega_0 < \omega^*$*. Furthermore, if* $\omega_0 < \omega^*$ *then there exists a state* $\hat{\omega}$ *with* $\omega_0 < \hat{\omega} < \omega^*$ *such that*

$$\Psi^*(\omega_t) < \widetilde{\Psi}(\omega_t) \quad \textit{whenever} \quad \omega_0 \le \omega_t < \hat{\omega},$$
$$\Psi^*(\omega_t) > \widetilde{\Psi}(\omega_t) \quad \textit{whenever} \quad \omega_t > \hat{\omega},$$

whereas if $\omega_0 > \omega^*$*, then there exists a state* $\hat{\omega}$ *with* $\omega^* < \hat{\omega} < \omega_0$ *such that*

$$\Psi^*(\omega_t) > \widetilde{\Psi}(\omega_t) \quad \textit{whenever} \quad \hat{\omega} < \omega_t \le \omega_0,$$
$$\Psi^*(\omega_t) < \widetilde{\Psi}(\omega_t) \quad \textit{whenever} \quad \omega_t < \hat{\omega}.$$

Recall that if $\kappa > \widetilde{\kappa}$, then μ gives a faster *structural* evolution of political power, while $\widetilde{\mu}$ is more gradual. The intuition is the same as in Part (iii) of Proposition 1. Decision makers respond to a more distorted political institution μ, by choosing *more* conservative responses. Initially, this Burkean incentive effect outweighs the structural effect from the authority functions. Hence, the more distorted institution μ initially produces a slower evolution of power and policy than the less distorted one $\widetilde{\mu}$. However, in the long run (including but not restricted to steady states), structural features take over and the more distorted polity produces a faster evolution of leaders and policies. This is illustrated in Figure 3.

3.4. *Cyclical Faustian Dynamics*

Now consider $\kappa < 0$, the case of countervailing distortion. Since μ slopes downward, increased public investment places power in more fiscally conservative citizen types.

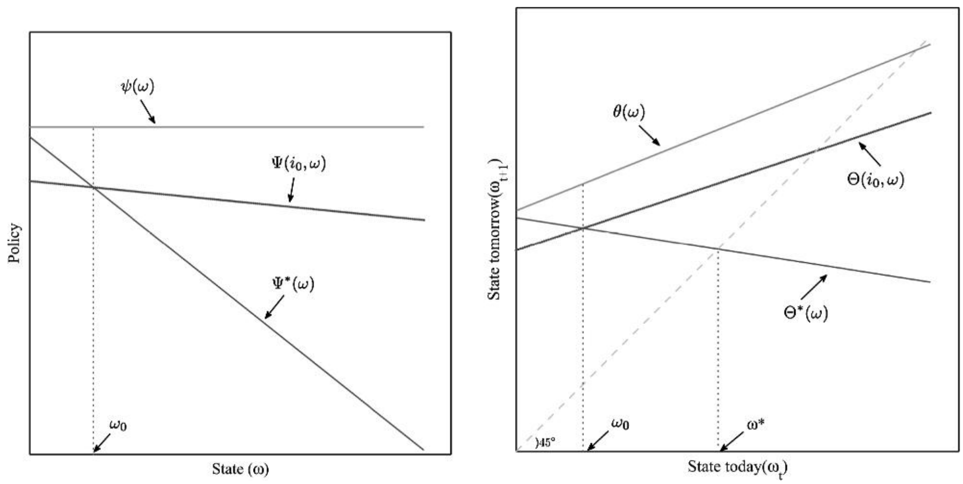

FIGURE 4
Cyclical dynamics in the PE and PA equilibria

Proposition 3. *Consider any authority function μ satisfying equation (9). If* $0 > \kappa \geq -\frac{1-d}{\delta} - \frac{1+\delta}{1-\delta}\left[(1-d)^2 + \frac{3+\delta}{1+\delta}(1-d) + \frac{1}{\delta}\right]$, *then the following hold:*

(i) *There exist a decreasing affine PE equilibrium policy rule $\Psi^*(\omega_t)$ and a non-increasing, affine hypothetical rule $\Psi(i_0, \omega_t)$, each of which are unique in the corresponding class of affine equilibria.*

(ii) *The PE equilibrium path of states $\{\omega_t\}$ converges to a unique steady state ω^*. If $\kappa > -\frac{1-d}{\delta}$, then the convergence is monotonic. However, if $\kappa < -\frac{1-d}{\delta}$, then the economy follows a dampened cycle converging to ω^* such that $\omega_t < \omega^*$ if and only if $\omega_{t+1} > \omega^*$. In either case, if $\omega_0 < \omega^*$, then $\omega_0 < \omega_t$ implies that $\Psi^*(\omega_t) < \Psi(i_0, \omega_t) < \psi(\omega_t)$, whereas if $\omega_0 > \omega^*$, then $\omega_t < \omega_0$ implies that $\Psi^*(\omega_t) > \Psi(i_0, \omega_t) > \psi(\omega_t)$.*

Faustian dynamics can therefore produce political cycles when the authority function is sufficiently distorted downward. The intuition, roughly, is that the evolution of political power counters the evolution of public sector capital. Hence, when fiscally liberal types choose high expenditures, leading to increases in capital stock, this induces a steep drop in the index i that determines the progressivity of the political type. More fiscally conservative types then lower expenditures that, in turn, produce more liberal types and so on.

Since μ slopes downward, then whenever the government's capital stock increases, political power moves downward towards more fiscally conservative types. Consequently, the preservation effect induces the initial, liberal, leader to decrease expenditures and hence *slow* the evolution of political authority as it moves downward. See Figures 4 and 5.

4. A GENERAL MONOTONE MODEL

This section returns to the basic set-up in Section 2. For tractability, we restrict attention to increasing authority functions. As before, we compare the PE equilibrium Ψ^*, the PA equilibrium ψ, and the hypothetical rule $\Psi(i_0, \cdot)$.

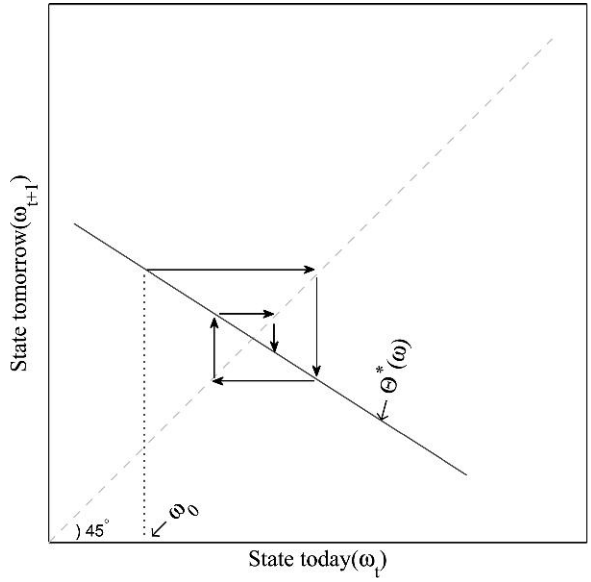

FIGURE 5
Cycles with countervailing bias

The main results characterize properties of smooth-limit equilibria, which we define below. A natural decomposition of the Euler equation is established in which the "Burkean" incentives of decision makers to slow the process of political change are isolated and identified. Note that the public investment model of the previous section is a special case of the general model examined here. All the results of this section apply there as well. The proofs are in the Appendix.

Smoothness (differentiability) plays a crucial role in our characterization. We use it to examine properties of the Euler equations, roughly following an approach dating back to Basar and Olsder (1982) for dynamic stochastic games.[16] Results in Judd (2004) suggest that smoothness is a natural selection device when multiple equilibria exist. Formally, a PE equilibrium Ψ^* and PA equilibrium ψ are *smooth-limit equilibria* if (i) Ψ^* and ψ are differentiable in the state, (ii) the resulting policies and $\Psi^*(\omega_t)$ and $\psi(\omega_t)$ lie in the interior of the feasible policy space, and (iii) Ψ^* and ψ are the limit of smooth, finite horizon PE and PA equilibria, respectively Property (iii) is not necessary in the following characterization of the Euler equation. It is only used later when time iteration of the value function is the most convenient way of establishing monotonicity in the state.

Issues of equilibrium existence are not explored here. There are, in fact, general existence results for smooth Markov equilibria, but these require stochastic shocks that are absent from our presentation.[17]

16. More recently, this approach has been adapted to dynamic macro policy problems by Klein, Krusell, and Ríos-Rull (2008), Krusell and Smith (2003), Krusell, Kuruscu, and Smith (2002), and Judd (2004) and to dynamic political games by Jack and Lagunoff (2004).

17. A previous draft of this paper had shocks, but we took them out as they added little to the main ideas. Aside from shocks, the remaining assumptions do not appear to violate any of the known existence results as far we are aware. See, for instance, Amir (1996), Curtat (1996), Horst (2005), Lagunoff (2008), and Novak (2007).

4.1. *Distortion-Adjusted Euler Equation*

The applied model in the previous section introduced the idea of a distortion function. The distortion function represents the wedge in one's long-run pay-offs arising from the conflict between one's own preferences and that of the decision maker. This conflict distorts the marginal pay-off away from its critical value. In the general case, the distortion function is given by

$$\Delta(i,\omega_t;\Psi^*) \;=\; D_{a_t}u(i,\omega_t,\Psi^*(\omega_t)) \;+\; \delta\, D_{a_t}Q(\omega_t,\Psi^*(\omega_t))\cdot D_{\omega_{t+1}}V(i,\omega_{t+1};\;\Psi^*) \qquad (14)$$

with $\omega_{t+1} = Q(\omega_t,\Psi^*(\omega_t))$. Generally, $\Delta(i,\omega_t;\Psi^*)$ describes the marginal distortion away from the i's critical value when some possibly different citizen type makes the policy decision in state ω_t. Of course, in the special case where $i = i_t = \mu(\omega_t)$, *i.e., i is* the leader, then there is no distortion. In that case, $\Delta(i,\omega_t;\Psi^*) = 0$ describes a first-order condition for $i = i_t$. Under PA, it also follows that $\Delta(i_0,\omega_t;\psi) = 0$ since there is no distortion when i_0 holds power forever. The distortion equation (14) can be written as

$$\delta D_{\omega_{t+1}}V_i \;=\; \big[D_{a_t}Q\big]^{-1}\cdot\big[\Delta(i,\omega_t;\Psi^*) \;-\; D_{a_t}u_i\big]. \qquad (15)$$

Next, consider an arbitrary citizen type $i \in I$ (not necessarily the leader) in the PE equilibrium. His continuation value function in state ω_{t+1} is

$$V(i,\omega_{t+1};\Psi^*) \;=\; u(i,\omega_{t+1},\Psi^*(\omega_{t+1})) \;+\; \delta\, V(i,\omega_{t+2};\Psi^*) \qquad (16)$$

with $\omega_{t+2} = Q(\omega_{t+1},\Psi^*(\omega_{t+1}))$. To save on notation, we use the abbreviated notation $u_i = u(i,\cdot)$ and $V_i = V(i,\cdot)$. Differentiating this value function V_i with respect to ω_{t+1} yields

$$D_{\omega_{t+1}}V_i \;=\; D_{\omega_{t+1}}u_i \;+\; D_{\omega_{t+1}}\Psi^*\cdot D_{a_{t+1}}u_i \;+\; \delta\big[D_{\omega_{t+1}}Q \;+\; D_{\omega_{t+1}}\Psi^*\cdot D_{a_{t+1}}Q\big]\cdot D_{\omega_{t+2}}V_i. \qquad (17)$$

Iterating equation (15) forward one period while holding i fixed and then substituting it back into equation (17) yields the expression

$$D_{\omega_{t+1}}V_i \;\;=\;\; R(i_t,\omega_{t+1};\;\Psi^*) \;+\; P(i_t,\omega_{t+1};\;\Psi^*), \qquad (18)$$

where

$$R(i_t,\omega_{t+1};\;\Psi^*) \equiv\; D_{\omega_{t+1}}u_{i_t} \;-\; D_{\omega_{t+1}}Q\cdot\big[D_{a_{t+1}}Q\big]^{-1}\cdot\big[D_{a_{t+1}}u_{i_t}\big] \qquad (18a)$$

and

$$P(i_t,\omega_{t+1};\;\Psi^*) \equiv \Big[D_{\omega_{t+1}}Q\cdot\big[D_{a_{t+1}}Q\big]^{-1} \;+\; D_{\omega_{t+1}}\Psi^*\Big]\cdot\Delta(i_t,\omega_{t+1};\Psi^*). \qquad (18b)$$

Using these definitions, we substitute equation (18) into the first-order condition $\Delta(i_t,\omega_t;\Psi^*) = 0$ of the date t leader i_t to obtain the *distortion-adjusted Euler equation*

$$D_{a_t}u_{i_t} \;+\; \delta\, D_{a_t}Q\cdot\big[R(i_t,\omega_{t+1};\;\Psi^*) \;+\; P(i_t,\omega_{t+1};\;\Psi^*)\big] \;=\; 0, \qquad (19)$$

where $\omega_{t+1} = Q(\omega_t,\Psi^*(\omega_t))$. The distortion-adjusted Euler equation is fundamental to our characterization of the general model. It captures the basic decomposition of motives of any leader when power is PE. Each leader weighs the marginal cost $D_{a_t}u_{i_t}$ against two types of marginal effects.

Starting first with the second effect, the term $P(i_t,\omega_{t+1};\;\Psi^*)$ is closely identified with the Faustian trade-off. Intuitively, it describes the marginal loss in i_t's pay-off that is brought about when the policy induces a sequence of different, and clearly less desirable, political leaders in the future. These future leaders choose policies that distort one's dynamic marginal pay-off away from its critical value. These distortions begin with the decision a_{t+1} made by i_{t+1} whose preferences may differ from i_t. We refer to $P(i_t, \omega_{t+1};\ \Psi^*)$ as the *preservation effect* because, as we later show, it induces Burkean conservatism by decision makers. In the public investment model, the preservation effect is given by $(K+1-d)\,\Delta(i_t, \omega_{t+1};\ \Psi^*)$, which is negative, a fact that we later show is robust. Note that the preservation effect vanishes in the PA equilibrium. That is, $P(i_t, \omega_{t+1};\ \psi) = 0$ due to the envelope theorem.

The preservation effect may be partly offset by a standard dynamic programming term in the form of $R(i_t, \omega_{t+1};\ \Psi^*)$ defined in equation (18a). Qualitatively, the term R arises in the typical Euler equations. Roughly, it describes the marginal effect of a change in next period's state ω_{t+1} on next period's pay-offs after adjusting policy so that the subsequent state ω_{t+2} remains at the level chosen by i_{t+1} in equilibrium (since changes in ω_{t+2} come from next period's leader i_{t+1}, and hence are part of the preservation effect).

The first term in equation (18a) describes the direct gain in i_t's pay-off next period from a change in ω_{t+1}. The second term describes the indirect gain from reduced cost of the policy in $t+1$. Specifically, a unit increase in ω_{t+1} lowers the required policy a_{t+1} by $D_{\omega_{t+1}} Q \cdot [D_{a_{t+1}} Q]^{-1}$ in order to achieve the state ω_{t+2} that i_t anticipates will be induced by i_{t+1}'s policy in equilibrium.[18] In turn, this leads to a change in pay-off equaling this second term. While the functional form R is standard, its magnitude differs across PA and PE regimes. We refer to the difference $R(i, \omega_{t+1}; \Psi^*) - R(i, \omega_{t+1}; \psi)$ as the *reformation effect* because it reflects the net incentive distortion due to the productivity differences of a current policy choice in PE relative to PA. A higher level of future investment under Ψ^* may increase one's incentive to invest more today. From equation (13), the reformation effect in the public investment model, for instance, is calculated to be $(1-d)\,(\Psi^*(\omega_{t+1}) - \psi(\omega_{t+1}))$. Since ψ is stationary in that model, the reformation effect is increasing in the state. Using Proposition 1, the reformation effect is negative at states below a cut-off $\hat{\omega}$ and positive at states above it.

Because the preservation effect vanishes under PA, the Euler equation in the PA equilibrium is given by

$$D_{a_t} u_{i_0} + \delta\, D_{a_t} Q \cdot R(i_0, \omega_{t+1}; \psi) = 0, \tag{20}$$

where $\omega_{t+1} = Q(\omega_t, \psi(\omega_t))$. Consequently, the net effect on continuation pay-offs of PE power, relative to PA, is given by

$$\overbrace{R(i, \omega_{t+1};\ \Psi^*) - R(i, \omega_{t+1}; \psi)}^{\text{Reformation effect}} + \overbrace{P(i, \omega_{t+1};\ \Psi^*)}^{\text{Preservation effect}}. \tag{21}$$

With the aid of a supermodularity assumption defined below, we use these terms to identify features of the PE equilibrium.

4.2. *Supermodularity and Monotonicity*

A central feature of the public investment model is monotonicity. In one case, political power shifts from lower to higher marginal valuation types, and this occurs when equilibrium policy rules are increasing in the state. In the second case, the paths exhibit cycles, with dampened

18. Note that both terms in the expression for R affect only pay-offs in $t+1$. This is because the effect on periods $t+2$ onward depends on the marginal variation of the policy rules that come from variations in the identity of the decision makers. These variations appear in the preservation effect and would be fully "enveloped out" in the standard, single-agent dynamic programming problems.

oscillations between high and low valuation types. This occurs when equilibrium policy rules are decreasing in the state.

The monotonicity of the policy rule in these cases owes not so much to the specific functional forms, but (as we later show) rather to the fact that these functional forms are *supermodular*. This is no accident. Supermodularity is the more general version of a collection of commonly used assumptions (e.g., increasing differences, strategic complements) that are generally used to establish monotonicity and/or well-defined monotone comparative statics of endogenous decision rules. Here it is central in our efforts to characterize Faustian trade-offs. In smooth (differentiable) models, the definition is straightforward. A smooth function $f:\mathbb{R}^m \to \mathbb{R}$ is *supermodular (spm)* iff $D_{x_j} D_{x_k} f \geq 0$ for all $j,k = 1,\ldots,m$, and f is *strictly spm* if the inequality is strict.

First, define $a_t = L(\omega_t, \omega_{t+1})$ implicitly from the transition equation $Q(\omega_t, L(\omega_t, \omega_{t+1})) = \omega_{t+1}$. The function $L(\omega_t, \omega_{t+1})$ is interpreted as the policy cost of generating tomorrow's state ω_{t+1} given current state ω_t.

(A1) *Supermodularity (spm)*. The composite flow pay-off defined by $\tilde{u}(i,\omega_t,\omega_{t+1}) \equiv u(i,\omega_t, L(\omega_t,\omega_{t+1}))$ is supermodular in $(i,\omega_t,\omega_{t+1})$ and strictly supermodular in (i,ω_t).

The composite flow pay-off captures both the static pay-off effect of a change in policy and its future consequence through L. Hence, by placing joint restrictions on u and Q through the composite pay-off $\tilde{u}$, Assumption A1 is weaker than separate spm assumptions on u and Q. Still, it is not unrestrictive. As with all spm assumptions, Assumption A1 applies only to environments with sufficiently strong complementarities between endogenous variables and exogenous parameters and between different sets of endogenous variables. It holds in most models of capital accumulation via taxation by a government since a government's current investment typically increases the productivity of its future investment. In the public investment model, for instance, Assumption A1 is easily verified since $\tilde{u} = f(i)\omega_t - 1/2(\omega_{t+1} - (1-d)\omega_t)^2$.

On the other hand, spm often does not hold in situations where there are free rider problems as in, for instance, cases where public good provision is decentralized and voluntary. In such cases, an individual's marginal incentive to contribute diminishes as the sum of others' contributions grows larger. The present model does not have this problem since public provision is centralized through a pivotal voting mechanism of one kind or another.

Theorem 1. *Suppose that Assumption A1 holds. Then, in any smooth-limit PE equilibrium Ψ^*, citizen type i's dynamic pay-off, $u(i,\omega_t,a_t) + \delta V(i, Q(\omega_t,a_t); \Psi^*)$, is supermodular in i and a_t.*

Application of the theorem, combined with standard results in voting theory (for instance, Gans and Smart, 1996) shows that the median voter theorem applies.[19] Of course, for a given μ this median may be weighted as in equation (8). Theorem 1 implies then that there exists a (possibly weighted) state-dependent voting rule such that the rule admits a pivotal voter $\mu(\omega_t)$ in state ω_t.

A secondary consequence of spm is that it allows for a straightforward comparison of PE to the PA equilibrium based on the reformation effect alone.

19. Assumption A1 implies that the pay-off satisfies single crossing in i and a_t. Hence, the application of the result of Gans and Smart applies. See also Roberts (1998, 1999) for another application of spm to a dynamic political economy model of club admissions.

Theorem 2. *Suppose Assumption A1 holds. For each citizen type i and each state ω_t, $\Psi^*(\omega_t) \geq \psi(\omega_t)$ iff $R(i,\omega_t;\Psi^*) \geq R(i,\omega_t;\psi)$.*

Under Assumption A1, a well-ordered evolution of political authority exists in PE equilibria.

Theorem 3. *Suppose Assumption A1 holds. Let Ψ^* be a smooth-limit PE equilibrium and let $\Theta^*(\omega_t) \equiv Q(\omega_t, \Psi^*(\omega_t))$ be the associated PE equilibrium transition rule. Then, Θ^* is increasing, and for any state $\omega_t \geq \omega_0$, $\Psi^*(\omega_t) \geq \Psi(i_0,\omega_t)$ with strict inequality if $\omega_t > \omega_0$.*

The theorem shows that the PE equilibrium transition rule is monotone in states, and the hypothetical PE rule is more conservative (in the natural order on policies) in every state than in the PE equilibrium. By itself, Theorem 3 does not say much about Faustian dynamics. However, under a simple initial condition, the following corollary asserts that Faustian dynamics are monotone.

Corollary. *Suppose that, in addition to the assumptions of Theorem 3, $\Theta^*(\omega_0) > \omega_0$. Then, the PE equilibrium paths of states $\{\omega_t\}$ and leaders $\{i_t\}$ are increasing: $\omega_{t+1} > \omega_t$ and $i_{t+1} > i_t$.*

The corollary asserts that the PE equilibrium path of states and decision-making types is increasing provided it starts off that way. Hence, current leaders knowingly lose power to more progressive decision types, and this evolution continues until either a steady state is reached or the largest (most progressive) type acquires power.

Finally, spm is used to show that one critical feature of the public investment model holds quite broadly. So-called "Burkean" incentives for conservative decision making are embodied in the preservation effect.

Theorem 4. *Suppose Assumption A1 holds. Let Ψ^* be a smooth-limit PE equilibrium. For each state ω_{t+1}, $P(i,\omega_{t+1};\ \Psi^*)$ is increasing in i and*

$$P(i,\omega_{t+1};\ \Psi^*) \ < \ 0 \quad \textit{if and only if} \quad i < \mu(\omega_{t+1}). \tag{22}$$

In particular, the result implies that political leaders act in such a way as to offset, at least partially, the loss of political authority resulting from their policy decisions. Hence, if the Faustian dynamics move towards more progressive leaders, then the preservation effect pushes the current leader towards a more conservative policy. If the evolution is towards less progressive leaders, then the current leader acts more progressively. Combining Theorem 4 with the initial condition that $\Theta^*(\omega_0) > \omega_0$ implies that the preservation effect is *negative* along the PE equilibrium path: $P(\mu(\omega_t),\omega_{t+1};\ \Psi^*) \ < \ 0$.

4.3. *Steady States*

By definition, the preservation effect vanishes in any steady state $\omega^* = \Theta^*(\omega^*) \equiv Q(\omega^*, \Psi^*(\omega^*))$ of a PE equilibrium. The identity of the leader clearly does not change once ω^* is reached. Consequently, the distortion-adjusted Euler equation for the steady-state leader $i^* = \mu(\omega^*)$ satisfies

$$D_{a_t} u(\mu(\omega^*),\omega^*,\Psi(\omega^*)) \ + \ \delta \ D_{a_t} Q \cdot R(\mu(\omega^*),\omega^*;\Psi^*) \ = \ 0. \tag{23}$$

From the definition of $R(\mu(\omega^*),\omega^*;\Psi^*)$ in equation (18a), observe that Ψ^* enters R only through the value of steady-state policy $a^* = \Psi(\omega^*)$. equation (23) therefore provides a joint restriction on ω^* and a^*. Combined with the transition equation $\omega^* = Q(\omega^*, a^*)$, equation (23) gives a condition on the determination of ω^* purely in terms of economic primitives. This fact can be seen more clearly in terms of the composite pay-off $\tilde{u}(i, \omega_t, \omega_{t+1})$ defined in Assumption A1.

By definition, $\tilde{u}(i, \omega_t, Q(\omega_t, a_t)) = u(i, \omega_t, a_t)$ holds as an identity. By taking partial derivatives of this identity with respect to ω_t and a_t, it follows that $D_{a_t} u(i, \omega_t, a_t) = D_{\omega_{t+1}} \tilde{u}(i, \omega_t, \omega_{t+1})$ $D_{a_t} Q(\omega_t, a_t)$ and $R(i, \omega_{t+1}; \Psi^*) = D_{\omega_{t+1}} \tilde{u}(i, \omega_{t+1}, \omega_{t+2})$. Evaluating these at the steady-state values, equation (23) reduces to

$$D_{\omega_{t+1}} \tilde{u}(\mu(\omega^*), \omega^*, \omega^*) + \delta\, D_{\omega_{t+1}} \tilde{u}(\mu(\omega^*), \omega^*, \omega^*) = 0 \tag{24}$$

The steady-state equation (24) expresses a simple marginal trade-off in the steady state. In the public investment model, this trade-off was between present and future public sector capital. Similarly, $\omega^\circ = Q(\omega^\circ, \psi(\omega^\circ))$ is a steady state in the PA equilibrium and the steady-state equation is given by

$$D_{\omega_{t+1}} \tilde{u}(i_0, \omega^\circ, \omega^\circ) + \delta\, D_{\omega_{t+1}} \tilde{u}(i_0, \omega^\circ, \omega^\circ) = 0. \tag{25}$$

Theorem 5. *Suppose Assumption A1 holds, and let Ψ^* and ψ be smooth-limit PE and PA equilibria, respectively. Then*

(i) ω^* is a steady state of Ψ^* iff equation (24) is satisfied, and ω° is a steady state of ψ iff equation (25) is satisfied.

(ii) The PE steady state ω^* is unique if

$$D_{\omega_{t+1}} \tilde{u}(\mu(\omega), \omega, \omega) + \delta\, D_{\omega_{t+1}} \tilde{u}(\mu(\omega), \omega, \omega) \quad \textit{is decreasing in the state } \omega \tag{26}$$

and the PA steady state ω° is unique if

$$D_{\omega_{t+1}} \tilde{u}(i_0, \omega, \omega) + \delta\, D_{\omega_{t+1}} \tilde{u}(i_0, \omega, \omega) \quad \textit{is decreasing in the state } \omega. \tag{27}$$

(iii) Suppose that equation (27) holds for each i. Then, $\omega^* > \omega^\circ$ iff $\omega^* > \omega_0$.

Properties (i)–(iii) are all satisfied in the stylized model. Hence, the conclusions of the theorem apply, indicating a degree of robustness of the steady-state properties of that model.

Property (iii) is perhaps the most significant of the three. It relates the long run in the PE equilibrium to the long run under PA. It suggests that in the long run at least, progressive change in policy is not hindered by political expediency.

5. RELATED LITERATURE AND DISCUSSION

The dynamic link between policy and power shows up in a small but growing number of papers. For purposes of relating these to the present paper, we found it most useful to separate them into two groups. One consists of models of one- or two-period lived agents; the other consists of models with longer-lived agents. As it turns out, there is an important difference that our general model allows us to identify.

Milesi-Ferretti and Spolaore (1994) and Besley and Coate (1998) are early contributors to the first category of papers. They posit interesting two-period models in which a policy maker's first-period decision influences voter choices in the second. A "political failure", as Besley and Coate describe it, occurs when PE loss of political control in the future leads to inefficient policy choices in the present.

Similarly, Bourguignon and Verdier (2000) explore a PE mechanism that works through education and its effect on political participation. Policies that change the level and distribution of education also change political participation across different groups. The PE mechanism of Dolmas and Huffman (2004) works through immigration. Immigration policy is determined by majority vote, and immigration changes the identity of the median voter in the subsequent period. Campante (2007) examines a related mechanism that works through campaign contributions. In this case, policies that change income distribution alter the composition of contributions that, in turn, determine who is elected.

Hassler et al. (2003) investigate the evolution of the welfare state in a parametric overlapping-generations model. A majority vote determines the level of transfers to unsuccessful agents. Because the population sizes of different types are endogenously determined by individual investment decisions, their model can generate a shift of political power even with majority voting.[20]

These studies highlight the broad array of mechanisms through which Faustian trade-offs occur. We view the present model as complementary to these in that it suggests that equilibria in these frameworks have elements in common. Our findings also suggest some possible missing ingredients. Because agents in these models live (at most) two periods, the Faustian trade-off is essentially one-shot. This means that a date t agent need not worry about the distortionary effect his policy has on distribution of power beyond $t+1$. In other words, these models have a reformation effect but *no* preservation effect. This is significant because the two effects often work in opposite directions. Our results suggest that the time paths of models with and without the preservation effect would be quite different.

Recent papers by Azzimonti (2005) and Ortega (2005) do have preservation effects in models with infinitely lived agents. Like the model of Dolmas and Huffman, Ortega (2005) studies a natural PE mechanism in the form of immigration. Ceteris paribus, current residents want to admit immigrants with complementary skills. On the other hand, such immigrants are future voters who will vote to admit future immigrants whose skills are substitutes to those of the current residents. Azzimonti (2005) posits an interesting model of dynamic inefficiency in government. An inefficiency arises because the dominant faction loses power to the other due to political shocks. She endogenizes the switching likelihoods between the two factions by introducing probabilistic voting. When the shocks to voters' ideological preferences for one group are asymmetric, then increases in public spending change voters' relative preferences between the groups, and so the identity of the pivotal voter changes as well.

Both these papers have something akin to both reformation and preservation effects. Azzimonti, in fact, emphasizes a decomposition of motives in an Euler equation related to the one in our stylized model.[21] However, the focus of these models is elsewhere, and both models' assumption of two political types/factions make the Burkean conservatism of their decision makers difficult to identify. The present study therefore recasts these papers in a new light, allowing a clear view of subtle attributes that two seemingly different models have in common.

Clearly, one would not want to argue that all policy choices involve Faustian trade-offs. In fact, a parallel literature has arisen that "uncouples" policy from political power by allowing the voters an explicit choice over political institutions. For instance, Acemoglu and Robinson (2000, 2001, 2006), Cervellati et al. (2006), Jack and Lagunoff (2006a,b), Persson and Tabellini (2009), and Lagunoff (2008, 2009) all examine models of explicit institutional (de jure) choices

20. Although, in their model, endogenous change in political power occurs mainly through private sector investment decisions rather than directly from current policies. A related model and PE mechanism is studied in Hassler et al. (2005).

21. Interestingly, her decomposition also includes exogenous inconsistency arising due to shocks rather than due to non-stationarity.

by current elites or majorities as a way of reversing or mitigating the deleterious effects of current policy on one's future political fortunes. Similarly, Roberts (1998, 1999) and Barbera, Maschler, and Shalev (2001) uncouple the policy–political choice by placing attributes of a future pivotal or marginal voter directly in the preferences of the current voter. In this sense, the policy itself *is* the composition of political power.

These "uncoupling" models make sense when current elites have the flexibility to isolate or reverse the consequences of their policy choices. Our mechanism is appropriate when this flexibility is lacking.

This contrast is apparent in a recent model of Acemoglu and Robinson (2008). Building on an earlier framework laid out in Acemoglu, Johnson, and Robinson (2005), they model the policy decisions of an elite that explicitly preserve *de facto* political power when exogenous, *de jure* changes in the political system move the country towards democracy. They identify "captured democracies" as those in which an elite's investments succeed in preserving power. The key difference between their model and ours is that in our model, policies generate political change, while in theirs, policies are used by elites to undo (exogenous) political change. Acemoglu and Robinson look to 20th-century Latin America for numerous instances of captured democracies. On the other hand, the collapse of the Soviet Union, and the role of glasnost in facilitating the change in power, suggest a Faustian trade-off at work. A decision maker (Gorbachev) turned "Burkean" as he attempted moderate reforms that eventually led to a, perhaps unavoidable, loss of his own power.

Finally, we return to the policy decision of Lyndon Johnson that was initially presented as an example of Faustian trade-offs. It was clearly part of a broader trend towards more progressive civil rights laws. The effects of Faustian trade-offs on incentives were apparently widespread in this case. Rodriguez and Weingast (2006) describe, for instance, how the 1964 Civil Rights Act was watered down to minimize the political impact for Northern legislators.[22]

6. CONCLUDING REMARKS

This paper analyses the dynamics of a "Faustian trade-off" between policy and political power. We characterize this trade-off in terms of a distortion-adjusted Euler equation that illustrates the two main motives of a political actor: the desire for gradualism on the one hand and the need for policy reformation on the other. We demonstrate how the trade-off works in both a stylized model of public investment and in a general monotone model.

The parametric results focus on the interaction between political bias and distributional change for creating a Faustian trade-off. When the resulting distortion is reinforcing, it gives rise to equilibria with monotone dynamics. By contrast, under a countervailing distortion, one can have both monotone and cyclical dynamics; however, only cyclical dynamics can arise if the distortion is strong enough.

In either case, Faustian trade-offs turn political leaders into "Burkean conservatives" who moderate their decisions in order to influence the future political evolution. This Burkean influence on individual incentives is a key consequence of the Faustian trade-off. However, the Burkean influence must be weighed against the dynamic change in political types when assessing the overall effect on the equilibrium path. In the short run, the Burkean influence dominates when the PE equilibrium is compared with the PA model. In the long run, however, the "type effect" dominates. This indicates that there are critical features in the transition dynamics of the Faustian model that would not be evident by focusing only on steady-state properties.

22. As Rodriguez and Weingast describe it, concessions in the Act were made to garner support from Northern Republicans. However, many Democrats as well as Republicans feared political fallout from Northern whites at the time. See, for instance, Stewart (1997).

A few modeling choices warrant further discussion. First, decision makers are assumed to be exclusively policy driven. Their desire for power is therefore purely instrumental. (This is in keeping with the original depiction of Faust as a well-intentioned character.) There are likely many historical examples of leaders who desire power for its own sake. It would not be difficult to incorporate “power-hungry” leaders into the model; however, this extension would be, in our view, rather prosaic.

Second, we omit stochastic shocks. Decision makers in the model choose not so much *whether* to lose power, but by *how much* and *to whom*. Consequently, we omit the case where leaders are uncertain about the political ramifications of their policies. As it turns out, shocks do not fundamentally change the nature of the Faustian trade-off. They do introduce, however, risk aversion into the motives of the leader and, for this reason, would be a useful addition to future work.

On a related point, note that the model looks only at endogenous changes in political power. Of course, there may also be incentive effects due to *exogenous* political change. These exogenous sources of change may be due to shocks, but they may also be built into the authority rule. Exogenous sources of political change would move the analysis closer to traditional models of dynamically inconsistent policy choice—for instance, hyperbolic $\beta-\delta$ policy models[23] as well as the famous fiscal policy models, of Persson and Svensson (1989) and Alesina and Tabellini (1990). In these models, a marginal effect somewhat similar to the preservation effect arises due to the conflict between current and future decision makers as calendar time changes the identity of the decision maker. Azzimonti’s (2005) parametric model shows this explicitly.

Third, the stylized (parametric) model generates some fairly specific results on public investment, growth, and political change. Because the investment is assumed to be financed by lump-sum taxation, no distortions arise other than that induced by the Faustian trade-off. This is by design, given the focus of the paper. However, recent papers of Battaglini and Coate (2007, 2008) have made inroads into our understanding of the dynamic political economy of distortionary taxation. At this stage, there is still much work to do, and in light of the present results, explorations on the interaction between Faustian and tax distortions seem well worth the effort.

Finally, throughout the analysis, we keep the political institution exogenous in the analysis. We focus only on the de facto evolution of the political power within a stable (*de jure*) political institution. This allows us to examine the consequences of exogenous changes in political institutions. By construction, the framework does not answer the question of why a certain political institution is chosen and what determines the evolution of the de jure political institution. Future work could investigate the interaction of the PE political power and PE political institutions.

APPENDIX

Proof of Parts (i) in Propositions 1 and 3. For brevity, we combine Parts (i) of Propositions 1 and 3 since the argument does not depend per se on whether $\kappa > 0$ or $\kappa < 0$.

We first conjecture a solution Ψ^* of the affine form $\Psi^*(\omega) = (d-K)\omega^* + K\omega$, where K is a constant (in ω), though we later show how it varies with κ. ω^* is the steady state that depends on κ and κ_0. The conjecture is used to characterize both Ψ^* and Ψ, the hypothetical rule. We establish a solution for coefficients (K,ω^*) and establish uniqueness. We then characterize the case with $\kappa = 0$ and show that $K = 0$ there. This case gives us the PA equilibrium ψ.

23. See Laibson (1997), Harris and Laibson (2001), Krusell, Kuruscu, and Smith (2002), Krusell and Smith (2003), Amador (2003), and Judd (2004).

Step 1. *Verifying the functional forms*. Using the affine form as our "guess", the flow utility is

$$u(i,\omega,a) = f(i)\omega - \frac{1}{2}\left((d-K)\omega^* + K\omega\right)^2$$
$$= f(i)\omega^* - \frac{1}{2}d^2\omega^{*2} + \left(f(i) - d\omega^* K\right)(\omega-\omega^*) - \frac{1}{2}K^2(\omega-\omega^*)^2.$$

For the purpose of solving for the equilibrium, we can drop the constant term. The continuation utility for an arbitrary i is

$$V\left(i,\omega_t;\ \Psi^*\right) = \sum_{s=0}^{\infty}\delta^s u\left(i,\omega_{t+s},\Psi^*(\omega_{t+s})\right)$$
$$= \sum_{s=0}^{\infty}\delta^s\left[\left(f(i) - d\omega^* K\right)(\omega_{t+s}-\omega^*) - \frac{1}{2}K^2(\omega_{t+s}-\omega^*)^2\right]$$
$$= \frac{f(i) - d\omega^* K}{1-\delta(K+1-d)}(\omega_t-\omega^*) - \frac{1}{2}\frac{K^2}{1-\delta(K+1-d)^2}(\omega_t-\omega^*)^2,$$

where the last line follows from the fact that $\omega_{t+s}-\omega^* = (K+1-d)^s(\omega_t-\omega^*)$. The last equality above requires convergence of the infinite sum that, in turn, requires $K+1-d < \frac{1}{\delta}$ and $(K+1-d)^2 < \frac{1}{\delta}$, which combines to $K+1-d < \frac{1}{\sqrt{\delta}}$. The hypothetical problem confronting an arbitrary citizen type i is

$$\max_{a_t}\ \left\{u(i,\omega_t,a_t) + \delta V\left(i,(1-d)\omega_t + a_t;\ \Psi^*\right)\right\},$$

which, when evaluated at the parametric assumptions, produces the first-order condition

$$-a_t + \delta\left[\frac{f(i) - d\omega^* K}{1-\delta(K+1-d)} - \frac{K^2}{1-\delta(K+1-d)^2}\left((1-d)\omega_t + a_t - \omega^*\right)\right] = 0. \tag{A1}$$

The first-order condition equation (A1) determines the hypothetical PE policy rule:

$$\Psi(i,\omega_t) = \frac{1}{1+\frac{\delta K^2}{1-\delta(K+1-d)^2}}\left[\delta\frac{f(i)}{1-\delta(K+1-d)}\right.$$
$$\left. + \delta\omega^* K\left(\frac{K}{1-\delta(K+1-d)^2} - \frac{d}{1-\delta(K+1-d)}\right) - \frac{\delta K^2(1-d)}{1-\delta(K+1-d)^2}\omega_t\right]. \tag{A2}$$

We return to the hypothetical equilibrium later. To determine the PE equilibrium, substitute $f(\mu(\omega)) = \kappa_0 + \kappa\omega$ in equation (A2) to derive

$$\Psi^*(\omega_t) = \frac{1}{1+\frac{\delta K^2}{1-\delta(K+1-d)^2}}\left[\frac{\delta\kappa_0}{1-\delta(K+1-d)}\right.$$
$$+ \delta\omega^* K\left(\frac{K}{1-\delta(K+1-d)^2} - \frac{d}{1-\delta(K+1-d)}\right)$$
$$\left. + \left(\frac{\delta\kappa}{1-\delta(K+1-d)} - \frac{\delta K^2(1-d)}{1-\delta(K+1-d)^2}\right)\omega_t\right]. \tag{A3}$$

Hence, in order for $\Psi^*(\omega_t) = (d-K)\omega^* + K\omega_t$ to be a PE equilibrium, we must have

$$K = \frac{\frac{\delta\kappa}{1-\delta(K+1-d)} - \frac{\delta K^2(1-d)}{1-\delta(K+1-d)^2}}{1+\frac{\delta K^2}{1-\delta(K+1-d)^2}} \quad \text{and} \tag{A4}$$

$$(d-K)\omega^* = \frac{\frac{\delta\kappa_0}{1-\delta(K+1-d)} + \delta\omega^* K\left(\frac{K}{1-\delta(K+1-d)^2} - \frac{d}{1-\delta(K+1-d)}\right)}{1+\frac{\delta K^2}{1-\delta(K+1-d)^2}}. \tag{A5}$$

We therefore have two equations and two unknowns, K and ω^*. In what follows, we verify that there exists a unique pair (K,ω^*) that satisfies equations (A4) and (A5).

Step 2°. *Existence and uniqueness of the pair* (K,ω^*). The equation for K in equation (A4) can be expressed as

$$K+\frac{\delta K^2(K+1-d)}{1-\delta(K+1-d)^2}-\frac{\delta\kappa}{1-\delta(K+1-d)}=0. \tag{A6}$$

Define $\widehat{K}=K+1-d$. Multiply both sides of the equation (A6) by

$$\left(1-\delta(K+1-d)^2\right)(1-\delta(K+1-d))$$

and after some messy algebra, the equation (A6) becomes

$$F(\widehat{K})\equiv\delta(1-d)\widehat{K}^3+\left(\kappa\delta-(2-d)-\delta(1-d)^2\right)\widehat{K}^2$$
$$+\left(\frac{1}{\delta}+(1-d)+(1-d)^2\right)\widehat{K}-\left(\kappa+\frac{1}{\delta}(1-d)\right)=0. \tag{A7}$$

Observe that the function F is of the form $F(\widehat{K})=a_0(\kappa)+a_1\widehat{K}+a_2(\kappa)\widehat{K}^2+a_3\widehat{K}^3$ (given the expression above, it should be clear that a_0 and a_2 vary with κ, whereas a_1 and a_3 do not). We then have

$$F(1-d)=-\kappa\left(1-\delta(1-d)^2\right),$$
$$F(1)=(1-\delta)\left(d(\frac{1}{\delta}-1+d)-\kappa\right),$$
$$F(-1)=\left(\kappa+\frac{1-d}{\delta}\right)(\delta-1)-(1+\delta)\left[(1-d)^2+\frac{3+\delta}{1+\delta}(1-d)+\frac{1}{\delta}\right],$$
$$F(0)=-\left(\kappa+\frac{1}{\delta}(1-d)\right).$$

Suppose first that $0<\kappa<d\left(\frac{1}{\delta}-1+d\right)$ as required in Proposition 1. Then, $F(1-d)<0$ and $F(1)>0$. Then, from the intermediate value theorem, there exists a $\widehat{K}^*$ such that $1-d<\widehat{K}^*<1$ (or equivalently $0<K^*<d$) and $F(\widehat{K}^*)=0$.

Suppose next that $0>\kappa>-\frac{1-d}{\delta}-\frac{1+\delta}{1-\delta}\left[(1-d)^2+\frac{3+\delta}{1+\delta}(1-d)+\frac{1}{\delta}\right]$ as required in Proposition 2. Then, $F(-1)<0$ and $F(1-d)>0$. Once again, we apply the intermediate value theorem to show that there exists a $-1<\widehat{K}^*<(1-d)$ (or equivalently $-(2-d)<K^*<0$) such that $F(\widehat{K}^*)=0$.

To show uniqueness of the solution $\widehat{K}^*$ in either the case of $\kappa>0$ or $\kappa<0$, it suffices to show that $F\left(\widehat{K}\right)$ is concave, *i.e.*, $F''\left(\widehat{K}\right)=2\left(a_2(\kappa)+3a_3\widehat{K}\right)<0$ for $-1\le\widehat{K}\le 1$. Towards this goal, it suffices to show that $a_2(\kappa)+3a_3<0$ (since $a_3>0$), *i.e.*, $\left[\kappa\delta-(2-d)-\delta(1-d)^2\right]+3\delta(1-d)<0$. The latter is equivalent to the equation

$$\kappa\le d\left(\frac{1}{\delta}-1+d\right)+2\left(\frac{1}{\delta}-1\right)(1-d),$$

which is always true since $\kappa<d\left(\frac{1}{\delta}-1+d\right)$. We conclude that $\widehat{K}^*$, and hence $K^*=\widehat{K}^*-1+d$, is unique.

Having established a unique solution, K^*, we now solve for the steady state, ω^*, from equations (A5) and (A6). After some algebra, we obtain

$$\omega^*=\frac{\kappa_0}{d\left(\frac{1}{\delta}-(1-d)\right)-\kappa}. \tag{A8}$$

We have therefore established a unique pair (K^*,ω^*) with K^* as the slope of Ψ^* and ω^* as the steady state satisfying equation (A8). As the solution to $F(K+1-d)=0$, notice that K^* varies with κ. We write $K^*=B(\kappa)$ to emphasize the dependence on κ. By the definition of F in equation (A7), it is clear that $B(0)=0$, which then yields an equation for the PA equilibrium ψ. As for the hypothetical PE rule, Ψ, its solution form is also affine. The coefficients of Ψ can be recovered from K^* and ω^* evaluated at their respective solutions. The slope is non-positive and equal to zero iff $\kappa=0$ or $d=1$. ‖

Rest of the proof of Proposition 1. We now turn to Part (ii) of Proposition 1. To prove that convergence to the steady state is monotone, observe that

$$\Theta^*(\omega_t) \equiv (1-d)\omega_t + \Psi^*(\omega_t) = (1-d+K^*)\omega_t + (d-K^*)\omega^*. \tag{A9}$$

Since $0 < K^* < d$, if $0 < \kappa < d\left(\frac{1}{\delta}-1+d\right)$, then convergence towards the steady state is monotonically increasing if $\omega_0 < \omega^*$ and monotonically decreasing if $\omega_0 > \omega^*$.

The rest of Parts (ii) and (iii) can be readily verified from straightforward algebra. We omit the details and only give a sketch for the case $\omega_0 < \omega^*$. The comparison of Ψ^* and $\Psi(i_0, \omega)$ follows directly from the affine solution and the fact that $f(\mu(\omega))$ is increasing in ω. The comparison of ψ and $\Psi(i_0, \omega)$ follows from the higher slope of ψ and the fact that $\Psi(i_0, \omega) < \psi(i_0, \omega)$, where the latter can be checked easily from the solution. Given Part (ii) and the fact that the steady state is higher under PE equilibrium, Part (iii) then follows from the intermediate value theorem. ‖

Rest of the proof of Proposition 3. For Part (ii), an inspection of equation (A9) reveals that $D_\omega\Theta^* > 0$ iff $\widehat{K}^* = 1-d+K^* > 0$. Recalling the definition of F in the proof of Proposition 1, it is clear that $F(0) < 0$ and $F(1-d) > 0$ if $0 > \kappa > -(1-d)/\delta$, and so the zero of F must satisfy $(1-d) > \widehat{K}^* > 0$. On the other hand, if $-(1-d)/\delta > \kappa > -\frac{1-d}{\delta} - \frac{1+\delta}{1-\delta}\left[(1-d)^2 + \frac{3+\delta}{1+\delta}(1-d) + \frac{1}{\delta}\right]$, then $F(0) > 0$ and $F(-1) < 0$, and so the zero of F must satisfy $-1 < \widehat{K}^* < 0$. In the latter case, the Faustian dynamics constitute a dampened cycle whereby, on path, $\omega_t < \omega^*$ iff $\omega_{t+1} > \omega^*$.

The rest of Part (ii) follows from a similar argument as the corresponding part of Proposition 1, with the main difference being a negative κ. Hence, we skip the details here. ‖

Proof of Proposition 2. We restrict attention to the case where $\omega_0 < \omega^*$, *i.e.*, the initial state lies below the steady state. The logic when $\omega_0 > \omega^*$ is symmetric.

It is easy to show that the steady state with κ is larger than that of $\widetilde{\kappa}$ whenever $\kappa > \widetilde{\kappa}$. Given this and the affine form of equilibrium, to show the final result it suffices to show that $\Psi^*(\omega_0) < \widetilde{\Psi}^*(\omega_0)$. Let $K^* = B(\kappa)$ and $\widetilde{K}^* = B(\widetilde{\kappa})$ denote the respective slope of PE equilibria Ψ^* and $\widetilde{\Psi}^*$. We want to show that

$$\left(d-\widetilde{K}^*\right)\frac{\widetilde{\kappa}_0}{d\left(\frac{1}{\delta}-1+d\right)-\widetilde{\kappa}} + \widetilde{K}^*\omega_0 > \left(d-K^*\right)\frac{\kappa_0}{d\left(\frac{1}{\delta}-1+d\right)-\kappa} + K^*\omega_0,$$

which is equivalent to

$$\frac{d-B(\widetilde{\kappa})}{d\left(\frac{1}{\delta}-1+d\right)-\widetilde{\kappa}} < \frac{B(\kappa)-B(\widetilde{\kappa})}{\kappa-\widetilde{\kappa}}. \tag{A10}$$

In words, the last inequality requires that the slope between the points $(\widetilde{\kappa}, B(\widetilde{\kappa}))$ and $(\kappa, B(\kappa))$ is larger than that between $(\widetilde{\kappa}, B(\widetilde{\kappa}))$ and $(d(\frac{1}{\delta}-1+d), d)$ whenever $\kappa > \widetilde{\kappa}$.

To proceed, we need to further characterize the properties of the solution $K^* = B(\kappa)$. But since K^* and $\widehat{K}^*$ differ only by a constant, we examine properties of the solution $\widehat{B}(\kappa) \equiv \widehat{K}^* = \widehat{B}(\kappa) + 1 - d$ below. Recall that the unique solution $\widehat{K}^* = \widehat{B}(\kappa)$ is defined implicitly by the equation $F\left(\widehat{K}\right) = 0$. Notice that, at the solution $\widehat{K}^*$, $F\left(\widehat{K}^*\right)$ satisfies

$$F'\left(\widehat{K}^*\right) = a_1 + 2a_2(\kappa)\widehat{K}^* + 3a_3\left(\widehat{K}^*\right)^2 > 0,$$

$$F''\left(\widehat{K}^*\right) = 2\left(a_2(\kappa) + 3a_3\widehat{K}^*\right) < 0.$$

Since $F'\left(\widehat{B}(\kappa)\right) > 0$, $\widehat{B}(\kappa)$ is continuously differentiable from the implicit function theorem. In addition, we know that

$$\widehat{B}'(\kappa) = \frac{1-\delta\left(\widehat{B}(\kappa)\right)^2}{a_1 + 2a_2(\kappa)\widehat{B}(\kappa) + 3a_3\left(\widehat{B}(\kappa)\right)^2} > 0.$$

Taking the derivative again and after some algebra, we have

$$\widehat{B}''(\kappa) = \frac{-2\left(1-\delta\left(\widehat{B}(\kappa)\right)^2\right)\left[3\delta a_3\left(\widehat{B}(\kappa)\right)^3 + 3\delta a_2(\kappa)\left(\widehat{B}(\kappa)\right)^2 + (2\delta a_1 + 3a_3)\widehat{B}(\kappa) + a_2(\kappa)\right]}{\left[a_1 + 2a_2(\kappa)\widehat{B}(\kappa) + 3a_3\left(\widehat{B}(\kappa)\right)^2\right]^3}.$$

Use the fact $a_0(\kappa) + a_1\widehat{B}(\kappa) + a_2(\kappa)\left(\widehat{B}(\kappa)\right)^2 + a_3\left(\widehat{B}(\kappa)\right)^3 = 0$ to get

$$\widehat{B}''(\kappa) = \frac{2\left(1-\delta\left(\widehat{B}(\kappa)\right)^2\right)\left(1-\delta\left(1-d^2\right)\right)\left[\widehat{B}(\kappa) - b(\kappa)\right]}{\left[a_1 + 2a_2(\kappa)\widehat{B}(\kappa) + 3a_3\left(\widehat{B}(\kappa)\right)^2\right]^3},$$

where $b(\kappa) = \frac{-\delta d^2 - 2(1-\delta)d + (1-\delta) + 4\delta\kappa}{1-\delta(1-d^2)}$. Consequently, the sign of $\widehat{B}''(\kappa)$ is the same as that of $\widehat{B}(\kappa) - b(\kappa)$.

In the following part, we show that there exists a unique $0 < \overline{\kappa} < d(\frac{1}{\delta} - 1 + d)$ such that (i) $\widehat{B}(\kappa) > b(\kappa)$ for $\kappa < \overline{\kappa}$; and (ii) $\widehat{B}(\overline{\kappa}) = b(\overline{\kappa})$ for $\kappa = \overline{\kappa}$; and (iii) $\widehat{B}(\kappa) < b(\kappa)$ for $\kappa > \overline{\kappa}$. To start with, it is easy to see that $\widehat{B}(0) > b(0)$ and $\widehat{B}(d(\frac{1}{\delta} - 1 + d)) \leq b\left(d\left(\frac{1}{\delta} - 1 + d\right)\right)$. From the intermediate value theorem, there exists a $\overline{\kappa}$ such that $\widehat{B}(\overline{\kappa}) = b(\overline{\kappa})$.

Now we show that $\widehat{B}(\kappa) > b(\kappa)$ for $\kappa < \overline{\kappa}$ and $\widehat{B}(\kappa) < b(\kappa)$ for $\kappa > \overline{\kappa}$. To show this, let

$$G(\kappa) = F(b(\kappa)) = a_3 (b(\kappa))^3 + a_2(\kappa)(b(\kappa))^2 + a_1 b(\kappa) + a_0(\kappa).$$

The derivative of $G(\kappa)$ can be calculated as

$$G'(\kappa) = \left[3a_3 (b(\kappa))^2 b'(\kappa) + 2a_2(\kappa) b(\kappa) b'(\kappa) + \delta (b(\kappa))^2 + a_1 b'(\kappa) - 1\right],$$

which equals to a quadratic form in $b(\kappa)$,

$$\frac{6}{1-\delta(1-d^2)}\left[\frac{1}{2}\delta\left[\delta d^2 - 4\delta d + 1 + 3\delta\right](b(\kappa))^2 \right.$$
$$\left. -\delta\left[\delta d^2 - 2(1+\delta)d + (3+\delta)\right] b(\kappa) + \frac{1}{2}\left[\delta d^2 - 4\delta d + 1 + 3\delta\right]\right].$$

To see the sign of $G'(\kappa)$, first notice that the discriminant for the quadratic equation inside the bracket is

$$\delta^2\left[\delta d^2 - 2(1+\delta)d + (3+\delta)\right]^2 - \delta\left[\delta d^2 - 4\delta d + 1 + 3\delta\right]^2$$
$$= \delta(\delta - 1)\left(1 - \delta(1-d)^2\right) < 0.$$

Combining this with the positive quadratic coefficient, *i.e.*, $\delta d^2 - 4\delta d + 1 + 3\delta = \delta d^2 + (1 - \delta d) + 3\delta(1-d) > 0$, we know that $G'(\kappa) > 0$ for every κ. As a result, $G(\kappa) < 0$ for $\kappa < \overline{\kappa}$ and $G(\kappa) > 0$ for $\kappa > \overline{\kappa}$. Therefore, $\widehat{B}(\kappa) > b(\kappa)$ for $\kappa < \overline{\kappa}$ and $\widehat{B}(\kappa) < b(\kappa)$ for $\kappa > \overline{\kappa}$.

Translating back to our original coefficient, $B(\kappa)$, we have thus established $B'(\kappa) > 0$, and $B''(\kappa) > 0$ for $\kappa < \bar{\kappa}$ and $B''(\kappa) < 0$ for $\kappa > \bar{\kappa}$. In addition, it is easy to check that $B'(0) = \frac{1}{\frac{1}{\delta} - 1 + d}$, $B(0) = 0$, and $B(d(\frac{1}{\delta} - 1 + d)) = d$. These properties together imply that $B(\kappa) \geq \frac{\kappa}{\frac{1}{\delta} - 1 + d}, \forall \kappa < d(\frac{1}{\delta - 1 + d})$. Geometrically, $B(\kappa)$ is an increasing function, convex below $\bar{\kappa}$ and concave above $\bar{\kappa}$. In addition, $B(\kappa)$ lies above the line $\frac{\kappa}{\frac{1}{\delta} - 1 + d}$, with a unique tangent point at $\kappa = 0$.

Now we are ready to prove the inequality (A10) from a geometric argument. Given the shape of $B(\kappa)$ established earlier, a simple graph shows that the inequality (A10) always holds for $\kappa < d(\frac{1}{\delta - 1 + d})$. This concludes the proof of the proposition. ||

Proof of Theorem 1. The proof is contained in the proof of theorem 3.

Proof of Theorem 2. From definition, $u(i, \omega, a) = \tilde{u}(i, \omega, Q(\omega, a))$. Using this fact to replace u with $\tilde{u}$ in equation (18a), after some algebra it follows that

$$R(i, \omega_t, \Psi^*) = D_{\omega_t} \tilde{u}(i, \omega_t, \Theta^*(\omega_t)), \tag{A11}$$

where $\Theta^*(\omega_t) = Q(\omega_t, \Psi^*(\omega_t))$ for any Ψ^* (including PA equilibrium $\psi(\omega)$). From Assumption (A1), $R(i, \omega_t, \Psi^*(\omega_t)) > R(i, \omega_t, \psi(\omega_t))$ if and only if $Q(\omega_t, \Psi^*(\omega_t)) > Q(\omega_t, \psi(\omega_t))$. From the monotonicity of $Q(\omega, a)$ in a, $Q(\omega_t, \Psi^*(\omega_t)) > Q(\omega_t, \psi(\omega_t))$ if and only if $\Psi^*(\omega_t) > \psi(\omega_t)$. This concludes the proof. ||

Proof of Theorem 3. For any arbitrary, smooth, strict supermodular continuation value $U(i, \omega)$, define

$$H(i, \omega_t, a_t, U) = u(i, \omega_t, a_t) + \delta U(i, Q(\omega_t, a_t)). \tag{A12}$$

$H(i, \omega_t, a_t, U)$ is the pay-off function of a citizen type i in state ω_t when his continuation is defined by $U(i, \omega_{t+1})$. Let $\Psi(i, \omega, U) \in \arg\max_a H(i, \omega_t, a_t, U)$ and let $\Psi^*(\omega, U) = \Psi(\mu(\omega), \omega, U)$.

To prove the monotonicity property, it is more convenient to work with a related representation as

$$\widetilde{H}(i, \omega_t, \omega_{t+1}, U) = \tilde{u}(i, \omega_t, \omega_{t+1}) + \delta U(i, \omega_{t+1}), \tag{A13}$$

where $\tilde{u}(i,\omega_t,\omega_{t+1})$ is defined in Assumption A1. Define $\Theta(i,\omega,U)\in\arg\max_{\omega_{t+1}}\widetilde{H}(i,\omega_t,\omega_{t+1},U)$ and $\Theta^*(\omega,U)=\Theta(\mu(\omega),\omega,U)$.

From the definition, it is immediate that $H(i,\omega_t,a_t,U)=\widetilde{H}(i,\omega_t,Q(\omega_t,a_t),U)$, $\Theta(i,\omega,U)=Q(\omega,\Psi(i,\omega,U))$, and $\Theta^*(\omega,U)=Q(\omega,\Psi^*(\mu(\omega),\omega,U))$. In addition, since $D_aQ(\omega,a)>0$, $H(i,\omega_t,a_t,U)$ is supermodular in (i,a) if and only if $\widetilde{H}(i,\omega_t,\omega_{t+1},U)$ is supermodular in (i,ω_{t+1}).

We now proceed with the proof. From Assumption A1, $\widetilde{H}(i,\omega_t,\omega_{t+1},U)$ is smooth, supermodular in $(i,\omega_t,\omega_{t+1})$, and strictly supermodular in (i,ω_{t+1}). By the Topkis monotonicity theorem, $\Theta(i,\omega_t,U)$ is non-decreasing in (i,ω_t) and strictly increasing in i for any U. Hence, we have shown that $\Theta^*(\omega,U)=\Theta(\mu(\omega),\omega,U)$ is increasing in ω. Combined with Assumption A1, this implies that

$$D_iD_\omega\widetilde{H}(i,\omega,\Theta^*(\omega,U),U)=D_iD_\omega\tilde{u}+\delta D_\omega\Theta^*\cdot D_iD_{\omega'}U(i,\omega')>0.$$

As a result, the map $U\mapsto H(i,\omega,\Theta^*(\omega,U),U)$ maps from functions with strictly increasing differences in i and ω to functions of the same.

Consider then the finite-horizon PE equilibrium with horizon T. Let $\Theta_T=\{\Theta^*_{t,T}\}_{t=1}^T$ denote the PE equilibrium transition in the T-period model, and let $U_{t,T}$ denote the value function in each period t. Notice that $U_{T,T}=\tilde{u}$, which satisfies strictly increasing differences in (i,ω) by Assumption A1.[24] A simple backward induction argument establishes that $U_{t,T}$ satisfies strictly increasing differences for all t. From the definition of the smooth limit equilibrium, $||U_{t,T}-V(\cdot\,;\,\Theta^*)||\to 0$ as $T\to\infty$, and $V(\cdot\,;\,\Theta^*)$, the infinite-horizon PE equilibrium continuation value, has increasing differences in i and ω. In fact, by repeating the step of the previous paragraph, it follows that $V(\cdot\,;\,\Theta^*)$ must have strictly increasing differences in i and ω.

Consequently, the solution $\Theta(i,\omega)$ must be strictly increasing in i, and so Θ^* must be strictly increasing in ω. This completes the proof of monotonicity of Θ^*.

This argument established $\Theta^*(\omega_t)=\Theta(\mu(\omega_t),\omega_t)>\Theta(i_0,\omega_t)$ for all $\omega_t>\omega_0$ or by definition

$$Q(\omega_t,\Psi^*(\omega_t))=\Theta^*(\omega_t)\;>\;\Theta(i_0,\omega_t)=Q(\omega_t,\Psi(i_0,\omega_t)).\tag{A14}$$

From the monotonicity of $Q(\omega,a)$ in a, we have $\Psi^*(\omega_t)>\Psi(i_0,\omega_t)$. ‖

Proof of Theorem 4. From definition, $\Theta(\omega)=Q(\omega,\Psi^*(\omega))$, with the derivative

$$D_\omega\Theta(\omega)=D_\omega Q+D_aQD_\omega\Psi^*.\tag{A15}$$

Combine equations (A15) and (18b) to get

$$P(i,\omega_{t+1};\,\Psi^*)=(D_{a_{t+1}}Q)^{-1}\cdot D_{\omega_{t+1}}\Theta^*\cdot\Delta(i,\omega_{t+1};\Psi^*).\tag{A16}$$

We know that $D_{a_{t+1}}Q>0$ and from the proof of Theorem 3, $D_{\omega_{t+1}}\Theta^*>0$. Therefore, $P(i,\omega_{t+1};\,\Psi^*)<0$ follows if and only if $\Delta(i,\omega_{t+1};\Psi^*)<0$.

Let $V^*(i,\omega)\equiv V(i,\omega;\,\Psi^*)$ be the equilibrium continuation value. Using the definition of H in equation (A12) and that of the distortion function Δ in equation (14), we have

$$D_{a_{t+1}}H(i_t,\omega_{t+1},\Psi^*(\omega_{t+1})),V^*)=\Delta(i_t,\omega_{t+1};\Psi^*).\tag{A17}$$

Using the proof of Theorem 3, which establishes strictly increasing differences of H in i and a, it follows that $\Delta(i,\omega_{t+1};\Psi^*)$ is increasing in i. It implies that whenever $i<i_{t+1}=\mu(\omega_{t+1})$, then

$$\Delta(i,\omega_{t+1};\Psi^*)\;<\;\Delta(i_{t+1},\omega_{t+1};\Psi^*)\;=\;0,\tag{A18}$$

where the latter equality follows from type i_{t+1}'s first-order condition in state ω_{t+1}. This concludes the proof. ‖

Proof of Theorem 5. Part (i). The PA case follows from the standard argument. For the PE equilibrium, the necessity part is obvious. To see the sufficiency, notice that the equation coincides with the steady-state equation of the PA equilibrium with $i_0=\mu(\omega^*)$. Since a PA decision maker faces fewer constraints than a PE one, if a PA authority decides to keep the state constant, it must be the optimal choice of a PE authority.

Part (ii) is obvious given Part (i).

24. In the last period T, $\underline{\omega}$ is chosen by the decision maker since there is no future return to the costly investment.

We now prove Part (iii), *i.e.*, $\omega^\circ < \omega^*$ if and only if $\omega_0 < \omega^*$. Consider $i_0 = \mu(\omega_0)$ and $i^* = \mu(\omega^*)$. From the strict increasing difference between i and (ω_t, ω_{t+1}), for each $(\omega_t, \omega_{t+1}, \omega_{t+2})$

$$D_{\omega_{t+1}}\widetilde{u}\left(i^*, \omega_t, \omega_{t+1}\right) + \delta D_{\omega_{t+1}}\widetilde{u}\left(i^*, \omega_{t+1}, \omega_{t+2}\right)$$
$$> D_{\omega_{t+1}}\widetilde{u}\left(i_0, \omega_t, \omega_{t+1}\right) + \delta D_{\omega_{t+1}}\widetilde{u}\left(i_0, \omega_{t+1}, \omega_{t+2}\right)$$

if and only if $i_0 < i^*$. Evaluate at $(\omega_t, \omega_{t+1}, \omega_{t+2}) = (\omega^\circ, \omega^\circ, \omega^\circ)$ to get

$$\begin{aligned} &D_{\omega_{t+1}}\widetilde{u}\left(i^*, \omega^\circ, \omega^\circ\right) + \delta D_{\omega_{t+1}}\widetilde{u}\left(i^*, \omega^\circ, \omega^\circ\right) \\ &\quad > D_{\omega_{t+1}}\widetilde{u}\left(i_0, \omega^\circ, \omega^\circ\right) + \delta D_{\omega_{t+1}}\widetilde{u}\left(i_0, \omega^\circ, \omega^\circ\right) \\ &\quad = 0 \\ &\quad = D_{\omega_{t+1}}\widetilde{u}\left(i^*, \omega^*, \omega^*\right) + \delta D_{\omega_{t+1}}\widetilde{u}\left(i^*, \omega^*, \omega^*\right) \end{aligned}$$

if and only if $i^* > i_0$. Since $D_{\omega_{t+1}}\widetilde{u}(i, \omega, \omega) + \delta D_{\omega_{t+1}}\widetilde{u}(i, \omega, \omega)$ is decreasing in ω, the inequality holds if and only if $\omega^\circ < \omega^*$. To summarize, we have just proven that $\omega^\circ < \omega^*$ if and only if $i_0 < i^*$, *i.e.*, $\omega_0 < \omega^*$. ‖

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