Bank Competition and Lending Policies over the Business Cycle*

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Abstract

Two empirical regularities of bank lending policies stand out: both interest rate spreads on loans and lending standards become lower during booms than in recessions. I develop a repeated game model of bank competition to explain these two facts, stressing procyclical competition of the banking sector as the driving force. Facing private information on borrowers, banks compete by choosing both the interest rates on loans and the lending standards, which are identified with the screening intensities used in the costly screening process. Over time, aggregate shocks affect a bank's payoff. In the equilibrium, better business conditions during booms increase bank's incentive to deviate ceteris paribus, thus forcing banks to compete more to shrink the profit margin and to restore the equilibrium incentive constraint. As a result, banks charge lower interest rates and impose looser standards during booms, while the opposite happens during recessions.

Keywords: Bank competition; Costly screening; Bank lending policy; Business cycles.

JEL codes: E32, G21, L13.

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I Introduction

A consensus among economists has emerged in the aftermath of the recent subprime crisis. The excessively lax mortgage lending standards used by banks and other types of lenders in the years preceding the crisis made too much credit be extended to borrowers whose ability of repaying debt was highly problematic. The resulting tremendous volume of credit flowing into the housing market helped fuel the historic housing boom, and the feedback effect of rapid appreciations in housing value in turn masked most of the unsound lending practices until the eruption of the crisis. Despite the consensus on the central role of the lax lending standards in paving the way for the crisis, there are still a lot of debates on why the lending standards became so low before the crisis.

To better understand the mechanisms of the relaxation of the lending standards before the crisis, I take one step back by focusing on the dynamics of bank lending polices over the business cycle. There are two empirical regularities concerning the bank lending dynamics: both the interest rate spreads on loans and the lending standards are lower during booms than in recession, i.e., bank lending polices are countercyclical.[1] I develop a model to study the dynamics of bank lending policies, including both the lending standards and the interest rates on loans, over the business cycle. By considering an environment in which the banking sector is imperfectly competitive, I allow bank competition to drive changes in the lending policies over time. I argue that competition matters a lot for the determination of the bank lending polices. Indeed, the model generates endogenous, procyclical bank competition, which in turn leads to countercyclical lending polices: as banks compete more during booms, they reduce the interest rates on loans and relax the lending standards.

In the baseline framework, a given number of symmetric banks and a continuum borrowers play a lending game in each period, and the lending game is repeated over time. Facing private information on a borrower's creditworthiness, banks rely on a costly screening technology to distinguish good borrowers from bad ones by choosing the screening intensities used in their screening processes.[2] The screening intensity determines the precision of the signal obtained on

[1]For the dynamics of interest rate spreads, see, e.g., Bernanke and Blinder (1992); for the dynamics of the lending standards, see Asea and Blomberg (1998), Berger and Udell (2004), and Lown and Morgan (2006).

[2]As noted by Rajan (1994), the lending standards reduce to the simple NPV rule and hence remain constant whenever information is perfect. In view of this, some form of asymmetric information is necessary for a meaningful discussion of variations in the lending standards.

a borrower in the screening process, and I identify a bank's lending standards with its screening intensity. Based on the signal obtained in the screening process, a banks makes the lending decision on any borrower it faces. Meanwhile, banks also choose the interest rates on the loans extended to the approved borrowers. I show that only pooling equilibrium exists in the lending game. Moreover, for a given level of the interest rate on loans, a bank chooses the screening intensity optimally. The resulting efficient screening intensity varies positively with the interest rate, as higher interest rate implies higher profitability on loans. This result provides an underpinning of the positive correlation between the two components of the bank lending policies.

There are three types of aggregate shocks affecting a bank's payoff over time: a pure quantity credit demand shock, a collateral value shock, and a risk distribution shock. These shocks captures different aspects of business cycle fluctuations. Building on a series of preliminary results established for the lending game, the repeated game is highly tractable even with the presence of the aggregate shocks so that I can analytically characterize the optimal symmetric subgame perfect equilibrium. As in the seminal work of Rotemberg and Saloner (1986), for a range of values of banks' common discount factor and the number of banks, the optimal equilibrium displays following feature: banks compete more during booms in the sense that the forgone profit of the banking sector is higher during booms than in recessions. The underlying intuition is quite simple. For all three types of aggregate shocks, a higher shock realization during booms increases a bank's payoff, hence its incentive to deviate, ceteris paribus. Banks therefore need to compete more to reduce the prevailing profit, for otherwise the equilibrium incentive constraint will be violated. Putting differently, bank competition is procyclical along the equilibrium path.

Procyclical competition translates into time-varying bank lending policies. For all three shocks, more competition during booms forces banks to charge lower interest rates, resulting in lower interest rate spreads as the risk-free rate is normalized to zero. Thus banks' interest rate policy is always countercyclical. The equilibrium dynamics of the lending standards has a more delicate structure. For the case of the credit demand shock, the equilibrium screening intensity becomes lower during booms unambiguously. As the shock is purely quantitative and does not affect the unit payoff of a loan, a lower interest rate during booms leads to a lower screening intensity. In contrast, both the collateral value shock and the risk distribution shock increase the profitability of a loan during booms, ceteris paribus, and the increase may not be more than offset by a lower equilibrium interest rate, thus the equilibrium screening intensity may not be lower during booms. However, I can still prove that under some additional parametric restrictions, equilibrium screening intensity becomes lower during booms. In sum, for all three types of aggregate shocks and a wide range of parameter combinations, procyclical competition results in countercyclical lending policies. This conclusion is robust to various modifications of the basic framework.

In the rest of the introduction, I review related literature. In Section II, I lay out the basic model setup. In Section III, I establish a series of results for the lending game within each period. In Section IV, I characterize the equilibrium dynamics of the model. In Section V, I discuss the robustness of the basic model framework and check the welfare implications of the model. The proofs of all results in the main text are relegated to Appendix A; the supplementary results that require lengthy discussion are collected in the Online Appendix.

Related literature This paper contributes to the literature of bank lending and competition in several ways.

The first contribution is on the modeling of the stage lending game. I model the lending process as an extensive form game with incomplete information. As the uninformed party, i.e., banks, move first, the game has the flavor of a signalling game, supplemented with a third stage in which banks can reject borrowers based on the screening results. In this regard, the model is closely related to the models of Hellwig (1987) and Hillas (2002).[3] A major difference between my work and theirs is that these authors rely on the stability refinement of Kohlberg and Mertens (1986) for obtaining a pool equilibrium, whereas I establish directly that only pooling equilibrium is possible and rely on the undefeatedness criterion of Mailath et al. (1993) to get a fairly reasonable equilibrium of the lending game. By explicitly assuming that borrowers are privately informed, this paper also differs from a number of works in which the information is imperfect but neither borrowers nor banks are informed a priori.[4]

A large number of works in bank competition follow the path-breaking paper of Broecker (1990) in modeling the screening and pricing process, which makes the model resemble a common-value auction model as a borrower applies for credit from every bank and each bank's signal about the

[3]After I obtained all the main results, I came across the unpublished work of Hillas (2002) which has a very similar setup as the stage lending game in this paper. Hillas' setup differs from mine in an important way that the screening intensity is not variable but given exogenously at a constant cost.

[4]See Manove et al. (2001) and recent works by Burke et al. (2012) and Wang (2015). This alternative information structure circumvents a major difficulty caused by the strategic interactions between informed borrowers and uninformed banks. Arguably, assuming borrowers have private information gives more binds for the notion of the lending standards as identified by the screening intensity.

borrower is imperfectly correlated.[5] However, the discreteness of the signal on each borrower leads to the existence of mixed strategy equilibrium only,[6] which represents a major difficulty in doing comparative static analyses and in extending the model to the dynamic case. In contrast, following the signalling game literature, I impose the exclusivity assumption that a borrower can apply for credit from only one bank in each period, and this guarantees the existence of pure strategy equilibria,[7] which helps for the tractability of the dynamic model. As I argue below, this is not a restrictive assumption for the model economy, and is perhaps a more realistic way for modeling the lending process.

The second contribution is on the dynamics. To my best knowledge, this paper is the first one to develop of a tractable repeated game model of bank competition in which banks choose both the interest rates and the lending standards with the presence of the aggregate shocks.[8] This allows a rigorous study of the jointly endogenous dynamics of bank competition and lending policies over the business cycle. As already argued, the framework developed in this paper gives rich and sharp predictions on the dynamics of the banking sector, many of which can hardly be obtained in a static framework.

II The Model

Time is discrete and infinite. There are $N < \infty$ symmetric banks and a continuum of borrowers, indexed by $i \in N \equiv \{1, \ldots, N\}$ and $j \in [0, 1]$. Both are risk-neutral and infinitely lived. In every period t, borrowers seek credit from competing banks in a credit market.

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[5] See the subsequent work of Cao and Shi (2001), Ruckes (2004), and Hauswald and Marquez (2003, 2006).

[6] When the signal is continuously valued, existence of a purely strategy equilibrium is restored. See Riordan (1993). Yet the equilibrium strategy is still a complicated function of the signal.

[7] Notably, Thakor (1996) modifies the extensive form structure of Broecker (1990) in a way to make the model admits a signalling game structure and proceeds to obtain pure strategy equilibrium.

[8] There are prior works on dynamic bank competition. Bagliano et al. (2000) and Chami and Cosimano (2001, 2010) are among the first to model bank competition as using repeated games, yet they rely on a reduced form demand function and do not model the lending process, thus the lending standards. Gorton and He (2008) propose a repeated game of bank competition with screening, yet their model is left unsolved due to tractability issue. Gehrig and Stenbacka (2004, 2011) also construct a dynamic model of bank competition with screening. However, their model is deterministic where as my model features stochastic aggregate shocks. More importantly, there is no direct strategic interactions among banks (they call financiers) as banks are randomly matched with borrowers (they call entrepreneurs); and banks' actions affect one another only indirectly through the change of borrower composition over time. Corbae and D'Erasmo (2013, 2014) develop quantitative models of banking industry dynamics, yet the banks in their models do not choose the lending standards directly.

II.A Borrower and Project

At the beginning of time t, each one of ex ante identical borrowers is hit by an idiosyncratic investment opportunity shock θ, taking one of two values in $\{\theta^g, \theta^b\} \subset [0,1]$ with $\Pr(\theta^g) = \bar{\mu}$ and $\Pr(\theta^b) = 1 - \bar{\mu}$. Identify Θ with $\{g, b\}$, indicating "good" and "bad"; hence a borrower can be one of two types. Investment shock is iid across borrowers and over time, so that the proportion of good borrower is always $\bar{\mu}$. Coming with shock θ is a one period investment project. For one unit investment, a θ project produces x units perishable output when succeeds, with probability θ, or c units when fails, with probability $1-\theta$, where $0 \le c < 1 < x$. The random output is iid over projects. All projects are indivisible and have a common size z, so a project either can not start or receives an investment z and produces zx or zc units of output.

By normalizing economy-wide risk free rate to 0, the expected net present value of a θ project is $\text{NPV}^\theta = \theta x + (1-\theta)c - 1$. Distinction between good and bad project is that $\text{NPV}^g > 0 > \text{NPV}^b$, hence good borrower is creditworthy while bad borrower is not. This is part (*i*) of the following assumption.

Assumption 1. (*i*) $\theta^g > (1-c)/(x-c) > \theta^b$, and (*ii*) $1 > \bar{\mu} > \theta^b/(\theta^g + \theta^b)$.

Part (*ii*) simply states that the proportion of good borrowers can not be too low.

The realization of θ to a borrower is private information. Banks do not know individual borrower's type; yet its distribution, together with other parameters of the model, is common knowledge to both banks and borrowers.

II.B Financing and Screening

Borrowers receive no endowment, neither do they possess a storage technology which transforms previous period surplus into current period resource. As a result they rely on bank lending to finance their projects. All borrowers are protected by limited liability, so that lending to bad ones can never be profitable as $\text{NPV}^b < 0$. In addition, borrower's type is private information, therefore banks have incentive to screen out bad borrowers before extending credits. Each bank owns a costly and noisy screening technology, namely creditworthiness test, which can generate information on borrower's type. This technology is symmetric across banks and works as follows.

For any borrower applying for credit from a bank, a test with *screening intensity* $q \in \mathbb{Q} \equiv \left[\frac{1}{2}, 1\right]$ generates a random signal $\phi \in \Phi \equiv \{G, B\}$ satisfying

$$\Pr(\phi = G|\theta = g) = \Pr(\phi = B|\theta = b) = q.$$

By Bayes law, posterior probabilities $\Pr(g|G)$ and $\Pr(g|B)$ are

$$\nu^G(q,\mu) \equiv \frac{q\mu}{q\mu + (1-q)(1-\mu)} \quad \text{and} \quad \nu^B(q,\mu) \equiv \frac{(1-q)\mu}{(1-q)\mu + q(1-\mu)}$$

for an arbitrary prior $0 < \Pr(g) = \mu < 1$. An important element of this model is that q is chosen by the testing bank at an upfront cost $zC(q)$, where the assumptions on the unit cost function $C(q)$ will be introduced below. Intensity q fully determines the accuracy of a test: the higher q, the higher $\nu^G(q,\mu)$ and a better signal G as for g. In this way, it helps the bank to determine the statistical creditworthiness of a borrower so that the lending decision of approval or denial may be made conditional on test result ϕ.

II.C Loan Contract

Banks post loan contracts for which borrowers apply. The general form of a loan contract is $\ell = (r, \lambda, q)$, where q denotes screening intensity. Ex post screening and upon approval, a one period loan of size z with terms given by (r, λ) is granted, where r is the (gross) interest rate to be repaid when a borrower is solvent and C is the collateral value seized by bank in default. Limited liability implies that r is bounded from above by x. Moreover, banks have access to perfectly elastic supply of funds at zero risk-free rate, so that r is bounded from below by 1.

Given $r \in [1, x]$, a θ borrower defaults if and only if the project fails. Limited liability then implies $\lambda \leq c$, so that without loss of generality I focus on loans in the form (r, c), or simply r, ex post of screening.[9] Thus upon approval, expected payoff from a unit loan r to a θ borrower is $u^\theta(r) = \theta(x - r) \geq 0$ to the borrower and $\eta^\theta(r) = \theta r + (1-\theta)c - 1$ to the bank. It follows that $\eta^g(r) > \eta^b(r)$, and $\eta^b(r) < 0 \; \forall r \in [1, x]$. Since $\eta^g(x) = \text{NPV}^g > 0$ and $\eta^g(1) < 0$, there is a unique $\underline{r} > 1$ such that $\eta^g(\underline{r}) = 0$. Evidently, no bank will offer a loan with $r < \underline{r}$. Let $\mathscr{R} \equiv [\underline{r}, x]$, and let $\mathscr{C} = \mathscr{R} \times \mathbb{Q}$ denote the contract space, with $\ell = (r, q) \in \mathscr{C}$ denoting a generic loan contract.

I assume that each bank i offers only one contract $\ell^i \in \mathscr{C}$ at the beginning of each period. This is with out of generality as I show below that the unique equilibrium in this framework is a pooling equilibrium in which every bank offers the same contract. Let $\mathscr{L} = \{\ell^1, \ldots, \ell^N\}$ denote the set of loans available in the market during each period, and let $\boldsymbol{\ell} = (\ell^1, \ldots, \ell^N)$ denote the corresponding vectorization.

[9]For any loan $(r, \lambda) \in [1, x] \times [0, c]$ satisfying $\theta r + (1-\theta)\lambda - 1 \geq 0$, there is an $r' \in [1, x]$ such that (r', c) replicates payoffs to both the bank and the borrower as they are both risk-neutral.

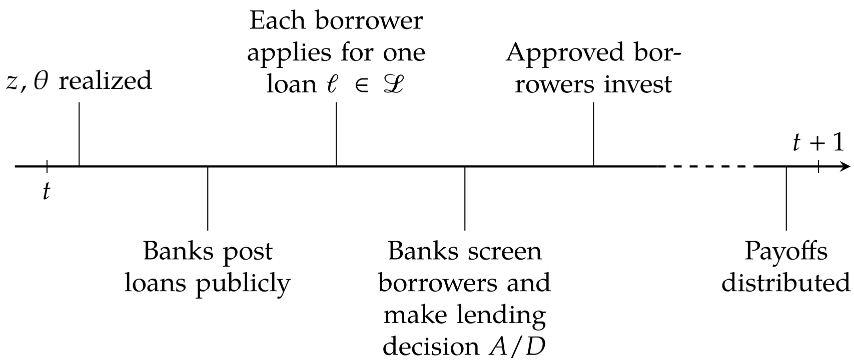

Figure 1: Timeline within a period

II.D Lending Game within a Period

Timing After the realization of θ at the beginning of time t, borrowers and banks meet in the credit market and take actions in the following order:

1. All Banks post loans simultaneously and publicly.

2. Each borrower selects *one* loan $\ell \in \mathscr{L}$ to apply for. If the loan is offered by multiple banks, the borrower randomly picks one bank.

3. Seeing a borrower at a loan $\ell = (r, q)$, the bank screens the borrower at intensity q paying cost $zC(q)$ and makes lending decision of approval (A) or denial (D) after observing test result ϕ.

If the bank approves the application, then the borrower receives a loan of size z at interest rate r for investment, and payoffs are distributed to both the borrower and the bank at the end of time t. If a loan application is denied by the bank, then the borrower needs to wait until next period to apply again. The timeline within a period is illustrated in Figure 1.

By posting loan contracts in the form of $\ell = (r, q)$, banks effectively commit to both interest rate and screening intensity. This also makes explicit that banks have two choice variables, r and q, where q corresponds to lending standard (see discussion in the next Section). Accordingly, when selecting a loan, borrower also takes into account effect of q on the probability of approval. That the loan granted upon approval is of size z means that one investment project can be financed by one bank through one loan, thus multiple creditors are not allowed. This is consistent with my focus on lending to relatively small borrowers and on loans of relatively small size.[10] The assumption

[10]Household loans (mortgage, auto and consumption loans) and loans to small business are almost always financed and managed by a single lender. The average size of commercial and industrial loans, including those to large and

that each borrower applies for only one loan during a period tends to capture the reality that loan application/screening is a nontrivial process, which requires considerable time and effort from both borrowers and banks. Although this exclusivity assumption is a standard one in the literature of adverse selection with screening/signaling,[11] I relax it in Section V.C to check the robustness of the model.

A Game-Theoretic Formulation In each period, the timing of the lending process, in together with information and payoff structure, determines an extensive form game with incomplete information, which I call the lending game henceforth.[12] I shall analyze this lending game in the next Section, using sequential equilibrium of Kreps and Wilson (1982) as the solution concept. Since banks post loan contracts $\mathscr{L}$ simultaneously and publicly, the subsequent lending process consists of a proper subgame of the lending game, and any sequential equilibrium is thereby subgame perfect. As a result, equilibrium payoff of bank i can be expressed as a function of the contract offerings $\boldsymbol{\ell}_t$ by all banks at the beginning of time t, which serves as i's profit function $\Pi^i(\boldsymbol{\ell}_t; s_t)$ in the repeated game among the N banks. The second argument in the profit function denotes the aggregate shock affecting the economy at t, on which I turn to discuss next.

II.E Dynamic Setup

At the beginning of each period, before credit market opens, an aggregate shock s_t hits the economy. The shock shifts parameters of the economy and is public information, so that both borrowers and banks may condition their strategies on its realization. I specify the precise form of this shock in Section IV; for now, it suffices to assume that s_t lies in a compact subset $\mathcal{S}$ of some finite dimensional Euclidean space $\mathbb{R}^d$ and follows some stochastic process. Since s_t is payoff relevant, it enters into bank's profit function $\Pi^i(\boldsymbol{\ell}_t; s_t)$.[13]

medium firms, is about 1 million over 1997 to 2013 and is very often financed and managed by one bank. In contrast, the average size of a syndicated loan with multiple lenders is more than 300 million (Sufi, 2007).

[11]See, e.g., seminal papers by Rothschild and Stiglitz (1976), Wilson (1977), and Riley (1979). One clarification: "screening" in this literature typically refers to separating different types of agents through designing incentive compatible contracts by the uninformed party (Stiglitz and Weiss, 1990), which differs from "screening" as an information production technology in this paper (and the literature on bank screening more generally).

[12]In particular, this is a three-stage game with a similar structure to the one analyzed in Hellwig (1987). See also Grossman (1979) for a similar timing structure without a good-theoretic formulation.

[13]There are 6 payoff relevant parameters: x, c, θ^g, θ^b, and z for individual project payoff; and $\bar{\mu}$ for project/borrowre type distribution. In principle, screening cost function $C(\cdot)$ may also shift over time, yet I do not consider this possibility here.

Conditional on s_t, borrowers and banks play the lending game at t; thus over time, the credit market interaction is described by a repeated game with aggregate shock, of which the one period lending game becomes the stage game. What simplifies the analysis is that borrower's type is iid over time and storage by borrower is not feasible, therefore borrower's decision problem is a static one. Given $\boldsymbol{\ell}_t$, once borrower's action is determined in the lending process (as a subgame) of the stage lending game, bank i's profit $\Pi^i(\boldsymbol{\ell}_t; s_t)$ is determined. As a result, the repeated game reduces to a game where N banks compete with each other for the market share of borrowers over time by choosing ℓ_t^i, conditional on the entire history up to t, taking $\Pi^i(\cdot)$ as the relevant stage payoff.

I proceed to analyze the stage lending game first and derive the profit function $\Pi^i(\cdot)$, after which I analyze the repeated game and characterize credit market dynamics.

III Stage Lending Game

Suppose that, at the beginning of current period, a realization of the aggregate shock s becomes public information which pins down all payoff relevant parameters of the economy. Then the credit market opens and the market outcome is determined in a sequential equilibrium of the stage lending game between borrowers and banks. In a sequential equilibrium, there is a common belief system $\mu(\ell)$, $\forall \ell \in \mathscr{L}$, about the probability of good type when a bank sees a borrower applying for a loan ℓ, such that the strategy profile of banks and borrowers consists a Nash equilibrium and the belief system $\mu(\cdot)$ is consistent with the strategy profile. More specific, bank i's strategy has two parts: choice of a loan $\ell^i \in \mathscr{C}$ to offer and lending decision A or D for each borrower approached under belief $\mu(\cdot)$. And strategy of either type of borrowers is simply choosing loans $\ell \in \mathscr{L}$ to apply for. Also, banks and borrowers can use mixed strategy.[14]

In the rest of this Section, I analyze sequential equilibria of this lending game in the backward order: bank's lending decision first, followed by borrower's choice, then a characterization of efficient contracts, i.e., contracts with optimal screening choice, and lastly bank's profit function.

III.A Lending Decision

Consider that a borrower applying for a loan $\ell = (r, q) \in \mathscr{C}$ offered by a bank. Let $\mu = \mu(\ell)$ denote the prior belief of the probability of having a good borrower. When $\mu = 0$ or 1, bank's lending decision is trivial: denial in the first and approval in the second case. When $0 < \mu < 1$ and screening intensity is q, posterior belief of a good borrower equals to $\nu^\phi(q, \mu)$ conditional on

[14] I consider only symmetric strategy among borrowers of the same type, where mixed strategy is allowed.

receiving test result $\phi \in \Phi$.[15] Accordingly, the expected payoff from a unit loan with interest rate r to a ϕ borrower is

$$\eta^{\phi}(\ell, \mu) = \nu^{\phi}(q, \mu)\eta^{g}(r) + \big(1 - \nu^{\phi}(q, \mu)\big)\eta^{b}(r).$$

As bank moves last in the lending game and screening cost becomes sunk cost by then, bank's lending decision is the simple NPV rule: approval if $\eta^{\phi}(\ell, \mu) \geq 0$ and denial if $\eta^{\phi}(\ell, \mu) < 0$. Following lemma shows the outcome of bank's lending decision conditional on test result ϕ.

Lemma 1. *For a given loan $\ell = (r, q)$, either one of three cases happens: (i) bank approves both G and B borrowers; (ii) bank approves G while denies B borrowers; or (iii) bank denies both G and B borrowers.*

The proof is in Appendix A (p.41). Roughly speaking, η^{G} and η^{B} are either both positive when bank's prior μ is close to 1 or both negative when μ is close to 0, whereas for interim values of μ, $\eta^{G} > 0 > \eta^{B}$.

III.B Borrower's Choice

Let $p^{\theta}(\ell)$ denote the approval probability of a θ borrower at loan $\ell = (r, q)$ with prior belief $\mu(\ell)$. A direct implication of Lemma 1 is that p^{θ} can be of three cases: (*i*) $p^{g} = p^{b} = 1$; (*ii*) $p^{g} = p^{b} = 0$; or (*iii*) $p^{g} = q$ and $p^{b} = 1 - q$. The last follows because the probability of getting a G signal in screening is q for a g borrower and $1 - q$ for a b borrower.

Fix the set of loan offers $\mathscr{L}$ by all banks where $\mathscr{L}$ contains at least two loans, and consider the subgame, i.e., stage 2–3. Any sequential equilibrium of the lending game is subgame perfect in this subgame. In particular, given the belief system $\mu(\cdot)$ and bank's lending decision based on $\mu(\cdot)$, a θ borrower chooses loans in stage 2 so as to maximize the expected payoff, i.e., $\max_{\ell \in \mathscr{L}} U^{\theta}(\ell)$, where

$$U^{\theta}(\ell) \equiv u^{\theta}(r)p^{\theta}(\ell) = [\theta(x - r)]p^{\theta}(\ell)$$

and $\ell = (r, q)$. Furthermore, borrower's choice needs to be consistent with the belief system such that if g borrower chooses ℓ with probability α and b borrower chooses ℓ with probability β, then $\mu(\ell) = \alpha\bar{\mu}/[\alpha\bar{\mu} + \beta(1 - \bar{\mu})]$.

If $\mathscr{L}$ and $\mu(\cdot)$ is such that approval probability $p^{\theta}(\ell) = 0 \forall \ell \in \mathscr{L}$ and $\theta \in \Theta$, then there will be no active lending in equilibrium.[16] To focus on economically interesting cases, I introduce following

[15]Since I do not exclude a priori the possibility of mixed strategy adopted by borrowers when $\mathscr{L}$ contains more than one loans, μ can take any value in $[0, 1]$. Otherwise, μ can take only three values: 0, 1, and $\bar{\mu}$.

[16]If $\bar{\mu}\text{NPV}^{g} + (1 - \bar{\mu})\text{NPV}^{b} < 0$, a possibility I do not exclude, and $q \approx 1/2$, $\mu(\ell) \approx \bar{\mu}\ \forall \ell = (r, q) \in \mathscr{L}$, then no bank finds it profitable to lend.

definition.

DEFINITION 1. An active lending equilibrium is a sequential equilibrium in which $p^{\theta}(\ell) > 0$ for some $\theta \in \Theta$ and $\ell \in \mathscr{L}$.

Evidently, in an active lending equilibrium, $p^{g}(\ell_1)$ must be positive for some $\ell_1 = (r_1, q_1)$, whereas it may happen that $p^{b}(\ell_1) = 0$ as $q_1 = 1$. Subsequently, I only look at active lending equilibria.

In principle, there may exist three types of equilibrium outcome in the subgame. In a pooling equilibrium, both types of borrowers choose the same loan; in a separating equilibrium, two types of borrowers choose different loans; and in a mixed equilibrium, at least two loans are selected with positive probability by one type of borrowers, and at least one of them is selected by both types. The next lemma asserts that only pooling outcome is possible.[17]

LEMMA 2. *Suppose* $r < x$ *and* $\frac{1}{2} < q < 1$ $\forall \ell = (r, q) \in \mathscr{L}$. *Then only pooling equilibrium outcome is consistent with an active lending equilibrium.*

The proof is in Appendix A (p.41). Outcome in the subgame other than the pooling one may be consistent with an active lending equilibrium when some loan contract in $\mathscr{L}$ lies on the boundary of contract space $\mathscr{C}$. Imposing $r < x$ as a condition is only for technical reason, i.e., ensuring borrower's payoff to be strictly positive whenever approved. This condition can be dropped if I modify the setup slightly such that a borrower enjoys some positive control rent whenever his investment is funded.[18] Loans with $q = \frac{1}{2}$ correspond to no effective screening, whereas $q = 1$ means perfect screening. Neither case is at odds with the basic premise of this paper that bank's screening is nontrivial but not perfect. Moreover, I demonstrate below that $\frac{1}{2} < q < 1$ in any active lending equilibrium.

III.C EFFICIENT SCREENING INTENSITY

Lemma 2 makes clear that equilibrium outcome in the lending process subgame can only be pooling. As a result, belief is fixed at $\mu(\ell) = \bar{\mu}$ for any loan $\ell \in \mathscr{L}$ selected by borrowers in an active lending equilibrium. As a result, bank's payoff from a unit loan $\ell = (r, q)$ to a ϕ borrower

[17] Critically, the non-existence of mixed equilibrium outcome depends on the presumption that banks choose approval whenever $\eta^{\phi}(\ell) = 0$, in particular $\eta^{B}(\ell) = 0$. If banks can randomize between approval and denial when $\eta^{B}(\ell) = 0$, then it is not hard to construct mixed equilibrium outcomes.

[18] Such control rent is private payoff and is not contractible.

simplifies to $\eta^{\phi}(\ell) \equiv \eta^{\phi}(\ell, \bar{\mu})$. Whenever borrowers select a loan $\ell = (r, q)$ in equilibrium, bank's unit payoff, gross of screening cost, is

$$\eta(\ell) = \Pr(G) \max\left\{\eta^{G}(\ell), 0\right\} + \Pr(B) \max\left\{\eta^{B}(\ell), 0\right\},$$

where $\Pr(G) = q\bar{\mu} + (1-q)(1-\bar{\mu})$ and $\Pr(B) = 1 - \Pr(G)$, and $\max\left\{\eta^{\phi}(\ell), 0\right\}$ reflects bank's lending decision at ℓ. It follows that bank's net unit payoff at a loan $\ell = (r, q)$ can be written as $\eta(r, q) - C(q)$, where $C(q)$ is the unit cost of screening.[19]

For any given interest rate $r \in \mathscr{R}$, $\eta(r, q) - C(q)$ defines a function of q over $\mathbb{Q}$. In particular, different q may give different net payoff to the lending bank, and a set of *efficient* screening intensities q may be defined so as to maximize the net payoff for a given r. To provide an economically meaningful characterization, I explore additional properties of $\eta(r, q)$ and impose further restrictions on $C(q)$ in turn.

Properties of $\eta(r, q)$ Evidently, $\eta(r, q) \geq 0$ and is continuous over $\mathscr{C}$. To further characterize $\eta(r, q)$, it is useful to define two more quantities. Let

$$\Delta(r) = \bar{\mu}\eta^{g}(r) - (1 - \bar{\mu})\eta^{b}(r).$$

As $\eta^{g}(r) \geq 0 > \eta^{b}(r)\ \forall r \in \mathscr{R}$, $\Delta(r) > 0$. This quantity turns out to be essential for determining efficient screening. Next, let

$$\bar{\eta}(r) = \bar{\mu}\eta^{g}(r) + (1 - \bar{\mu})\eta^{b}(r),$$

be the unit payoff from lending at r to all borrowers indiscriminately. Following lemma shows that $\eta(r, q)$ is piece-wise linear in q for any given r, with $\Delta(r)$ as the marginal benefit of screening for q above certain threshold. The proof is in Appendix A (p.42).

Lemma 3. $\forall r \in \mathscr{R}$ *there is a cutoff value* $q^{C}(r) \in \mathbb{Q}$ *such that*

$$\eta(r, q) = \begin{cases} \max\{\bar{\eta}(r), 0\}, & \text{if } q \leq q^{C}(r); \\ \Delta(r)q + (1 - \bar{\mu})\eta^{b}(r), & \text{if } q \geq q^{C}(r). \end{cases}$$

Parametric restrictions on $C(q)$ I assume that $C(\cdot)$ is twice continuously differentiable with $C\left(\frac{1}{2}\right) = C'\left(\frac{1}{2}\right) = 0$ and $C'(\cdot) \geq 0$, $C''(\cdot) > 0$ over $\mathbb{Q}$. Moreover, I impose following parametric restrictions on $C(\cdot)$.

Assumption 2. $C(1) < \min\left\{\bar{\mu}\eta^{g}(x), -(1 - \bar{\mu})\eta^{b}(x)\right\}$ and $C'(1) > \Delta(x)$.

[19] For a loan $\ell = (r, q)$ of size z, screening cost is $zC(q)$ and bank's payoff is $z\eta(r, q)$.

The first restriction is equivalent to

$$\eta(x,1) - C(1) = \bar{\mu}\eta^g(x) - C(1) > \max\{\bar{\eta}(x), 0\} = \eta\big(x, \tfrac{1}{2}\big),$$

which ensures that active screening $q > \frac{1}{2}$ is better than trivial screening at least for $r = x$. The second restriction says that marginal cost of screening gets higher than marginal benefit when q is close to 1 for $r = x$.

Now I can define and characterize efficient screening intensity. For any $r \in \mathcal{R}$, let

$$\pi(r) = \max_{q \in \mathbb{Q}} \eta(r,q) - C(q)$$

be the maximum of the net unit payoff, and $\mathcal{E}\mathcal{Q}(r)$ be set of maximizers. In words, $\pi(r)$ represents the profit rate when lending at interest rate r with efficient screening intensity given by $\mathcal{E}\mathcal{Q}(r)$. The next result demonstrates that $\mathcal{E}\mathcal{Q}(r)$ has a simple structure and satisfies nice properties.

Lemma 4. *Under Assumption 1–2, there exists a unique* $r^0 \in \text{Int}\mathcal{R}$ (*interior of* $\mathcal{R}$) *and a function* $q^e(r)$ *such that*

$$\mathcal{E}\mathcal{Q}(r) = \begin{cases} \frac{1}{2}, & \text{if } \underline{r} \le r < r^0; \\ \frac{1}{2} \cup q^e(r), & \text{if } r = r^0; \\ q^e(r), & \text{if } r^0 < r \le x. \end{cases}$$

Moreover, $q^e(r)$ *is continuous, strictly increasing, and satisfies* $\frac{1}{2} < q^e(r) < 1$ *over* $\mathcal{R}$; *and* $\pi(r)$ *equals to* 0 *over* $[\underline{r}, r^0]$ *and is continuous and strictly increasing over* $[r^0, x]$.

The proof is in Appendix A (p.43). Essentially, banks find it profitable to screen at $q^e(r) > \frac{1}{2}$ if and only if the corresponding interest rate is not too low, i.e., $r \ge r^0$. Otherwise, expected payoff from lending is not enough to recoup screening cost, so banks prefer to not screening at all. Furthermore, $q^e(r)$ is determined simply by the first order condition (ignoring the kink of $\eta(r,q)$ caused by $\max\{\bar{\eta}(r), 0\}$)

$$\Delta(r) = C'(q^e),$$

which makes clear why $q^e(r)$ is strictly increasing: $C'(\cdot)$ is increasing as $C''(\cdot) > 0$; and $\Delta(\cdot)$ is increasing because $\Delta'(r) = \bar{\mu}\theta^g - (1-\bar{\mu})\theta^b$ is positive by part (*ii*) of Assumption 1.

The result that efficient screening intensity $q^e(r)$ is increasing in r over $[r^0, x]$ is crucial for the main results of this paper. To better illustrate the underlying intuition, first note that whenever marginal benefit of screening gets higher, bank is better off by increasing q to equate marginal cost with marginal benefit. Second, marginal benefit $\Delta(r)$ is increasing in r if and only if $\bar{\mu}$ is greater than the lower bound given by part (*ii*) of Assumption 1. The reason is as follows. At the margin, a one unit increase in r changes $\Delta(r)$ in two ways: it increases $\Delta(r)$ by increasing the direct gain of lending to additional good borrowers, which is $\bar{\mu}\theta^g$ on average; but it also reduces $\Delta(r)$ by decreasing the indirect gain of preventing loss from lending to additional bad borrowers, which is $(1-\bar{\mu})\theta^b$ on average, as higher r makes loan loss to bad borrowers smaller. Therefore, the overall effect depends on the magnitude of $\bar{\mu}$.

III.D Efficient Contract Space

For any contract (r, q) with $r < r^0$ and efficient screening intensity $q = \frac{1}{2}$, the associated screening activity is trivial. Such cases are economically uninteresting,[20] henceforth I focus on what I call the efficient contract space

$$\mathscr{C}^e = \{(r, q) \in \mathscr{C} | q = q^e(r), r \in [r^0, x]\}.$$

As a corollary of the previous two results, the next lemma describes bank's lending decision at a loan contract within $\mathscr{C}^e$ (see Appendix A, p.43 for the proof). This also shows that contract in $\mathscr{C}^e$ is always consistent with an active lending equilibrium.

Lemma 5. *If $r \geq r^0$, banks deny borrowers with test result B while approve borrowers with test result G after the screening process at $(r, q^e(r))$.*

Focusing on the subspace $\mathscr{C}^e$ instead of $\mathscr{C}$ amounts to restrict bank's choice to $\mathscr{C}^e$, which raises two concerns about the legitimacy of such a restriction. The first concern is that whether this restriction will effectively restrict the possibility of strategic interactions among banks in a meaningful way. The answer is no. This comes from the observation that profit rate of lending at a loan $\ell \in \mathscr{C}^e$ ranges continuously from 0 to $\pi(x)$, the maximum that is achievable from lending at any loan over the entire contract space $\mathscr{C}$.[21] As a result, every profit rate level that may arise when banks compete with each other by choosing contracts from $\mathscr{C}$ can be realized by restricting bank's choice to $\mathscr{C}^e$.

Perhaps an even more important concern is that whether it is reasonable to have an active lending equilibrium at a loan outside $\mathscr{C}^e$. First of all, one can easily construct such an equilibrium.

[20] Zero profit is always achievable at $(r^0, q^e(r^0))$, where screening is nontrivial. Moreover, for $r < r^0$, $\bar{\eta}(r)$ may be negative, which then implies that no lending takes place as the corresponding efficient screening intensity is $q = \frac{1}{2}$. This can never happen when $r \geq r^0$, as made clear in the next result.

[21] That is, $\pi(x) = \max_{(r,q)\in\mathscr{C}} \eta(r, q) - C(q)$. This follows from the fact that $\max_{(r,q)\in\mathscr{C}} \eta(r, q) - C(q) = \max_{r\in\mathscr{R}} \max_{q\in\mathbb{Q}} \eta(r, q) - C(q) = \max_{r\in\mathscr{R}} \pi(r)$ and that $\pi(r)$ is increasing in r.

Consider a loan $\ell_1 = (r_1, q_1) \in \text{Int}\mathscr{C} \setminus \mathscr{C}^e$ but is close to $\mathscr{C}^e$, and let the belief system be such that $\mu(\ell_1) = \bar{\mu}$ and $\mu(\ell) = 0 \; \forall \ell \neq \ell_1$. This is clearly an active lending equilibrium as all borrowers only apply for ℓ_1. However, I argue that such an equilibrium is not reasonable, since the commitment of screening borrowers at ℓ_1 with intensity q_1 is not credible. The argument is simple. On the one hand, $q_1 \neq q^e(r_1)$ given that ℓ_1 is not in $\mathscr{C}^e$. On the other hand, since all borrowers apply for ℓ_1, $\mu(\ell_1) = \bar{\mu}$ for sure. Recall that $q^e(r_1)$ generates the highest net payoff to any bank lending with r_1 and facing a borrower distribution $\bar{\mu}$. Thus any bank which has offered ℓ_1 strictly prefers to deviate from q_1 to $q^e(r_1)$ in the subsequent screening process upon each borrower it has; and there can be no punishment as the lending game ends after the bank's deviation right away. Such a deviation is prohibited only because I assume no change on q can be made by a bank after the contract (r, q) is offered as a whole at the beginning of a period. In other words, I effectively assume that all banks possess a commitment technology on both r and q. Yet, by the nature of screening, the verification of a bank's actual screening intensity would be very difficult, if not entirely impossible, to implement by either a borrower or some third party. This is in clear contrast to a bank's commitment to r. To sum up, even though a bank is assumed to be able to commit to any contract in $\mathscr{C}$, its commitment to a contract not in $\mathscr{C}^e$ is much less credible, thus I shall restrict the analysis to $\mathscr{C}^e$ below.[22]

III.E Profit Function

Having argued that it is appropriate to restrict bank's choice set to be $\mathscr{C}^e$, I move on to explore borrower's choice over $\mathscr{C}^e$ and deduce profit function of each bank. At any efficient contract $\ell = (r, q^e(r)) \in \mathscr{C}^e$, expected payoffs to good and bad borrowers are

$$U^g(r) = u^g(r)q^e(r) \quad \text{and} \quad U^b(r) = u^b(r)(1 - q^e(r)),$$

where I use r to index ℓ for simplicity. It is clear that $U^b(r)$ is decreasing in r, as both $u^b(r)$ and $1-q^e(r)$ are decreasing. However, monotonicity of $U^g(r)$ is ambiguous without further assumption. To restore monotonicity of $U^g(r)$, I impose following parametric restriction.[23]

[22] The foregoing argument on the (in)credibility issue serves as an informal motivation for restriction on $\mathscr{C}^e$. In Section V.B, I show in a more formal way that equilibrium loan contracts must belong to $\mathscr{C}^e$ for a game with a slightly different extensive form but of the same normal form as the one considered here. Therefore, the invariance principle elucidated by Kohlberg and Mertens (1986) suggests that no equilibrium contract outside $\mathscr{C}^e$ satisfies their stability criterion. In this sense, "reasonable" equilibria can emerge only in $\mathscr{C}^e$.

[23] I discuss the robustness of the main results in Section V.D when this assumption is dropped. Assumption 2 and 3 together impose restrictions on the magnitude, the derivative, and the curvature of $C(\cdot)$, which raises a concern of

Assumption 3. $C''(\cdot) \geq 2[\bar{\mu} - (1-\bar{\mu})\theta^b/\theta^g]\text{NPV}^g$ over $\mathbb{Q}$.

Lemma 6. *Under Assumption 3, $U^g(r)$ is decreasing in r.*

The proof is in Appendix A (p.43). Observe that $\bar{\mu} - (1-\bar{\mu})\theta^b/\theta^g = (\bar{\mu}\theta^g - (1-\bar{\mu})\theta^b)/\theta^g$ which is positive by part (*ii*) of Assumption 1, thus restriction on $C''(\cdot)$ imposed by Assumption 3 is not a void one. However, this parameter restriction is not a severe one either, as it is only a sufficient condition for $U^g(r)$ being decreasing. In addition, provided that $\bar{\mu}\theta^g - (1-\bar{\mu})\theta^b$ and NPV^g are relatively small, the Assumption is not difficult to satisfy.

This result seems to suggest that apparently both types of borrowers prefer loan contract $(r, q^e(r))$ with lower interest rate r. There is nonetheless a gap for this result to actually hold in equilibrium. The problem is that approval probability $q^e(r)$ and $1 - q^e(r)$, for good and bad borrowers respectively, at $\ell = (r, q^e(r))$ is derived under the belief $\mu(\ell) = \bar{\mu}$, which is only pinned down in such a way if ℓ is selected by borrowers in an equilibrium. In contrast, by the same argument at the end of the previous subsection, any contract $\ell_1 = (r_1, q^e(r_1)) \in \mathscr{C}^e$ can be supported as the only equilibrium outcome by a belief system $\mu(\ell) = 0 \; \forall \ell \neq \ell_1$ over $\mathscr{C}^e$, even though a contract $\ell_2 = (r_2, q^e(r_2))$ with $r_2 < r_1$ may be available in the market as well.[24]

To deal with the problem of multiple equilibria, I provide two arguments for why ℓ_1 should be discarded as a reasonable equilibrium outcome whenever ℓ_2 is present. At a formal level, the only equilibrium that survives the undefeatedness refinement criterion of Mailath et al. (1993) is the one in which all borrowers select the loan $\ell_{\min}$ with the minimum interest rate among all loans available in the market.[25] In addition, this equilibrium also Pareto dominates any other pooling equilibria from borrower's perspective, which is an appealing property in itself. At a less

whether the parameter space satisfying all assumptions is empty or not. The answer is no. I address this issue in Online Appendix B.

[24]To be more precise, $\forall \ell = (r, q^e(r)) \in \mathscr{C}^e$, there exits $0 < x(\ell) < \bar{\mu} < y(\ell) < 1$ such that $p^g(\ell) = p^b(\ell) = 1$ for $\mu(\ell) \geq y(\ell)$, $p^g(\ell) = q^e(r)$ and $p^b(\ell) = 1 - q^e(r)$ for $x(\ell) \leq \mu(\ell) < y(\ell)$, and $p^g(\ell) = p^b(\ell) = 0$ for $\mu(\ell) < x(\ell)$. As a result, to support ℓ_1 as the unique equilibrium contract, out-of-equilibrium belief needs only to satisfy $\mu(\ell) < x(\ell) \; \forall \ell \neq \ell_1$.

[25]In the current context, the undefeatedness criterion selects equilibrium which is not defeated by any another equilibrium in the following way. Given loan contracts $\mathscr{L} \subset \mathscr{C}^e$ offered by banks, Lemma 2 shows that only pooling equilibrium is possible in this subgame. A pooling equilibrium at $\ell \in \mathscr{L}$ defeats another pooling equilibrium at $\ell' \in \mathscr{L}$, if both types want to deviate from ℓ', and at least one type strictly wants to do so, when the equilibrium belief at ℓ in the latter equilibrium is replaced by the equilibrium belief $\bar{\mu}$ in the former one. Clearly, given ℓ_1 and ℓ_2 such that $r_2 < r_1$, if $\mu(\ell_2) = \bar{\mu}$ in an pooling equilibrium at ℓ_1, then both good and bad borrowers strictly prefer ℓ_2 to ℓ_1. Therefore the pooling equilibrium at ℓ_2 always defeats the one at ℓ_1.

formal level, it seems not overly counter-intuitive to assume that, whenever a bank offers a loan contract $\ell' = (r', q^e(r')) \in \mathscr{C}^e$ that undercuts all other loan contracts in the market, i.e., $r' < r$ s.t. $(r, q^e(r)) \in \mathscr{L}$, it's belief about borrower's distribution at ℓ' should be such that it will approve at least G borrowers at ℓ'; for otherwise there is no incentive for the bank to make such an offer. But once the bank plans to approve at least G borrowers, then ℓ' will attract all borrowers.

To summarize, given the set of contracts $\mathscr{L}$ offered by all banks in stage 1, there is a unique pooling equilibrium in which both good and bad borrowers select $\ell_{\min} = (r_{\min}, q^e(r_{\min}))$, i.e., the contract with the lowest interest rate $r_{\min} = \min\{r|(r, q^e(r)) \in \mathscr{L}\}$.[26] Under the assumption that borrowers applying for the same contract offered by multiple banks randomly choose one bank, equal splitting of market share follows. Thus given $\boldsymbol{\ell}$, bank i's profit function has the following form:

$$\Pi^i(\boldsymbol{\ell}) = \begin{cases} z\pi(\ell_{\min})/N(\ell_{\min}), & \text{if } \ell^i = \ell_{\min}, \\ 0, & \text{otherwise,} \end{cases}$$

where $N(\ell_{\min})$ is the number of banks offering $\ell_{\min}$. With a slight abuse of notation, I rewrite bank's profit rate $\pi(r)$ as

$$\pi(\ell) = \pi(r, q^e(r)) \equiv \bar{\mu} q^e(r)\eta^g(r) + (1 - \bar{\mu})(1 - q^e(r))\eta^g(r) - C(q^e(r))$$

to emphasize that bank chooses both interest rate r and screening intensity q, despite the that q equals to $q^e(r)$. Implicitly, $\Pi^i(\cdot)$ is also a function of the aggregate shock s; and I will make this explicit in the next section.

Before proceeding to the repeated game, I briefly discuss two extreme cases of the static lending game.

Zero profit   Suppose that there are more than one bank. Since $\pi(r)$ is strictly increasing in r over $[r^0, x]$, and both types of borrowers strictly prefer a loan with lower interest rate, it is always profitable for banks to undercut one another as long as $\ell_{\min}$ specifies an interest rate $r_{\min} > r^0$.[27] As a result, the only equilibrium of the static lending game has banks offer zero profit efficient

[26] An immediate implication is that, for any bank i, the only contract that matters for i's payoff, were i allowed to offer multiple contracts, would be the contract with the lowest interest rate. This justifies my assumption that each bank offers only one contract in each period.

[27] This feature of the model coincides with the prototypical model of Bertrand competition with homogeneous output and constant marginal cost. Such a correspondence is only a superficial one; the lengthy derivation of the profit function should have made this point evident.

contract $\ell^0 = (r^0, q^0)$ with $q^0 = q^e(r^0)$ in stage 1 which results in a pooling equilibrium at ℓ^0 in the subgame of stage 2 to 3.

Monopoly Suppose that there is only one bank. Then the equilibrium of the lending game features a pooling equilibrium at the monopoly profit efficient contract $\ell^m = (r^m, q^m)$ with $r^m = x$ and $q^m = q^e(x)$, as profit rate $\pi(r)$ is increasing in r over $[r^0, x]$.[28]

IV Equilibrium Dynamics

With borrower's equilibrium behavior subsumed into bank's profit function $\Pi^i(\cdot)\ \forall i \in N$, the lending game between banks and borrowers reduces to a game among N banks with $\Pi^i(\cdot)$ specifying the payoff associated with bank's action profile $\boldsymbol{\ell} = (\ell^1, \ldots, \ell^N)$ conditional on the aggregate shock s. Despite the fact that the entire lending game is repeated over time, it suffices to consider strategic interactions only among banks over time, because borrower's investment shock θ, i.e., type, is iid over time. This fact greatly simplifies the analysis of equilibrium dynamics and leads to a setup similar to Rotemberg and Saloner (1986). In what follows, I first introduce some notations for the repeated game and define the equilibrium concept under a general specification of the aggregate shock process $\{s_t\}$. Then I solve for the equilibrium dynamics under various specific forms of $\{s_t\}$, which reflect various aspects of business cycle fluctuations. To avoid trivial case, I assume $N \geq 2$ throughout this section.

IV.A A Formal Setup

Both the banks and the borrowers observe the realization of $s_t \in \mathcal{S} \subset \mathbb{R}^d$ perfectly at the beginning of time t, and $\{s_t\}$ evolves over time as a stationary Markov process. I assume that s determines the payoff relevant parameter vector $(x, c, \theta^g, \theta^b, z, \bar{\mu})$ through a continuous, vector-valued function $\Xi(s)$. Later in this section, I consider a series of different forms of $\Xi(s)$ corresponding to different forms of aggregate shocks. To be able to use the results established in the previous section, I assume that Assumption 1–3 hold $\forall s \in \mathcal{S}$.[29] Thus the efficient contract space $\mathscr{C}^e(s) = \{(r, q^e(r;s)) | r^0(s) \leq r \leq x, s \in \mathcal{S})\}$ is well defined $\forall s \in \mathcal{S}$, with $\ell^0(s) = (r^0(s), q^0(s))$ and $\ell^m(s) = (r^m(s), q^m(s))$ denoting the zero profit and the monopoly contract. Since $\Xi(s)$ is continuous in s, $\mathscr{C}^e(s)$ is a continuous

[28] To be precise, when $r = x$, pooling is no longer the unique equilibrium outcome as borrower is indifferent between applying for a loan or not. For reasons detailed in the discussion following Lemma 2, I fix borrower's behavior by assuming that both good and bad borrowers still apply for ℓ^m when only this loan is offered.

[29] As shown in Appendix B, Assumption 1–3 are satisfied by the parameter vector $(x, c, \theta^g, \theta^b, z, \bar{\mu})$ over an open region. Given a continuous function $\Xi(s)$, the assumptions are satisfied $\forall s \in \mathcal{S}$ as long as $\mathcal{S}$ is not too "big."

correspondence, and $\ell^0(s)$ and $\ell^m(s)$ are continuous functions. Moreover, I write $\Pi^i(\boldsymbol{\ell}; s)$ explicitly as a function of s with $\boldsymbol{\ell} \in (\mathscr{C}^e(s))^N$; evidently, $\Pi^i(\boldsymbol{\ell}; s)$ is also continuous in s. Since s_t determines both the current value and the future distribution of the parameter vector, it is a state variable of the economy.

Let $h^t = (s_t, \boldsymbol{\ell}_{t-1}, s_{t-1}, \ldots, \boldsymbol{\ell}_0, s_0)$ denote the history up to time t with $h^0 = s_0$, $s_\tau \in \mathcal{S}$, and $\boldsymbol{\ell}_\tau \in (\mathscr{C}^e(s_\tau))^N$ $\forall \tau = 0, \ldots, t$. All banks observe past history perfectly. A pure strategy of bank i at t is a function $\sigma_t^i : h^t \mapsto \ell_t^i$ assigning for each history h^t a loan choice $\ell_t^i \in \mathscr{C}^e(s_t)$. Let $\sigma_t = (\sigma_t^1, \ldots, \sigma_t^N)$ denote the strategy profile at t, and $\sigma = \{\sigma_t\}_{t=0}^\infty$ denote the overall strategy profile. A strategy profile σ recursively determines $h^t = (s_t, \sigma_{t-1}(h^{t-1}), h^{t-1})$. Based on the distribution of $\{s_t\}$, σ induces a distribution over the set of all history h^t, and hence the expectation operator $\mathbb{E}^\sigma$. Moreover, let $\sigma|h^t$ denote the strategy profile induced by σ after history h^t, and $\mathbb{E}^\sigma[\cdot|h^t]$ denote the corresponding conditional expectation operator. Lastly, let $0 < \delta < 1$ denote the common discount factor for all banks,[30] then bank i's end-of-period expected payoff conditional on h^t can be written as

$$V^i(\sigma|h^t) = \mathbb{E}^\sigma\left[\sum_{\tau=0}^{\infty} \delta^\tau \Pi^i(\sigma_{t+\tau}(h^{t+\tau}); s_{t+\tau}) \middle| h_t\right],$$

with $\delta V^i(\sigma) = \delta \mathbb{E} V^i(\sigma|s_0)$ denoting the discount value before the realization of s_0.

As is standard, a subgame perfect equilibrium (SPE) is a strategy profile σ such that for any history h^t, $\sigma|h^t$ is a Nash equilibrium for the subgame starting from h^t. A symmetric subgame perfect equilibrium (SSPE) is an SPE in which all banks use the same strategy $\sigma^1 = \cdots = \sigma^N$. To save notation, let σ denote both individual bank's strategy and the strategy profile of all banks in an SSPE, i.e., $\sigma = (\sigma, \ldots, \sigma)$. Correspondingly, let $\boldsymbol{\ell} = \{\{\boldsymbol{\ell}_t(s_t)\}_{s_t \in \mathcal{S}}\}_{t=0}^\infty$ denote a (symmetric) action profile where $\boldsymbol{\ell}_t(s_t) = (\ell_t(s_t), \ldots, \ell_t(s_t))$. All banks receive the same expected discount payoff $V(\sigma)$ which is bounded from below by 0 and from above by $(1-\delta)^{-1} \max_{s \in \mathcal{S}} z(s)\pi(\ell^m(s); s) < \infty$. There could be many SSPE for the repeated game considered here and I shall focus on a particular one, the optimal SSPE.

Definition 2. An optimal SSPE is a strategy profile σ^* such that

$$V(\sigma^*) = V^* \equiv \sup\{V(\sigma) | \sigma \text{ is an SSPE}\},$$

[30]Since risk-free rate is 0 in this economy, I interpret $\delta < 1$ as reflecting some positive premium commanded by bank's owner/manager on its return over risk-free rate. Nevertheless, having zero risk-free rate and $\delta < 1$ only serves to simplify relevant algebra. It is straightforward to extend the benchmark setting allowing for a constant (gross) risk-free rate $r_f > 1$ with $\delta = 1/r_f$.

and an optimal (stochastic) path $\boldsymbol{\ell}^*$ is an action profile $\boldsymbol{\ell}^* = \{\{\ell_t^*(s_t)\}_{s_t \in \mathcal{S}}\}_{t=0}^{\infty}$ such that $V^* = V(\boldsymbol{\ell}^*) \equiv \mathbb{E}\sum_{t=0}^{\infty} \delta^t z(s_t)\pi(\ell_t^*(s_t); s_t)/N$.

I use standard results in the literature of repeated game to solve for an optimal SSPE.[31] First, observe that repeated play of $\ell^0(s_t)$ by all banks consists of an SSPE with zero payoff in each period. As each bank's minmax payoff is also zero, repeated play of $\ell^0(s_t)$ by all banks represents the (symmetric) optimal punishment strategy. Let $\boldsymbol{\ell}^*$ be an optimal path of some optimal SSPE. Then $\boldsymbol{\ell}^*$ can be supported by the following *simple strategy* profile:

- All banks choose $\ell_t^*(s_t)$ at t if no bank deviates from $\boldsymbol{\ell}^*$ at $t-1$.
- All banks revert to the optimal punishment strategy if a bank deviates from $\boldsymbol{\ell}^*$ at $t-1$.

Whenever one bank deviates from the optimal path $\boldsymbol{\ell}^*$ at $t-1$, all banks choose the zero profit contract $\ell^0(s_\tau)$ from t onwards forever, resulting in zero continuation value after any deviation. The optimal value V^* is clearly unique, and the optimal action profile is also unique as shown below. I therefore call the simple strategy profile described above *the* optimal SSPE σ^*.[32]

Since $\{s_t\}$ is a stationary Markov process, the optimal action profile is a time invariant function $\ell^*(s) = (r^*(s), q^*(s))$. Correspondingly, the optimal value achieved by $\boldsymbol{\ell}^*$ simplifies to $V^* = N^{-1}(1-\delta)^{-1}\mathbb{E}z(s)\pi(\ell^*(s); s)$, where $\mathbb{E}$ is evaluated under the stationary distribution of $\{s_t\}$. As a direct implication of the optimal SSPE profile σ^*, $\{\ell^*(s) \in \mathscr{C}^e(s) | s \in \mathcal{S}\}$ solves the following maximization problem

$$V^* = \max_{\{\ell(s) \in \mathscr{C}^e(s) | s \in \mathcal{S}\}} \frac{\mathbb{E}z(s)\pi(\ell(s); s)}{N(1-\delta)}$$

subject to the intertemporal incentive constraint (IIC) $\forall s \in \mathcal{S}$

$$\tfrac{1}{N} z(s)\pi(\ell(s); s) + \delta \mathbb{E}_s V(\ell(\cdot); s') \geq z(s)\pi(\ell(s); s),$$

where $V(\ell(\cdot); s_t) = \mathbb{E}_{s_t}\sum_{\tau=0}^{\infty} \delta^\tau z(s_{t+\tau})\pi(\ell(s_{t+\tau}); s_{t+\tau})/N$ denotes the continuation value under the action profile $\ell(\cdot)$ conditional on s_t. The LHS of the IIC is the sum of the current and the continuation

[31] See, e.g., Mailath and Samuelson (2006, ch.2, sec.6).

[32] This simple strategy coincides with the grim trigger strategy (reversion to the static Nash equilibrium following any deviation), a result typical for repeated Bertrand competition model with homogeneous output and constant marginal cost. For the repeated game considered here, there is a unique optimal action profile associated with the optimal value, whereas punishment strategy other the one specified in σ^* can be used for supporting the optimal action profile along the equilibrium path. One particular nonsymmetric optimal punishment strategy has one bank offers $\ell^0(s_{t+\tau})$ following any deviation from $\ell^*(s_t)$.

value of following $\ell(\cdot)$ conditional on s, and the RHS is the profit that a deviating bank can capture by undercutting other banks' choice $\ell(s) = (r(s), q^e(r(s)))$ an infinitesimal amount. Since any deviation entails zero continuation value, a bank optimally choose not to deviate if and only if the LHS is no less than the RHS.

In the above formulation, the maximization problem is non-linear in $\ell(s)$. However, observe that $\ell(s)$ enters into both the objective and the constraints via the value of the profit $v(s) = z(s)\pi(\ell(s); s)$, it follows that the maximization problem is a linear program in $\{v(s)\}$. In particular, $\ell(s)$ maps one-to-one to $v(s)$ so that $v(s)$ ranges over $[0, \bar{v}(s)]$ where $\bar{v}(s) = z(s)\pi(\ell^m(s); s)$.[33] Furthermore, as in Rotemberg and Saloner (1986), I focus mostly on the case in which $\{s_t\}$ is iid over time. Then the linear program associated with the optimal SSPE has the following simple form

$$\mathcal{P}: \qquad \max_{\{v(s)\in[0,\bar{v}(s)]|s\in\mathcal{S}\}} \mathbb{E}v(s) \quad \text{s.t.} \quad \chi(N,\delta)v(s) \le \mathbb{E}v(s) \quad \forall s \in \mathcal{S}.$$

The coefficient in the IICs is given by

$$\chi(N,\delta) = (N-1)\frac{1-\delta}{\delta}$$

$\forall N \ge 2$ and $0 < \delta < 1$. This quantity turns out to be a crucial characteristic of the overall competitive force of the banking sector.

The solution of $\mathcal{P}$, denoted by $\{v^*(s)|s \in \mathcal{S}\}$, relates to $\ell^*(s)$ through the one-to-one correspondence of $v(s) = z(s)\pi(\ell(s); s)$ and satisfies $V^* = \mathbb{E}v^*(s)$. In general, $\mathcal{P}$ is not easy to solve directly as the constrained set is an irregular polyhedron. However, the solution of $\mathcal{P}$ is closely related to the function defined by

$$\mathcal{P}_w: \qquad \mathcal{B}(w) \equiv \max_{\{v(s)\in[0,\bar{v}(s)]|s\in\mathcal{S}\}} \mathbb{E}v(s) \quad \text{s.t.} \quad \chi(N,\delta)v(s) \le w$$

$\forall w \in [0, \max_{s\in\mathcal{S}} \bar{v}(s)]$. The next lemma shows that the solution of $\mathcal{P}$ is actually the maximum fixed point of $\mathcal{B}(w)$; the proof is in the Appendix A (p.44).

Lemma 7. *Suppose $\{s_t\}$ is iid over time and has a distribution strictly positive over $\mathcal{S}$. Then V^* is the unique maximum fixed point of $\mathcal{B}(w)$.*

[33] More specifically, $v(s) = z(s)\pi(\ell(s); s) = z(s)\pi(r; s)$ with $r^0(s) \le r \le x$. As $\pi(r; s)$ is strictly increasing in r, $v(s)$ is one-to-one to $\ell(s) = (r, q^e(r; s))$ and ranges over $[0, \bar{v}(s)]$.

As a result, there is a unique solution $\{v^*(s)\}$ of $\mathcal{P}$, and this in turn verifies the claim that the optimal SSPE is unique.[34] The task of solving for $\mathcal{P}$ becomes finding the maximum fixed point of $\mathcal{B}(w)$. For any given w, the counterparts of IICs in $\mathcal{P}_w$ become state independent, i.e., $v(s) \leq w/\chi(N, \delta)$, which makes $\mathcal{P}_w$ easy to solve and $\mathcal{B}(w)$ easy to characterize. Consequently, solving for the maximum fixed point of $\mathcal{B}(w)$ reduces a simple discussion of w over different regions, yielding $\{v^*(s)\}$ as a by-product.[35]

In the rest of this section, I proceed by characterizing the optimal SSPE under three different specifications of iid $\{s_t\}$, corresponding to three different forms of business cycle shocks: the credit demand shock, the collateral value shock, and the risk distribution shock. Then I investigate the implications of the interaction among different forms of shocks. Lastly, I discuss briefly the case where $\{s_t\}$ is serially correlated.

IV.B Credit Demand Shock

I first consider the credit demand shock $z_t = \Xi(s_t)$ with the simplest distribution specification denoted by F_z: z_t is iid over time, takes one of two values $\{z_h, z_l\}$ with $z_h > 1 > z_l > 0$, $\Pr(z_h) = \gamma_h > 0$ and $\Pr(z_l) = \gamma_l = 1 - \gamma_h > 0$, and satisfies $\mathbb{E}z_t = 1$. In this case z_t is the only aggregate shock and all other payoff relevant parameters remain constant. A value $z_h > 1$ reflects a positive credit demand shock which shifts the inelastic credit demand schedule outwards during booms; and a two-state z_t means that the economy is either in boom or recession. Since the shock affects only the indivisible size of each project but no other payoff relevant parameters, the efficient contract space $\mathcal{C}^e$ is independent of z. In particular, the profit rate $\pi(\ell)$, the zero profit contract $\ell^0 = (r^0, q^0)$, and the monopoly contract $\ell^m = (r^m, q^m)$ are the same for all z. For simplicity I relabel $\mathcal{S}$ so that $s = h$ or l.

The following proposition fully characterizes the optimal SSPE where the credit demand shock is the only aggregate shock; the proof is in Appendix A (p.44).

Proposition 1. *Suppose $\{z_t\}$ is the only aggregate shock which satisfies F_z and denote by $\ell^*_s = (r^*_s, q^*_s)$ $\forall s \in \{h, l\}$ the action profile of the optimal SSPE.*

(a) *If $\chi(N, \delta) \leq 1/z_h$ then $\ell^*_s = \ell^m$ and $\pi(\ell^*_s) = \pi^m$ $\forall s = h, l$.*

[34] A strictly positive distribution over $\mathcal{S}$ ensures the uniqueness of the solution of $\mathcal{P}_w$. When $\mathcal{S}$ is a continuum set, the uniqueness is subject to the requirement that $v(s)$ be continuous over $\mathcal{S}$. See Liu (2014) for a more detailed discussion.

[35] Essentially, Lemma 7 is a restatement of the solution approach used in Rotemberg and Saloner (1986) for a general shock specification.

(*b*) *If* $1/z_h < \chi(N,\delta) \leq 1$ *then* $\ell_l^* = \ell^m$ *and* ℓ_h^* *is such that* $r^0 < r_h^* < r_l^* = r^m$ *and* $q^0 < q_h^* < q_l^* = q^m$. *Moreover* $0 < \pi(\ell_h^*) < \pi(\ell_l^*) = \pi^m$ *while* $z_h\pi(\ell_h^*) \geq z_l\pi(\ell_l^*)$

(*c*) *If* $1 < \chi(N,\delta)$ *then* $\ell_s^* = \ell^0$ *and* $\pi(\ell_s^*) = 0\ \forall s = h, l$.

This proposition makes clear that the property of the optimal SSPE depends on the value of the characteristic $\chi(N,\delta)$. Each of the three regions of $\chi(N,\delta)$ corresponds to a distinct category of overall competition. Intuitively, bank's long-term gain of joint profit maximization at monopoly level outweighs short-term gain of deviation when either the number of banks N is sufficiently small or the banks are sufficiently patient with δ close to 1, both of which lead to small $\chi(N,\delta)$. The opposite occurs if N is very large or δ is very small, as summarized by a large $\chi(N,\delta)$, in which case long-term cooperation is always vulnerable to short-term deviation whenever the gain of doing so is positive. For interim values of both N and δ, $\chi(N,\delta)$ falls in the region where joint profit maximization in booms is constrained by the increased gain from deviation, which in turn pushes the banks to compete more so as to keep the prevailing profit rate low enough for counter-balancing the greater incentive to deviate.

The last point can be seen more clearly from the intertemporal incentive constraint in a boom period. The IIC in state h for the optimal SSPE is

$$\frac{1}{N}z_h\pi(\ell_h) + \frac{\delta}{N(1-\delta)}[\gamma_h z_h\pi(\ell_h) + \gamma_l z_l\pi(\ell^m)] \geq z_h\pi(\ell_h),$$

where I have used the fact that $\ell_l^* = \ell^m$ when $1/z_h < \chi(N,\delta) \leq 1$. Were the banks try to maintain jointly monopoly profit rate by choosing $\ell_h = \ell^m$, the IIC would require $\frac{\delta}{N(1-\delta)}\pi(\ell^m) \geq \frac{N-1}{N}z_h\pi(\ell^m)$ as $\mathbb{E}z = 1$, so that the continuation value (LHS) is no less than the value of deviation (RHS). However, $1/z_h < \chi(N,\delta)$ implies that $\frac{N-1}{N}z_h > \frac{\delta}{N(1-\delta)}$, so that the value of deviation is necessarily greater than the continuation value if $\ell_h = \ell^m$. The banks stop undercutting each other only if $\pi(\ell_h)$ is low enough to restore the IIC, and any profit rate higher than this level is competed away.

Compare this with the IIC in state l, i.e., recessions. Were the banks to charge $\ell_l = \ell_h^*$ such that $\pi(\ell_h^*) < \pi(\ell^m)$, then the IIC would only require $\frac{\delta}{N(1-\delta)}\pi(\ell_l) \geq \frac{N-1}{N}z_l\pi(\ell_h)$. Yet $\chi(N,\delta) \leq 1 < 1/z_l$ implies that $\frac{N-1}{N}z_l < \frac{\delta}{N(1-\delta)}$, therefore the IIC is strictly non-binding, which means that there is still room for a higher profit rate in state l to be achieved in the optimal SSPE. The comparison between the IIC in the two states clearly indicates that it is the higher value of z_h, i.e., a positive demand shock, that causes a higher incentive to deviate during booms. The banks optimally choose to compete away any profit rate that is higher than $\pi(\ell_h^*)$, the highest level that is sustainable in the optimal SSPE. For this, the banks charge a lower interest rate $r_h^* < r_l^*$ and enforce a lower standard $q_h^* < q_l^*$ during booms.

As a result, procyclical lending policy emerges endogenously in the optimal SSPE when the banking sector characteristic $\chi(N, \delta)$ takes interim values. This procyclicality, especially in lending standard, is a fairly strong result, as the underlying demand shock z_t is a pure quantity shock which affects no risk attribute of the model economy. Because of bank's endogenous choice of lower lending standard, higher credit demand during booms ultimately leads to more risk on bank's balance sheet, as more bad projects are financed by the banking sector. This is made evident by the probability of good borrowers conditional on receiving good signal G hence being approved by banks, i.e., $\nu^G = \frac{\bar{\mu}q^*}{\bar{\mu}q^*+(1-\bar{\mu})(1-q^*)}$, an increasing function of q^*. The lower q^* is, the more likely a bad borrower receives a good signal G, and consequently more bad borrowers obtain credits. Given ν^G, the average success probability across all projects being financed is $\bar{\theta} = \nu^G\theta^g + (1-\nu^G)\theta^b$, and the average default probability $1-\bar{\theta}$ thereby is decreasing in q^* as $\theta^g > \theta^b$. Thus, a pure quantity demand shock endogenously leads to higher default risk in the economy by inducing the banks to lower their lending standards.

Procyclical lending policy induced by credit demand shock also highlights the importance of an intermediate degree of competitive force as captured by an interim value of $\chi(N, \delta)$.[36] When the competitive force in the banking sector is either too strong or too weak, captured by extreme values of $\chi(N, \delta)$, lending policy does not respond to credit demand shock but stays constant with either hight-rate/high-standard ℓ^m or low-rate/low-standard ℓ^0. In contrast, for intermediate competitive force= captured by interim values of $\chi(N, \delta)$, lending policy becomes responsive to the credit demand shock through the channel of competition. Procyclical competition thus drives procyclical lending policy in this case.

The above discussion suggests that procyclical lending policy $\ell_h^* \neq \ell_l^*$ depends on having a

[36]The term *competitive force* tends to describe the degree of the overall strategic rivalry among the banks, both across state and over time, as imposed by relatively slow-moving fundamentals like industry structure N and time preference δ. In this regard, the competitive characteristic $\chi(N, \delta)$ turns out to be a proper indicator of three different categories of the overall pattern of competition: jointly monopoly, zero profit perfect competition, and the one in between. In comparison, by *procyclical competition*, I refer to a situation in which the market outcome of banks' strategic interactions varies systematically across different states of the economy, but remains in between the two extremes of monopoly and perfect competition. Nonetheless, there is a connection between $\chi(N, \delta)$ and the market outcome in state h: when $1/z_h < \chi(N, \delta) \leq 1$, $\pi(\ell_h^*) = \frac{\gamma_l}{\chi(N,\delta)-\gamma_h}\frac{z_l}{z_h}\pi^m$ (see the proof of Proposition 1), which allows an interpretation that higher $\chi(N, \delta)$ leads to more competition in state h.

higher than usual demand shock $z_h > 1$ during booms. To further explore the impact of the magnitude of z_h on ℓ_h^*, I consider the following comparative static exercise. Keep on fixing z_l, γ_l, and $\mathbb{E}z = 1$. Let z_h and γ_h vary in a way such that $\gamma_h z_h = \bar{z} < 1$ remain constant.[37] The next proposition characterizes ℓ_h^* as a function of z_h; the proof is in Appendix A (p.45).

Proposition 2. *Suppose that* $\chi(N,\delta) \leq 1$ *is fixed and variation in* z_H *always satisfies* $1/z_h < \chi(N,\delta)$. *Then in the optimal SSPE* $\ell_h^* \in \mathscr{C}^e$ *is such that*

$$\pi_h^* \equiv \pi(\ell_h^*) = \frac{1-\bar{z}}{z_h\chi(N,\delta)-\bar{z}}\pi^m < \pi^m.$$

Moreover $\lim_{z_h\to\infty} \pi_h^* = 0$ *and* $\lim_{z_h\to\infty} \ell_h^* = \ell^0$.

The intuition underlies this proposition is simple. Since the deviation incentive is proportional to z_h in state h, the greater the z_h is, the more excess profit needs to be competed away so as to satisfy the IIC. In the limit, the banking sector becomes very close to perfect competition with zero profit when the economy is hit by a sufficiently strong credit demand shock. With credit demand shock evolving in such a pattern, the economy is jumping back and forth along the equilibrium path, where both interest rate and lending standard spike to a level as high as monopoly during recessions while plummet to the perfectly competitive level during large booms. With the aid of imperfect competition, the financial cycle resulted in such a way can be quite volatile.

IV.C Collateral Value Shock

In this subsection, I consider the aggregate collateral value shock $c_t = \Xi(s_t)$.[38] As for the case of credit demand shock, I continue to assume that $\{c_t\}$ satisfies a simple distribution specification denoted by F_c: c_t is iid over time, takes one of two values $\{c_h, c_l\}$ with $1 > c_h > c_l > 0$, $\Pr(c_h) = \gamma_h > 0$ and $\Pr(c_l) = \gamma_l = 1 - \gamma_h > 0$. This captures the idea that as asset prices pick up on average

[37] This ensures that when comparing ℓ_h^* of economies with different z_h, all those economies have the same mean credit demand and aggregate risk attributes in state l. One interpretation is that state l represents the normal time whereas state h stands for a period with unexpectedly high credit demand.

[38] To help fix idea, I provide a more specific interpretation of c. Consider a θ borrower starts a project with one unit initial investment. The initial investment is divided into two parts: one for fixed investment into tangible capital, be it machinery, equipment, and plant for a firm, or simply a house for a home buyer; the other for intangible capital, such as enhanced productivity of a home buyer achieved from having the house for better rest. Depending on the overall economic situation, the tangible capital commands a resale value of c at the end of the period, while the extra risky payoff from operating the project is either $x - c$ with probability θ or 0 with $1 - \theta$. By assuming limited liability, the only collateral the borrower needs to put up front is the project itself, from which the lending bank seizes the tangible capital in default at a value of c.

in booms, the value of given collateral also increases. In this case, the collateral value shock is the only aggregate shock, and all other parameters remain constant. In particular, the credit demand $z = 1$ is constant.

Even though the collateral value shock only takes two values c_h and c_l, it is useful to define relevant functions $\forall c \in [c_l, c_h]$. In particular, let $\eta^\theta(r;c) = \theta r + (1-\theta)c - 1$, $\Delta(r;c) = \bar{\mu}\eta^g(r;c) - (1-\bar{\mu})\eta^b(r;c)$, and $q^e(r;c) = (C')^{-1}(\Delta(r;c))$. As is easily verified, Assumption 1–3 still hold $\forall c \in [c_l, c_h]$ as long as they hold for $c = c_l$ and c_h. Thus $\mathscr{C}^e(c)$, $\ell^0(c)$, and $\ell^m(c)$ are all well-defined $\forall c$.

For subsequent use, I write the profit rate function $\pi(\ell;c) = \pi(r;c)$ explicitly as a function of r and c:

$$\pi(r;c) = \bar{\mu}q^e(r;c)\eta^g(r;c) + (1-\bar{\mu})(1-q^e(r;c))\eta^b(r;c) - C(q^e(r;c)).$$

As $q^e(r;c)$ solves $\max_q \bar{\mu}q\eta^g(r;c) + (1-\bar{\mu})(1-q)\eta^b(r;c) - C(q)$, the Envelope theorem implies that

$$\partial_c\pi(r;c) = \bar{\mu}q^e(1-\theta^g) + (1-\bar{\mu})(1-q^e)(1-\theta^b) > 0,$$
$$\partial_r\pi(r;c) = \bar{\mu}q^e\theta^g + (1-\bar{\mu})(1-q^e)\theta^b > 0,$$

with $q^e = q^e(r;c)$. It follows that $\pi_h^m = \pi(x;c_h) > \pi(x;c_l) = \pi_l^m$, and $r_h^0 < r_l^0$, for otherwise a contradiction results from inequality $\pi(r_h^0;c_h) \geq \pi(r_l^0;c_h) > \pi(r_l^0;c_l) = 0$. Let $\bar{\pi}^m = \gamma_h\pi_h^m + \gamma_l\pi_l^m$.

With this preparation, I can state the first main result for the case of the collateral value shock. The next proposition provides a partial characterization of the optimal SSPE; the proof is in Appendix A (p.45).

Proposition 3. *Suppose $\{c_t\}$ is the only aggregate shock which satisfies F_c and denote by $\ell_s^* = (r_s^*, q_s^*)$ $\forall s \in \{h, l\}$ the action profile of the optimal SSPE.*

(a) *If $\chi(N,\delta) \leq \bar{\pi}^m/\pi_h^m$ then $\ell_s^* = \ell_s^m$ and $\pi(\ell_s^*) = \pi_s^m$ $\forall s = h, l$.*

(b) *If $\bar{\pi}^m/\pi_h^m < \chi(N,\delta) \leq 1$ then $\ell_l^* = \ell_l^m$ and ℓ_h^* is such that $r_h^0 < r_h^* < r_l^* = r_h^m$. Moreover $0 < \pi(\ell_l^*;c_l) = \pi_l^m \leq \pi(\ell_h^*;c_h) < \pi_h^m$.*

(c) *If $1 < \chi(N,\delta)$ then $\ell_s^* = \ell_s^0$ and $\pi(\ell_s^*;c_s) = 0$ $\forall s = h, l$.*

Virtually the same intuition detailed following Proposition 1 applies here: as higher collateral value during booms increases deviation incentive, it is optimal for the banks to compete away any excess profit above $\pi(\ell_h^*)$ so to restore IIC when $\chi(N,\delta)$ takes interim values. This proposition predicts that when there is only the aggregate collateral value shock, the banking sector's profit rate $\pi(\ell_s^*;c_s)$ is weakly procyclical as long as the sector is no perfectly competitive, i.e., $\chi(N,\delta) \leq 1$.

In contrast, Proposition 1 predicts that when the only aggregate shock is a credit demand shock, the profit rate is weakly countercyclical under the same condition of $\chi(N,\delta)$. As a result, when both shocks are present, the cyclicality of banking sector profit rate is ambiguous. This suggests that an empirical measure of banking sector profit rate, however accurate, is not always a good indicator of cyclical competition when the real economy is subject to both a credit demand shock and a collateral value shock.

Unlike the case of credit demand shock, equilibrium interest rate is always countercyclical regardless of the competitive force, i.e., $\chi(N,\delta)$, of the banking sector. In particular, even if the competitive force is strong enough so that equilibrium profit rate is zero in both states, r_l^0 is still higher than r_h^0, whereas the zero profit interest rate is constant for the case of credit demand shock. This is so because a credit demand shock does not affect the riskiness of the economic fundamental, while a higher collateral value raises the overall profitability of all projects.

So far I have not discussed equilibrium dynamics of the lending standard, that is whether q_l^* is greater than q_h^* or not. The reason is that the impact of collateral value c upon efficient screening intensity could go either way under maintained assumptions. Simple calculus shows that

$$\frac{\partial q^e(r;c)}{\partial r} = \frac{\partial_r \Delta(r;c)}{C''(q^e(r;c))} = \frac{\bar{\mu}\theta^g - (1-\bar{\mu})\theta^b}{C''(q^e(r;c))},$$
$$\frac{\partial q^e(r;c)}{\partial c} = \frac{\partial_c \Delta(r;c)}{C''(q^e(r;c))} = \frac{\bar{\mu}(1-\theta^g) - (1-\bar{\mu})(1-\theta^b)}{C''(q^e(r;c))}.$$

As before, part (*ii*) of Assumption 1 requires that $\bar{\mu}\theta^g - (1-\bar{\mu})\theta^b > 0$, so $\partial_R q^e(r;c) > 0$; yet it puts no restriction on $\bar{\mu}(1-\theta^g) - (1-\bar{\mu})(1-\theta^b)$, so that $\partial_c q^e(r;c)$ can be either positive or negative. In order to work out a complete characterization of equilibrium dynamics of q_s^*, I first provide a partial characterization in the next lemma, of which the proof is in Appendix A (p.46).

LEMMA 8. *Suppose $c_l \le c_1 < c_2 \le c_h$ and $\ell_j = (r_j, q_j) \in \mathscr{C}^e(c_j)$ $j = 1,2$ is such that $\pi(\ell_1; c_1) = \pi(\ell_2; c_2)$. Then $r_1 > r_2$ and $q_1 > q_2$.*

An immediate implication of this result is that $q_l^* = q_l^0 > q_h^* = q_h^0$ when $\chi(N,\delta) > 1$, as both ℓ_l^0 and ℓ_h^0 results in zero profit. Like the interest rate, the equilibrium lending standard is also countercyclical when the competitive force is very strong in the banking sector. It is also straightforward to characterize $q_s^* = q_s^m \; \forall s = h,l$ when $\chi(N,\delta) \le \bar{\pi}^m/\pi_h^m$. According to the sign of

$\partial_c q^e(x;c)$, there is

$$
\begin{aligned}
q_l^* < q_h^*, &\quad \text{if } \bar{\mu}(1-\theta^g) - (1-\bar{\mu})(1-\theta^b) > 0, \\
q_l^* = q_h^*, &\quad \text{if } \bar{\mu}(1-\theta^g) - (1-\bar{\mu})(1-\theta^b) = 0, \\
q_l^* > q_h^*, &\quad \text{if } \bar{\mu}(1-\theta^g) - (1-\bar{\mu})(1-\theta^b) < 0.
\end{aligned}
$$

Thus when the competitive force in the banking sector is very weak, the cyclicality of the lending standard depends on the sign of $\bar{\mu}(1-\theta^g)-(1-\bar{\mu})(1-\theta^b)$. In particular, if the borrower distribution $\bar{\mu}$ is lower than a critical level $(1-\theta^b)/(2-\theta^g-\theta^b) < 1$, then the lending standard is countercyclical.

The remaining case is for the intermediate degree of competitive force in the banking sector. Let r_h^l denote the unique $r < x$ such that $\pi(r;c_h) = \pi(x;c_l)$. The next proposition gives a complete characterization of the equilibrium lending standard for this case; the proof is in Appendix A (p.46)

Proposition 4. *Suppose* $\bar{\pi}^m/\pi_h^m < \chi(N,\delta) < 1$. *When* $\bar{\mu} \le \frac{1-\theta^b}{2-\theta^g-\theta^b}$ *there is* $q_l^* > q_h^*$. *When* $\bar{\mu} > \frac{1-\theta^b}{2-\theta^g-\theta^b}$ *there exists an* $\hat{r}$ *such that* $r_h^l < \hat{r} < x$ *and*

(*a*) $q_l^* < q_h^*$ *if* $r_h^* > \hat{r}$;

(*b*) $q_l^* = q_h^*$ *if* $r_h^* = \hat{r}$;

(*c*) $q_l^* > q_h^*$ *if* $r_h^* < \hat{r}$.

Moreover $q_l^* > q_h^*$ *always holds for* $\chi(N,\delta)$ *sufficiently close to* 1.

Although the complete characterization of $q_s^*\ \forall s = h,l$ is somewhat delicate, for most parameter combinations the lending standard dynamics still features a countercyclical pattern, that is the banks tend to enforce a lower lending standard when the booming economy is associated with a higher collateral value. In particular, whenever the overall fraction of good borrowers is not too high or the competitive force of the banking sector is relatively strong, a countercyclical lending standard is guaranteed to emerge. Only with very high proportion of good borrowers and relatively weak competitive force of the bank sector will a higher collateral value be more than compensate a lower interest rate, such that the marginal benefit of screening becomes higher during booms, resulting in higher lending standard.

IV.D Risk Distribution Shock

In this subsection, I consider the risk distribution shock $\bar{\mu}_t = \Xi(s_t)$. As before, I assume that $\{\bar{\mu}_t\}$ satisfies a simple distribution specification denoted by $F_{\bar{\mu}}$: $\bar{\mu}_t$ is iid over time, takes one of two values $\{\bar{\mu}_h, \bar{\mu}_l\}$ with $1 > \bar{\mu}_h > \bar{\mu}_l > 0$, $\Pr(\bar{\mu}_h) = c_h > 0$ and $\Pr(\bar{\mu}_l) = c_l = 1 - c_h > 0$. This shock affects only the composition of good and bad borrowers in the economy over time.[39] The idea is that in general risk distribution should improve during booms, resulting in a better composition of good borrowers, so that $\bar{\mu}_h > \bar{\mu}_l$. In this case, the risk distribution shock is the only aggregate shock, and all other parameters, including $z = 1$ and c, remain constant over time.

As before, I define $\eta^\theta(r; \bar{\mu})$, $\Delta(r; \bar{\mu})$, and $q^e(r; \bar{\mu})$ $\forall \bar{\mu} \in [\bar{\mu}_l, \bar{\mu}_h]$, and all assumptions hold $\forall \bar{\mu}$ as long as they hold for $\bar{\mu} = \bar{\mu}_l$ and $\bar{\mu}_h$. Consequently, $\mathscr{C}^e(\bar{\mu})$, $\ell^0(\bar{\mu})$, and $\ell^m(\bar{\mu})$ are all well-defined. Analogous to the case of the collateral value shock, I write the profit rate $\pi(\ell; \bar{\mu}) = \pi(r; \bar{\mu})$ explicitly as a function of r and $\bar{\mu}$. It is easily shown that $\partial_r \pi(r; \bar{\mu}) > 0$ and $\partial_{\bar{\mu}} \pi(r; \bar{\mu})$, so that $\pi_h^m = \pi(x; \bar{\mu}_h) > \pi(x; \bar{\mu}_l) = \pi_l^m$ and $r_h^0 < r_l^0$. Let $\bar{\pi}^m = \gamma_h \pi_h^m + \gamma_l \pi_l^m$.

Although the maximization problem defining the optimal SSPE under the risk distribution shock appears to be similar to that of the collateral value shock, there turns out to be no simple characterization of equilibrium screening intensity without an additional parametric restriction.[40] The necessary restriction is $\eta^g(x) + \eta^b(x) \leq 0$. Under this restriction, I can obtain again a sharp characterization of the equilibrium. The next proposition states the result; the proof is in Appendix A (p.47).

Proposition 5. *Suppose $\{\bar{\mu}_t\}$ is the only aggregate shock which satisfies $F_{\bar{\mu}}$ and $\eta^g(x) + \eta^b(x) \leq 0$. Denote by $\ell_s^* = (r_s^*, q_s^*)$ $\forall s \in \{h, l\}$ the action profile of the optimal SSPE.*

(a) *If $\chi(N, \delta) \leq \bar{\pi}^m / \pi_h^m$ then $\ell_s^* = \ell_s^m$ and $\pi(\ell_s^*) = \pi_s^m$ $\forall s = h, l$. Moreover $q_h^m \leq q_l^m$ and $\pi_h^m > \pi_l^m$.*

(b) *If $\bar{\pi}^m / \pi_h^m < \chi(N, \delta) \leq 1$ then $\ell_l^* = \ell_l^m$ and ℓ_h^* is such that $r_h^0 < r_h^* < r_l^* = r_h^m$ and $q_h^0 < q_h^* < q_l^* = q_h^m$. Moreover $0 < \pi(\ell_l^*; \bar{\mu}_l) = \pi_l^m \leq \pi(\ell_h^*; \bar{\mu}_h) < \pi_h^m$.*

(c) *If $1 < \chi(N, \delta)$ then $\ell_s^* = \ell_s^0$ and $\pi(\ell_s^*; c_s) = 0$ $\forall s = h, l$. Moreover $r_h^0 < r_l^0$ and $q_h^0 < q_l^0$.*

Under the additional restriction $\eta^g(x) + \eta^b(x) \leq 0$, the overall pattern of the equilibrium dynamics is quite similar to that of the collateral value shock. In particular, both bank lending policy and banking sector profit rate are countercyclical for interim values of $\chi(N, \delta)$. As for the

[39] A supplementary assumption is that conditional on $\bar{\mu}_t$, the investment shock θ is still iid over borrowers and satisfies $\Pr(\theta^g) = \bar{\mu}_t$, so that an application of a suitable law of large number implies that the fraction of good borrowers is $\bar{\mu}_t$ at time t.

[40] The characterization of equilibrium interest rate parallels that of the collateral value shock, so does equilibrium profit rate, without any additional parametric restriction. For the problem of obtaining a simple characterization of screening intensity comparable to that of the collateral value shock, see Online Appendix C.

case of the credit demand shock and the collateral value shock, this countercyclicality is driven by procyclical competition within the banking sector. To sum up, countercyclical lending policy in both interest rate on loans and lending standard appears to be a robust equilibrium phenomenon, under all three aggregate shock specifications.

IV.E Interactions among Shocks

In this subsection, I consider the case in which two aggregate shocks are correlated. In particular, I assume both the credit demand shock and the collateral value shock are affecting the economy, so $(z_t, c_t) = \Xi(s_t)$ and satisfies the a simple distribution specification denoted by F_{zc}: s_t is iid over time, with the joint distribution given by

Prob.	$\varphi\gamma_h$	$(1-\varphi)\gamma_h$	γ_l
c_t	c_h	c_h	c_l
z_t	z_h	z_l	z_l

where $\gamma_l, \gamma_h > 0$, $\gamma_h + \gamma_l = 1$, $0 < \varphi < 1$, $c_h > c_l$, and $z_h > z_l$. It can be showed that the correlation coefficient between z_t and c_t increases from 0 to 1 as φ increases from 0 to 1.[41] This specification has the following interpretation. With probability γ_h (γ_l), the economy experiences a boom (recession) period, during the collateral value is high (low). Conditional on a boom period, with probability φ, the credit demand is high; otherwise it remains at a lower level during either a boom or a recession. This gives a very simple specification in which z_t and c_t are correlated to any degree.

To simplify the notation, let $s = h, x, l$ denote the high state (z_h, c_h), the mixed state (z_l, c_h), and the low state (z_l, c_l) respectively, and let the corresponding probabilities be ξ_s $\forall s = h, x, l$. As before, I assume Assumption 1–3 are satisfied in all states. Since z_t does not affect the unit payoff of a loan, the requirement is only binding for c_t. More specifically, the efficient contract space $\mathscr{C}_s^e$, the profit function $\pi(\ell_s; c_s)$, the zero profit contract ℓ_s^0, and the monopoly contract ℓ_s^m are well defined $\forall s = h, l$, with the convention that all objects defined for $s = x$ are the same for $s = l$. As for the case of only the collateral value shock, $\pi_h^m = \pi_x^m = \pi(\ell_h^m; c_h) > \pi(\ell_l^m; c_l) = \pi_l^m$. Moreover, let $\bar{v}_h = z_h \pi_h^m$, $\bar{v}_x = z_l \pi_h^m$, $\bar{v}_l = z_l \pi_l^m$, and $\bar{v} = \mathbb{E}\bar{v}_s = \xi_h \bar{v}_h + \xi_x \bar{v}_x + \xi_l \bar{v}_l$.

The next proposition characterizes the optimal SSPE where the collateral value shock interacts with the credit demand shock; the proof is in Appendix A (p.47).

[41] See the technical lemma D.1 in the online Appendix D.

PROPOSITION 6. *Suppose the aggregate shock* $s_t = (z_t, c_t)$ *satisfies distribution* $F_{z,c}$ *and* $\bar{\mu} \leq \frac{1-\theta^b}{2-\theta^g-\theta^b}$. *Denote by* $\ell_s^* \ \forall s = h, x, l$ *the action profile of the optimal SSPE.*

(a) *If* $\chi(N,\delta) \leq \bar{v}/\bar{v}_h$ *then* $\ell_s^* = \ell_s^m \ \forall s = h, x, l$.

(b) *If* $\bar{v}/\bar{v}_h < \chi(N,\delta) \leq [(\xi_h + \xi_x)\bar{v}_x + \xi_l \bar{v}_l]/\bar{v}_x$ *then* $\ell_s^* = \ell_s^m$ *for* $s = x, l$ *and* $\ell_h^* = (r_h^*, q_h^*)$ *is such that* $r_h^* < r_x^* = r_l^* = x$ *and* $q_h^* < q_x^* < q_l^*$.

(c) *If* $[(\xi_h + \xi_x)\bar{v}_x + \xi_l \bar{v}_l]/\bar{v}_x < \chi(N,\delta) \leq 1$ *then* $\ell_l^* = \ell_l^m$ *and* $\ell_s^* = (r_s^*, q_s^*) \ \forall s = h, x$ *are such that* $r_h^* < r_x^* < r_l^* = x$, *and* $q_h^* < q_x^* < q_l^*$.

(d) *If* $\chi(N,\delta) > 1$ *then* $\ell_s^* = \ell_s^0 \ \forall s = h, x, l$.

The proposition indicates that if $0 < \varphi < 1$, i.e., z_t and c_t are positively correlated, then the credit demand shock amplifies the impact of the collateral shock upon the lending policies for interim $\chi(N,\delta)$. The same result holds if I exchange the order of c_t and z_t in the distribution specification F_{zc}. Thus the two shocks are mutually reinforcing each other when their correlation is positive, making the lending policies even more countercyclical over time. Intuitively, when the correlation is positive, both shocks are likely to have high realizations together, and for certain values of $\chi(N,\delta)$, this makes the deviation incentive be even higher, thus more aggressive competition is required for restoring the intertemporal incentive constraint.

IV.F Persistent Shock

In this subseciton, I consider again the credit demand shock $z_t = \Xi(s_t)$ which takes one of two values $z_h > 1 > z_l > 0$ as before. Unlike the previous case in which $\{z_t\}$ is iid over time, now I assume the shock is persistent. In particular, $\{z_t\}$ is a stationary Markov chain with the transition matrix

$$P = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix},$$

with $0 < \alpha, \beta < \frac{1}{2}$, so that $\Pr(z_t = z_l | z_{t-1} = z_h) = \alpha$ and $\Pr(z_t = z_h | z_{t-1} = z_l) = \beta$. The stationary distribution of $\{z_t\}$ γ is a row vector $(\gamma_h, \gamma_l) = \left(\frac{\beta}{\alpha+\beta}, \frac{\alpha}{\alpha+\beta}\right)$, and I normalize z_s so that $\mathbb{E}z_t = \gamma_h z_h + \gamma_l z_l = 1$. Intuitively, the shock becomes more persistent as α and β become closer to 0. Indeed, the first-order autocorrelation coefficient between z_{t-1} and z_t equals to $1-(\alpha+\beta) > 0$.[42] This setup captures the idea that boom periods tend to persist over time. As before, $\mathscr{C}^e$, ℓ^0, ℓ^m, and $\pi(\ell)$ are independent of z, and hence do not change over time.

[42]See the technical lemma D.2 in the online Appendix D.

Even though the shock is persistent, the optimal action profile $\{\ell_s^*\}$ in the optimal SSPE depends only on the state of the economy. Under the one-to-one map $v_s = z_s\pi(\ell_s)$, the maximization problem associated with the optimal SSPE becomes a linear program. In contrast to the case of an iid shock, when $\{z_t\}$ is persistent, the expected continuation value from any time-invariant value profile $\{v_s\}$ implied by an action profile depends on the current state s. Let V_s denote the expected continuation value in state s. Exploiting the recursive structure of the setup, V_s is linked to v_s through the following matrix equation

$$V = \frac{1}{N}(P + \delta P^2 + \delta^2 P^3 + \cdots)v = \frac{1}{N}P(1-\delta P)^{-1}v,$$

where $V = (V_h, V_l)^\top$ and $v = (v_h, v_l)^\top$ are both column vectors.

Since $\gamma P = \gamma$, the expected discount value of any bank γV equals to $\frac{1}{N(1-\delta)}\gamma v$. As the IIC in state s is equivalent to $N(1-\delta)V_s \geq \chi(N,\delta)v_s$, the linear program associated with the optimal SSPE can be written in the following compact form:

$$\max_{v\in[0,z_h\pi^m]\times[0,z_l\pi^m]} \gamma v \quad \text{s.t.} \quad (1-\delta)P(1-\delta P)^{-1}v \geq \chi(N,\delta)v,$$

where $\pi^m = \pi(\ell^m)$. The following proposition characterizes the optimal SSPE; the proof is in Appendix A (p.48).

Proposition 7. *Suppose $\{z_t\}$ follows a two state Markov chain with the transition matrix P and denote by ℓ_s^* $\forall s = h, l$ the action profile in the optimal SSPE.*

(*a*) *If $\chi(N,\delta) \leq \bar{\chi}$ then $\ell_s^* = \ell^m$ and $\pi(\ell_s^*) = \pi^m$ $\forall s = h, l$.*

(*b*) *If $\bar{\chi} < \chi(N,\delta) \leq 1$ then $\ell_l^* = \ell^m$ and ℓ_h^* is such that $r^0 < r_h^* < r_l^* = r^m$ and $q^0 < q_h^* < q_l^* = q^m$. Moreover $0 < \pi(\ell_h^*) < \pi(\ell_l^*) = \pi^m$ while $z_h\pi(\ell_h^*) \geq z_l\pi(\ell_l^*)$*

(*c*) *If $1 < \chi(N,\delta)$ then $\ell_s^* = \ell^0$ and $\pi(\ell_s^*) = 0$ $\forall s = h, l$.*

The cutoff value $\bar{\chi} = \frac{\rho(1-\delta)}{1-\delta\rho} + \frac{1-\rho}{1-\delta\rho}\frac{1}{z_h}$ and satisfies $\frac{1}{z_h} \leq \bar{\chi} < 1$ $\forall 0 \leq \rho < 1$.

As demonstrated by this proposition, the qualitative feature of the optimal SSPE with a persistent shock remains the same as the case with an iid shock,[43] and similar results hold for persistent

[43]This result also complements the finding of Kandori (1991) who showed that countercyclical pricing still holds in the setting of Rotemberg and Saloner (1986) with a general Markov shocks for two limiting cases: (*a*) $\delta \to (N-1)/N$ so that $\chi(N,\delta) = 1$; and (*b*) $\delta \to 1$ and $N(1-\delta)$ remains a constant M between 0 and 1 so that $\chi(N,\delta)$ converges to $M(N-1)/N$ from above. See Liu (2014) for more discussion.

collateral value shock and risk distribution shock as well. Exactly the same intuition explained above for the countercyclical lending policies applies here: increased deviation incentive during booms forces the banking sector to compete more as long as $\chi(N,\delta)$ lies in an interim range. The main difference relative to the iid shock case is the diminished range of $\chi(N,\delta)$ displaying countercyclical lending policies. In the iid case, the lower bound for such $\chi(N,\delta)$ is $1/z_h$. In contrast, in the persistent case, the lower bound $\bar{\chi}$ is higher then $1/z_h$ and is increasing in ρ. Indeed, $\lim_{\rho\to 1}\bar{\chi} = 1$, hence the optimal SSPE displays either joint monopoly or perfect competition. Intuitively, as ρ converges to 1, uncertainty vanishes and the standard result of a deterministic repeated game emerges.

V Discussions of the Model

In this section, I will first discuss a number of extensions and variations of the benchmark setting presented in Section II, so that I can assess the robustness of the benchmark setting. In the last subsection, I will discuss the welfare prediction of the basic framework in both the static and the dynamic setup.

V.A Characteristics of Borrower and Project

I have considered the simplest case of a borrower's type distribution, a binary one, in the benchmark setting. This particular information structure can be easily generalized to the case where a borrower's type θ is continuous distributed over interval $[0,1]$ under some distribution function $F(\theta)$ (see, e.g., de Meza and Webb 1987). Keep the investment technology the same, i.e., for a θ borrower (project) the output equals to x with probability θ and c with probability $1-\theta$. It is clear that there is a cutoff level $0<\hat{\theta}<1$ such that $\text{NPV}^{\theta} = \theta x + (1-\theta)c - 1 \geq 0$ if and only if $\theta > \hat{\theta}$. Now, call any borrower with $\theta \geq \hat{\theta}$ a g borrower and with $\theta < \hat{\theta}$ a b borrower, and let $\bar{\mu} = 1 - F(\hat{\theta})$, $\theta^g \equiv \mathbb{E}[\theta|\theta \geq \hat{\theta}]$, and $\theta^b \equiv \mathbb{E}[\theta|\theta < \hat{\theta}]$. Then this generalized setting is identical to the benchmark setting as long as I assume that the screening technology continues to work the same, separately on g borrowers and b borrowers:

$$\Pr(\phi = G|\theta \geq \hat{\theta}) = \Pr(\phi = B|\theta < \hat{\theta}) = q,$$

for a given intensity q. Continue to assume that θ shock is iid across a continuum of borrowers, then a bank lending decision will again depends only on the screening result ϕ. This is so because the variation in the particular realization θ for an individual borrower is averaged out within all g or b borrowers, thus a bank's payoff from lending depends only on $\bar{\mu}$, θ^g, θ^b, and q as before.

Along the same line, the benchmark setting can be further generalized to accommodate a continuously distributed end-of-period output for each investment project. More specifically, let θ be distributed according $F(\theta)$ as well, and suppose that the output y of each θ borrower (project) is distributed over $[0, x]$ according to some distribution function $H(y, \theta)$. Moreover, θ is no longer assumed to stand for the success probability of a project, but instead becomes an index for the riskiness of projects. A particular way of indexation is to assume $H(y, \theta)$ satisfies the first order stochastic dominance in θ, so that $\text{NPV}^{\theta} = \int_0^x y \mathrm{d}H(y, \theta) - 1$ is continuously increasing in θ.[44] Let $\hat{\theta}$ be the threshold level above which NPV^{θ} becomes positive, and call borrowers with $\theta \geq \hat{\theta}$ type g while those with $\theta < \hat{\theta}$ type b. Assuming again limited liability on the part of borrowers, the unit expected payoff from lending to a θ borrower at interest rate $r \in [1, x]$ is $\eta^{\theta}(r) = r[1 - H(r, \theta)] + \int_0^r y \mathrm{d}H(y, \theta)$.[45] Thus the unit expected payoff from lending to either g or b borrowers, $\eta^{g}(r) \equiv \mathbb{E}[\eta^{\theta}(r) | \theta \geq \hat{\theta}]$ and $\eta^{b}(r) \equiv \mathbb{E}[\eta^{\theta}(r) | \theta < \hat{\theta}]$, can be computed accordingly. Under the iid assumption on the θ shock and the output of each individual project, and continue to use the screening technology as specified above, all results regarding screening and lending, hence the equilibrium behavior of the benchmark setting, are preserved.

V.B An Alternative Lending Process

In the benchmark setting, banks are required to post contracts which specify both r and q at the same time, which amounts to committing to a particular level of screening effort during the entire process of creditworthiness testing on any borrower who enters into such a process. Two implicit assumptions are buried into such a specification: first, a commitment technology is available to each bank; and second, each bank is willing to honor a commitment given the commitment technology. The latter point is equivalent to that a commitment by a bank is credible. In reality, credibility on a commitment to some particular screening intensity q by any bank is evidently questionable. The argument presented in Section III.D for restricting the analysis to the efficient contract space $\mathscr{C}^e$ just exploits this built-in weakness. However, a more fundamental point associated with the commitment problem is the availability of such a technology. In particular, one is compelled to

[44] Alternatively, as in Stiglitz and Weiss (1981) and Bester (1985), one may assume $H(y, \theta)$ satisfies the second order stochastic dominance in θ while NPV^{θ} is the same for all θ.

[45] Here I assume again that lending takes the form of a simple loan contract with gross interest rate r. Indeed, as output level y is publicly verifiable, an assumption I maintain here, all expected payoff that is achievable by an arbitrary (contingent) contract $R(y)$ is also achievable by a simple loan contract. When y is not verifiable, the optimal contract design problem arises, as analyzed in Townsend (1979), Diamond (1984), and Gale and Hellwig (1985).

think about the situation where the commitment technology regarding q does not exist altogether, which amounts to modify the lending process such that decisions on q can only be made by banks right before any screening activity.

From a game theoretic point of view, this is equivalent to a modification of the extensive form of the lending game. In particular, banks first offer contracts only in interest rates r, followed by borrowers choosing which contracts to apply for, and lastly banks make whatever screening and lending decisions on any borrowers they encounter. Given this modified game tree, sequential rationality requires the bank which is choosing q to take into account its choice of r already made in the previous information set. More specifically, let $\mu(r)$ denote the (common) prior belief on the probability that a borrower applying for the loan r is type g. For any choice of $q \in \mathbb{Q}$, let $\nu^{\phi}(q, r) = \nu^{\phi}(q, \mu(r))$ denote the bank's posterior conditional on the screening result ϕ. It follows that the unit expected payoff of the bank from lending to a ϕ borrower is

$$\eta^{\phi}(r, q) = \nu^{\phi}(q, r)\eta^{g}(r) + \big(1 - \nu^{\phi}(q, r)\big)\eta^{b}(r),$$

and thus the unit expected payoff from lending at r, gross of the screening cost, is

$$\eta(r, q) = \Pr(G)\max\big\{\eta^{G}(r, q), 0\big\} + \Pr(B)\max\big\{\eta^{B}(r, q), 0\big\},$$

where $\Pr(\phi)$ is a function of q and $\mu(r)$. Since choosing q is the last nontrivial decision for the bank (approval decision is trivial once knowing η^{ϕ}), sequential rationality implies that the bank will choose q to maximize the net payoff $\eta(r, q) - C(q)$.

Up to now, the above formulation of a bank's choice of q looks like the same as those already discussed in Section III.D; but there remains a crucial difference. In Section III.D, the belief $\mu(\ell)$ at any candidate equilibrium contract $\ell = (r, q) \in \mathscr{C}$ has been fixed at $\bar{\mu}$, by Lemma 2 in Section III.B. Yet an arbitrary belief system $\mu(\cdot)$ is used here. To pin down the belief system, at least along any equilibrium path, note that the conclusion of Lemma 2 is still applicable to this modified lending game. No matter in which order the banks make decisions on r and q, any borrower can always correctly predict the equilibrium choices, i.e., (r, q) pairs, by the banks; this is nothing but the premise of Nash equilibrium. Consequently, the same proof of Lemma 2 applies to pairs of (behavioral) strategies (r, q), and therefore $\mu(r, q) = \bar{\mu}$ for any (r, q) to emerge in a candidate equilibrium. Furthermore, since all borrowers make their choices of loans r before any bank chooses q, consistency of beliefs along the game tree implies that $\mu(r) = \mu(r, q)$ regardless of a bank's choice of q. Thus I conclude that along any equilibrium path, $\mu(r) = \bar{\mu}$ when r is a candidate equilibrium contract.

Once $\mu(r) = \bar{\mu}$ is pinned down along any equilibrium path of the modified lending game, the optimal choice of q by any bank at r has to be $q^e(r)$, i.e., the efficient screening intensity defined in Section III.D. As a final remark, because the modified lending game has the same normal form representation as the original lending game, the invariance principle of Kohlberg and Mertens (1986) implies that, for the original lending game, any "reasonable" equilibrium satisfying their stability criterion features a contract lying in the efficient contract space $\mathscr{C}^e$. This justifies my restriction to $\mathscr{C}^e$ in Section III.D.

V.C Other Forms of Screening Technologies

In this subsection, I will discuss in some detail the specification of the screening technology. The benchmark specification of the screening technology in Section II.B takes a very simple, symmetric form: $\Pr(G|g) = \Pr(B|b) = q$. Moreover, the screening technology is applied on a borrower by a bank only once in each period. This feature is closely related to the exclusivity assumption, that a borrower can apply for at most one loan in a given period. In what follows, I shall consider some alternative specifications of the screening technology.

Asymmetric Screening Given a binary-type-binary-signal structure, a general screening technology is described by the conditional probabilities $\Pr(G|g) = q_g$ and $\Pr(B|b) = q_b$. Examples of asymmetric screening include: Broecker (1990) where $q_g \neq q_b$ are both fixed numbers and the screening is costless; and Gehrig (1998) and Kanniainen and Stenbacka (1997) where $q_g(e)$ and $q_b(e)$ are two different functions of a common argument e, with per unit screening cost of $C(e)$.[46] By introducing proper assumptions, one can extend the benchmark setting by replacing the simple symmetric screening technology with a general form asymmetric screening technology. However, such an extension does not add much insight into the determination and the dynamics of the lending policies, especially the lending standards. Thus I shall not pursue it further.

Sequential Screening As already mentioned, the exclusivity assumption used in the benchmark setting is widely adopted in the literature on adverse selection problems. The central feature of the exclusivity assumption is that a borrower will be screened by only one bank within a period.

[46] For asymmetric screening technology with variable screening intensity and variable upfront cost, it is not meaningful to suppose that q_g and q_b can be chosen independently with independent cost function $C_g(q_g)$ and $C_b(q_b)$. The reason is that with the screening bank does not know a priori the private type of the borrower who is applying for a loan at this bank. Intuitively, variable cost of screening comes from the variation in effort (time, carefulness, etc.) that loan officers exert in reviewing an application. So that once they decide how much effort to exert, certain amount of upfront cost is incurred regardless of the underlying true type of an applicant.

Given that a borrower denied by a bank in the current period may apply again in the next period, the exclusivity assumption does not restrict the interpretational power of the benchmark model. Nonetheless, I shall briefly discuss some alternative setups with multiple screenings (within a period). A first alternative along this line is sequential screening.

A most straightforward extension to the benchmark setting is to allow a borrower to apply for another loan from the set of loans offered by all banks within the same period. This extension barely changes the results established in Section III. Since approved borrowers have no incentive to apply for a different loan — the loan they obtained is of the lowest interest rate — only denied borrowers will apply again. This pins down the prior on the borrower distribution held by the banks screening borrowers a second time, i.e., $\nu(B)$.[47] For completeness, one needs to specify as well how the second-round screening will work, conditional on knowing the results of the first-round screening. If the second-round screening is independent of the first round, then a new signal will be produced given the prior $\nu(B)$, and further posterior can be calculated. In the opposite case, if the second-round screening is perfectly correlated with the first-round, then no new information will be obtained and the lending decision will only be based on the prior $\nu(B)$. Either way, the equilibrium results of the benchmark setting remain mostly unchanged.

The preceding argument presumes that all the borrowers do choose the loan $\ell_{\min}$ in the first-round, even with the opportunity of applying for another loan later. The presence of the second-round screening opens the possibility that the borrowers, especially the good ones, may choose first to apply for a loan with higher than minimum interest rate and preserve the option of applying for $\ell_{\min}$ in the second round. However, such a strategy will not be beneficial to the borrowers, even under the assumption that the first-round screening results are verifiable to the screening banks in the second-round. The reason is that, anticipating the banks in the second-round free-ride costly first-round screening results, no banks will incur such costs in the first place, hence no useful information will ever be produced in the first-round (see Anand and Galetovic 2000 for an elaboration on this point in a setting similar to the two-rounds screening considered here). Furthermore, under the assumption that the first-round results are unverifiable to the banks in the second-round, the argument for the approved borrowers not to choose applying for another loan is even simpler: being pooled with the denied borrowers in the second round, the approved borrowers gain nothing more than what they can get in the first-round.

[47]Models featuring sequential screening (each time by one bank) in similar setups include Direr (2008) and Gehrig and Stenbacka (2004, 2011).

Simultaneous Screening   Simultaneous screening by multiple banks offers yet another alternative to the benchmark setting. In such a setup, a borrower applies for loans from multiple banks and these banks screen the borrower simultaneously, after which the borrower chooses the most favorable loan from a bank that approves the borrower. As for the case of sequential screening, one needs to specify how such simultaneous screening works. There are two natural specifications to consider. The first one is where the screening results are perfectly correlated across banks. However, such a specification is at odds with the premise that each bank chooses the screening intensity of its own screening process. The second specification is where the screening results are independent across banks, conditional on the true type of a borrower.[48] Such a specification is compatible with variable and costly screening intensity choice. However, by making the stage game analogous to a common value auction game, it complicates significantly the equilibrium pricing strategy, letting alone the equilibrium screening strategy. More specifically, when the screening result is a discrete signal, only mixed strategy equilibria exist;[49] when the screening result is a continuous signal, pure strategy equilibria may exist but are typically intricate functions of the signal realization.[50]

V.D Monotonicity of Borrower's Choice

Lemma 6 of Section III.E shows that a good borrower always prefers a loan with lower interest rate over the efficient contract space. This result is a direct implication of Assumption 3, which states that the second order derivative of the convex screening cost function, $C''(q)$, is bounded from below by $2[\bar{\mu} - (1-\bar{\mu})\theta^b/\theta^g]\text{NPV}^g$ for all $q \in \mathbb{Q} = [1/2, 1]$. Moreover, Assumption 1 implies $\bar{\mu} - (1-\bar{\mu})\theta^b/\theta^g > 0$, therefore the lower bound on $C''(q)$ is not trivial. In this sense, Assumption 3 is a strong assumption.

[48] Such a conditional independent screening specification has been widely used in the literature: first introduced in the seminal work of Broecker (1990), and followed by Riordan (1993), Thakor (1996), Cao and Shi (2001), Dinç (2000), Ruckes (2004), Hauswald and Marquez (2003, 2006), Ogura (2006).

[49] This is the case for the models of Broecker (1990), Cao and Shi (2001), Ruckes (2004), and Hauswald and Marquez (2003, 2006). The relationship banking model studied by Rajan (1992) also features a unique mixed strategy equilibrium where an inside bank and outside bank simultaneously bid for a borrower. The only exception is Thakor (1996), where a unique pure strategy equilibrium emerges. However, Thakor assumes a particular extensive form of the lending game, allowing a bank to know how many other banks have obtained good signals on a borrower before making final lending decision. This effectively breaks down the "winner's curse" problem that underlies the nonexistence of pure strategy equilibria of the papers just mentioned.

[50] See Riordan (1993), Dinç (2000), and Ogura (2006).

However, it is possible to weaken Assumption 3 and still have a good borrower to prefer a lower interest rate. Over the efficient contract space $\mathscr{C}^e = \{(r, q^e(r))\}$, the expected payoff for a good borrower getting a loan $(r, q^e(r))$ is $U^g(r) = u^g(r)q^e(r) = [\theta^g(x - r)]q^e(r)$, thus

$$\frac{\mathrm{d}U^g(r)}{\mathrm{d}r} = -\theta^g q^e(r) + u^g(r)\frac{\bar{\mu}\theta^g - (1 - \bar{\mu})\theta^b}{C''(q^e(r))}.$$

For the validity of all the results of the benchmark setting, it suffices to have $\mathrm{d}U^g(r)/\mathrm{d}r < 0$ over $[r^0, x]$. Since $u^g(x) = 0$, it follows that $\mathrm{d}U^g(x)/\mathrm{d}r = -\theta^g q^e(x) < 0$, which suggests that $\mathrm{d}U^g(r)/\mathrm{d}r$ is always negative for r close to x. Consequently, for any calibration such that r^0 is close to x, $\mathrm{d}U^g(r)/\mathrm{d}r$ is guaranteed to be negative over $[r^0, x]$ even without Assumption 3.

VI Concluding Remarks

Two empirical regularities of bank lending practices stand out: interest rate spreads on loans and lending standards both are lower during booms than in recessions. I provide a unified explanation of these two facts, stressing procyclical competition of the banking sector as the driving force. I first develop a game of bank lending with screening. Borrowers have private information about the creditworthiness of their projects. Banks rely on a screening technology to distinguish good projects from bad ones by choosing the screening intensity, which I identify with the lending standards. Because screening is costly, in the optimum the screening intensities chosen by banks are always less than perfect. Moreover, the screening intensity, and hence the lending standards, determined in this way are positively correlated with the profitability on loans. Next, this lending game is repeated over time, and a bank's payoff is affected by various aggregate shocks which capture various aspects of the business cycle. I show that in the optimal subgame perfect equilibrium of this repeated game, better business conditions during booms increase bank's incentive to deviate ceteris paribus, thus forcing banks to compete more to shrink the profit margin and to restore the equilibrium incentive constraint. As a result, banks charge lower interest rates and impose looser standards during booms, while the opposite happens during recessions.

Appendix

A Proofs

Denote by Int$\mathcal{X}$ the interior of a set $\mathcal{X}$ and $\partial_x f(x,y)$ the partial derivative of a differentiable function $f(x,y)$ w.r.p. to x. The following lemma summarizes various properties of the benchmark screening technology, where $\nu^{\phi}(q,\mu) = \Pr(g|G,q,\mu)$ for $\phi \in \Phi = \{G,B\}$.

Lemma A.1. *$\forall q \in \mathbb{Q}$ and $0 < \mu < 1$ it follows that*

(*a*) *$\nu^{\phi}(q,\mu)$ is continuous, $\nu^{\phi}\left(\frac{1}{2},\mu\right) = \mu\ \forall \phi \in \Phi$, $\nu^{G}(1,\mu) = 1$, $\nu^{B}(1,\mu) = 0$;*

(*b*) *$\partial_q \nu^{G}(q,\mu) > 0$, $\partial_q \nu^{B}(q,\mu) < 0$, $\partial_\mu \nu^{\phi}(q,\mu) > 0\ \forall \phi \in \Phi$; and*

(*c*) *$1 > \nu^{G}(q,\mu) > \mu > \nu^{B}(q,\mu) > 0\ \forall q \in \text{Int}\mathbb{Q}$.*

And when $q < 1$ there is $\nu^{\phi}(q,1) = 1$ and $\nu^{\phi}(q,0) = 0\ \forall \phi \in \Phi$.

Proof. Part (*a*) is straightforward to verify, and part (*c*) follows from part (*b*). For (*b*), note that $\nu^{G}(q,\mu) = \mu/\left(\mu + (1-\mu)\frac{1-q}{q}\right)$. Since $(1-q)/q = 1/q - 1$ is decreasing in q, $\nu^{G}(q,\mu)$ is increasing in q. For $\nu^{B}(q,\mu)$, note that $q/(1-q)$ is increasing in q, so that $\nu^{B}(q,\mu) = \mu/\left(\mu + (1-\mu)\frac{q}{1-q}\right)$ is decreasing in q. Analogous reasoning proves that $\nu^{\phi}(q,\mu)$ is strictly increasing in μ. When $q < 1$, the result follows trivially from the expression of $\nu^{\phi}(q,\mu)$. *Q.E.D.*

Proof of Lemma 1. Consider first the case where $r > \underline{r}$ and $q < 1$. As $r > \underline{r}$, $\eta^{g}(r) > 0 > \eta^{b}(r)$; and as $q < 1$, $\nu^{\phi}(q,\mu)$ is strictly increasing in μ with $\nu^{\phi}(q,1) = 1$ and $\nu^{\phi}(q,0) = 0$. Therefore, $\eta^{\phi}(\ell,\mu)$ is also strictly increasing in μ, and $\eta^{\phi}(\ell,\mu)$ is negative for μ sufficiently close to 0 and non-negative for μ sufficiently close to 1, $\forall \phi \in \Phi$. Moreover, as $\nu^{G}(q,\mu) > \nu^{B}(q,\mu)$, it follows that for interim μ, $\eta^{G}(\ell,\mu) > 0 > \eta^{B}(\ell,\mu)$. When $r = \underline{r}$, then $\eta^{g}(r) = 0$, so that $\eta^{\phi}(\ell,\mu) < 0\ \forall \phi$, unless either $\mu = 1$ with $\eta^{\phi}(\ell,\mu) = 0$ or $q = 1$ and $\mu < 1$ with $\eta^{G}(\ell,\mu) = 0$ and $\eta^{B}(\ell,\mu) < 0$. Finally, when $q = 1$, $\eta^{G}(\ell,\mu) = \eta^{g}(r) \geq 0$ and $\eta^{B}(\ell,\mu) = \eta^{b}(r) < 0\ \forall r \geq \underline{r}$ and μ. *Q.E.D.*

Proof of Lemma 2. First note that as $r < x$ borrower's expected payoff is strictly positive whenever approved.

As an immediate implication, separating outcome is impossible because all bad borrowers will be denied as they are separated from good ones.

The remaining possibility is mixed outcome. To fix notation, suppose this outcome is associated with two loans $\ell_j = (r_j, q_j)$ and common belief $\mu_j = \mu(\ell_j)$ for $j = 1,2$. Let $p_j^{\theta} = p^{\theta}(\ell_j)$ for $j = 1,2$

be corresponding approval probabilities. First I claim that it has to be the case where $p_1^g, p_2^g > 0$. Suppose on the contrary, $p_1^g = 0$, then no good borrower will choose ℓ_1. By the definition of mixed outcome, bad borrowers must choose ℓ_1 by positive probability, but then this implies $\mu_1 = 0$, which leads all banks to deny any borrower at ℓ_1, i.e., $p_1^b = 0$. Yet by assumption $q_2 < 1$, thus $p_2^b = 1 - q > 0$ or 1, so that all bad borrowers applies only for ℓ_2. This contradicts with ℓ_1, ℓ_2 be associated with a mixed outcome.

Since good borrowers are indifferent between ℓ_1 and ℓ_2, $u^g(r_1)p_1^g = u^g(r_2)p_2^g > 0$. Without loss of generality, suppose $r_2 < r_1$. Then $p_2^g < p_1^g$ as $r_2 < r_1$ implies $u^g(r_2) > u^g(r_1)$. Accordingly, there are two cases to consider.

Case 1: $p_1^g = 1$. It follows that $p_1^b = 1$, $p_2^g = q_2 < 1$, and $p_2^b = 1 - q_2 > 0$ in this case. For this to be an equilibrium outcome, it is necessary for bad borrowers to be indifferent, i.e., $u^b(r_2)(1 - q_2) = u^b(r_1)$. Also, $u^g(r_2)q_2 = u^g(r_1)$ for good borrowers. Rearranging these two equalities yields

$$\frac{x - r_1}{x - r_2} = 1 - q_2 < q_2 = \frac{x - r_1}{x - r_2},$$

as $q_2 > \frac{1}{2}$. This is clearly impossible.

Case 2: $p_1^g = q_1 < 1$. It follows that $p_1^b = 1 - q_1 > 0$, $p_2^g = q_2 < q_1$, and $p_2^b = 1 - q_2 > 1 - q_1$. Indifference between ℓ_2 and ℓ_1 implies $u^b(r_2)(1 - q_2) = u^b(r_1)(1 - q_1)$. This is impossible since $u^\theta(r_2) > u^\theta(r_1)\ \forall\theta$ and $1 - q_2 > 1 - q_1 > 0$. *Q.E.D.*

Remark. As noted in footnote 17, I assume that banks do not randomize between approval and denial when $\eta^B(\ell) = 0$, or equivalently, banks do not randomize between $p^\theta = 1$, $p^g = q = 1 - p^b$, and $p^\theta = 0$. This allows me to write down borrower's incentive constraint as $u^\theta(r_1)p_1^\theta = u^\theta(r_2)p_2^\theta$, and the result follows. Without this presumption, mixed equilibrium outcome exists as randomization of bank's lending decision convexifies borrower's payoff.

Proof of Lemma 3. Let $\nu^\phi(q) = \nu^\phi(q, \bar{\mu})$, and $\eta^\phi(r, q) = \nu^\phi(q)\eta^g(r) + (1 - \nu^\phi(q))\eta^b(r)$. For $r = \underline{r}$ and $\eta^g(r) = 0$, there is $\eta^\phi(r, q) < 0\ \forall q$ and ϕ except for $\eta^G(1) = 0$. Thus $\eta(\underline{r}, q) = 0$ and $q^c(\underline{r}) = 1$.

For $r > \underline{r}$, $\eta^g(r) > 0 > \eta^b(r)$. If r is such that $\bar{\eta}(r) \geq 0$, monotonicity of $\nu^\phi(\cdot)$ implies that $\eta^G(r, q) \geq \bar{\eta}(r) \geq 0$ over $\mathbb{Q}$ and there is a unique value of $q^c(r) < 1$ such that $\eta^B(r, q) < 0$ iff $q > q^c(r)$. As a result, for $q \leq q^c(r)$, no borrower is denied, hence $\eta(r, q) = \bar{\eta}(r)$; and for $q \geq q^c(r)$, B borrowers are denied so that

$$\eta(r, q) = \Pr(G)\eta^G(r, q) = \Delta(r)q + (1 - \bar{\mu})\eta^b(r).$$

In contrast, if r is such that $\bar{\eta}(r) < 0$, monotonicity of $\nu^{\phi}(\cdot)$ implies that $\eta^B(r,q) \le \bar{\eta}(r) < 0$ over $\mathbb{Q}$ and there is a unique value of $q^c(r) < 1$ such that $\eta^G(r,q) \ge 0$ iff $q \ge q^c(r)$. As a result, for $q < q^c(r)$, no borrower is approved, hence $\eta(r,q) = 0$; and for $q \ge q^c(r)$, G borrowers are approved, so that again $\eta(r,q) = \Delta(r)q + (1-\bar{\mu})\eta^b(r)$. *Q.E.D.*

Proof of Lemma 4. To circumvent complications due to the kink of $\eta(r,q)$, let

$$\tilde{\eta}(r,q) = \Delta(r)q + (1-\bar{\mu})\eta^b(r),$$
$$\tilde{\pi}(r) = \max_{q\in\mathbb{Q}} \tilde{\eta}(r,q) - C(q).$$

It is easily verified that $\pi(r) = \max\{\tilde{\pi}(r), \bar{\eta}(r), 0\}$. $\forall r \in \mathcal{R}$, $\tilde{\eta}(r,q)$ is linear in q and $C(q)$ is strictly convex, so that $\varpi(r,q) \equiv \tilde{\eta}(r,q) - C(q)$ is strictly concave in q. As $\Delta(r) > 0$, $\partial_q \varpi(r, \frac{1}{2}) = \Delta(r) > 0$. Moreover, $\Delta'(r) = \bar{\mu}\theta^g - (1-\bar{\mu})\theta^b > 0$ by part (*ii*) of Assumption 1, so that $\Delta(r)$ is strictly increasing in r, hence $\Delta(r) \le \Delta(x)$, which implies $\partial_q \varpi(r,1) = \Delta(r) - C'(1) \le \Delta(x) - C'(1) < 0$ by Assumption 2. As a result, first order condition associated with $\tilde{\pi}(r)$ holds as equality

$$\Delta(r) = C'(q^e).$$

This in turn defines $q^e(r) = (C')^{-1}(\Delta(r))$, which is increasing as both $C'(\cdot)$ and $\Delta(\cdot)$ are increasing function. Observe that $0 = C'\left(\frac{1}{2}\right) < \Delta(\underline{r}) \le \Delta(r) \le \Delta(x) < C'(1)$, it follows that $\frac{1}{2} < q^e(r) < 1$.

By the Envelope theorem, $\tilde{\pi}'(r) = \bar{\mu}q^e(r)\theta^g + (1-\bar{\mu})(1-q^e(r))\theta^b > 0$. Since $\frac{1}{2} < q^e(r) < 1$, $\bar{\eta}'(r) = \bar{\mu}\theta^g + (1-\bar{\mu})\theta^b > \tilde{\pi}(r)$. Combining with $\tilde{\pi}(x) \ge \eta(x,1) - C(1) > \max\{\bar{\eta}(x), 0\} \ge \bar{\eta}(x)$ by Assumption 2, this then implies that $\tilde{\pi}(r) > \bar{\eta}(r)$ over $\mathcal{R}$. As a consequence, $\pi(r) = \max\{\tilde{\pi}(r), 0\}$. Clearly, $\tilde{\pi}(r)$ is continuous and strictly increasing with $\tilde{\pi}(x) > 0$ and $\tilde{\pi}(\underline{r}) < 0$ as $\tilde{\eta}(\underline{r}, q) \le \tilde{\eta}(\underline{r}, 1) = 0$, therefore there exists a unique $r^0 \in \text{Int}\mathcal{R}$ such that $\tilde{\pi}(r^0) = 0$. If $r < r^0$, $0 = \pi(r) = \max_q \eta(r,q) - C(q)$, which is achieved at $q = \frac{1}{2}$ as $C\left(\frac{1}{2}\right) = 0$. If $r > r^0$, $\pi(r) = \tilde{\pi}(r)$ with the maximizing $q = q^e(r)$. If $r = r^0$, both $q = \frac{1}{2}$ and $q^e(r^0)$ achieve $\pi(r^0) = 0$. *Q.E.D.*

Proof of Lemma 5. For a loan contract $\ell = (r,q)$ with $r \ge r^0$ and $q = q^e(r) > \frac{1}{2}$, unit payoff from lending is

$$\eta(\ell) > \eta(\ell) - C(q) = \pi(r) > \max\{\bar{\eta}(r), 0\},$$

thus all B borrowers are denied credit as argued in the proof of Lemma 3 above. *Q.E.D.*

Proof of Lemma 6. The derivative of $U^g(r)$ is

$$\frac{\mathrm{d}U^g(r)}{\mathrm{d}r} = -\theta^g q^e(r) + u^g(r)\frac{\bar{\mu}\theta^g - (1-\bar{\mu})\theta^b}{C''(q^e(r))}.$$

Clearly, $\mathrm{d}U^g(r)/\mathrm{d}r < 0$ is equivalent to $C''(q^e(r)) > \big(\bar{\mu} - (1-\bar{\mu})\theta^b/\theta^g\big)u^g(r)/q^e(r)$. Since $q^e(r) > \frac{1}{2}$, the RHS of the last inequality is less than $2\big(\bar{\mu} - (1-\bar{\mu})\theta^b/\theta^g\big)u^g(r)$, which in turn is less than $2\big(\bar{\mu} - (1-\bar{\mu})\theta^b/\theta^g\big)\mathrm{NPV}^g$ as $u^g(r) \leq u^g(\underline{r}) = \mathrm{NPV}^g$, therefore Assumption 3 ensures that $\mathrm{d}U^g(r)/\mathrm{d}r < 0$. *Q.E.D.*

Proof of Lemma 7. Suppose $\{v^*(s)\}$ solves $\mathcal{P}$ and $V^* = \mathbb{E}v^*(s)$. I show first that $V^* = \mathcal{B}(V^*)$. On the one hand, since $V^* = \max \mathbb{E}v(s)$ subject to all constraints of $\mathcal{P}$, any point $\{v(s)\}$ in the constrained set of $\mathcal{P}$ satisfies $\chi(N,\delta)v(s) \leq \mathbb{E}v(s) \leq V^*$ and therefore belongs to the constrained set of $\mathcal{P}_{V^*}$, which implies that $V^* \leq \mathcal{B}(V^*)$. On the other hand, let $\{v^{**}(s)\}$ denote the solution of $\mathcal{P}_{V^*}$, then it satisfies $\chi(N,\delta)v^{**}(s) \leq V^* \leq \mathcal{B}(V^*) = \mathbb{E}v^{**}(s)$, which implies that $\{v^{**}(s)\}$ belongs to the constrained set of $\mathcal{P}$ and consequently $\mathcal{B}(V^*) \leq V^*$. I thereby conclude that V^* is a fixed point of $\mathcal{B}(\cdot)$. Moreover, it follows that V^* has to be the maximum fixed point, for otherwise a bigger fixed point of $\mathcal{B}(\cdot)$ solves $\mathcal{P}$ as well, thus contradicting the optimality of V^*.

Next, observe that the solution of $\mathcal{P}$ is unique as long as the solution of $\mathcal{P}_{V^*}$ is also unique. When $\mathcal{S}$ is a discrete set and the probability of each $s \in \mathcal{S}$ is positive, it is fairly evident that $\mathcal{P}_w$ $\forall w \in [0, \max_s \bar{v}(s)]$ has a unique solution: for $w > 0$, $\mathbb{E}v(s)$ is maximized at the extreme point $\{v^*(s)\}$ of the constrained set of $\mathcal{P}_w$ — a rectangular box — with $v^*(s) > 0$ $\forall s$; and for $w = 0$, $v^*(s) = 0$ $\forall s$. The same intuition holds when $\mathcal{S}$ is a continuum set with positive distribution over $\mathcal{S}$ and the solution of $\mathcal{P}_w$ is required to be continuous. A rigorous proof of this result is more involved and is omitted to save space. Also I omit the proof for the existence of a solution to both $\mathcal{P}$ and $\mathcal{P}_w$. Liu (2014) contains the omitted proofs for both results under assumptions more general than the ones imposed for this lemma. *Q.E.D.*

Proof of Proposition 1. Let $\mathcal{V}_f = [0, z_h\pi^m] \times [0, z_l\pi^m]$ denote the feasible set of (v_h, v_l). By Lemma 7, solving for the optimal SSPE is equivalent to finding the maximum fixed point of $\mathcal{B}(w)$ over $[0, z_h\pi^m]$ defined by the linear program $\mathcal{P}_w$:

$$\mathcal{B}(w) = \max_{(v_h,v_l)\in\mathcal{V}_f} \gamma_h v_h + \gamma_l v_l$$

$$\text{s.t.} \quad \chi(N,\delta)v_h \leq w \quad \text{and} \quad \chi(N,\delta)v_l \leq w.$$

Note that $\gamma_h z_h \pi^m + \gamma_l z_l \pi^m = \pi^m$ as $\gamma_h z_h + \gamma_l z_l = 1$. Let V^* denote the maximum fixed point and (v_h^*, v_l^*) denote the solution of $\mathcal{P}_{V^*}$.

(a) $\chi(N,\delta) \leq 1/z_h$. It suffices to consider $w \in [\pi^m, z_h\pi^m]$. Since $z_l < 1 < z_h$ and $\chi(N,\delta)z_h \leq 1$, $\chi(N,\delta)v_s \leq w$ is satisfied $\forall v_s \in [0, z_s\pi^m]$ and $s = h, l$. As $\mathcal{B}(w) = \pi^m$, the maximum fixed point is

$V^* = \pi^m$ and $v_h^* = z_h \pi^m$ and $v_l^* = z_l \pi^m$. Under the one-to-one mapping of $z_s \pi(\ell_s) = v_s$, it follows that $\ell_h^* = \ell_l^* = \ell^m$.

(*b*) $1/z_h < \chi(N, \delta) \leq 1$. It suffices to consider $w \in [z_l \pi^m, \pi^m]$. Since $z_h \chi(N, \delta) > 1$, $\chi(N, \delta) v_h \leq w$ is a binding constraint, and $v_h^* = w/\chi(N, \delta)$. Meanwhile, $z_l \chi(N, \delta) \leq z_l$, thus $v_l^* = z_l \pi^m$. It follows that $\mathcal{B}(w) = \gamma_h w/\chi(N, \delta) + \gamma_l z_l \pi^m$, and therefore the maximum fixed point is $V^* = \frac{\gamma_l z_l}{1-\gamma_h/\chi(N,\delta)} \pi^m$, with $v_h^* = \frac{\gamma_l}{\chi(N,\delta)-\gamma_h} z_l \pi^m$ and $v_l^* = z_l \pi^m$.

Under the one-to-one mapping of $z_s \pi(\ell_s) = v_s$, it follows that the $\ell_l^* = \ell^m$ and ℓ_h^* satisfies $r^0 < r_h^* < r^m$ and $q^0 = q^e(r^0) < q_h^* = q^e(r_h^*) < q^m = q^e(r^m)$, as $q^e(r)$ is strictly increasing in r by Lemma 4.

(*c*) $1 < \chi(N, \delta)$. Since $v_s \leq w/\chi(N, \delta) < w$, it follows that $\mathcal{B}(w) < w$ $\forall w > 0$. Thus the only fixed point is $V^* = 0$ with $v_s^* = 0$. Consequently $\ell_s^* = \ell_s^0$ $\forall s$. *Q.E.D.*

Proof of Proposition 2. This is a simple corollary of part (*b*) of the previous proposition. In the proof for that part I showed that $\frac{\gamma_l z_l}{\chi(N,\delta)-\gamma_h} \pi^m$. Substituting out $\gamma_l z_l = 1 - \gamma_h z_h = 1 - \bar{z}$ yields

$$v_h^* = \frac{1 - \bar{z}}{\chi(N, \delta) - \bar{z}/z_h} \pi^m,$$

and from the one-to-one mapping $z_h \pi_h^* = v_h^*$, it follows that

$$\pi_h^* = \frac{1 - \bar{z}}{z_h \chi(N, \delta) - \bar{z}} \pi^m.$$

By assumption, $z_h \chi(N, \delta) > 1$, therefore $0 < \pi(\ell_h^*) < \pi^m$. Moreover, as $\pi(\ell^e) = \pi(r, q^e(r))$ is strictly increasing over $[r^0, x]$ with a range of $[0, \pi^m]$, the above equation determines a unique $\ell_h^* \in \mathscr{C}^e$ such that $\pi(\ell_h^*) = \pi_h^*$. Lastly, as $\chi(N, \delta)$, $\bar{z}$, and π^m are all fixed, $\lim_{z_h \to \infty} \pi_h^* = 0$, and since $\pi(\ell^e) = 0$ only at ℓ^0, there is $\lim_{z_h \to \infty} \ell_h^* = \ell^0$. *Q.E.D.*

Proof of Proposition 3. Denote $\mathscr{V}_f = [0, \pi_h^m] \times [0, \pi_l^m]$ the feasible set of (v_h, v_l). Following Lemma 7, the associated linear program $\mathscr{P}_w$ is

$$\mathcal{B}(w) = \max_{(v_h, v_l) \in \mathscr{V}_f} \gamma_h v_h + \gamma_l v_l$$

$$\text{s.t.} \quad \chi(N, \delta) v_h \leq w \quad \text{and} \quad \chi(N, \delta) v_l \leq w,$$

$\forall w \in [0, \pi_h^m]$. This is almost identical to the one analyzed in the proof of Proposition 1 (p.44). Thus a similar procedure results in the characterization of $\pi(\ell_s^*; c_s)$ $\forall s = h, l$ stated in Proposition 3. When $\bar{\pi}^m/\pi_h^m < \chi(N, \delta) \leq 1$, $r_l^* = x > r_h^* > r_h^0$ follows directly from $0 < \pi(r_h^*; c_h) < \pi(x; c_h)$ and $\pi(r; c_h)$ is increasing in r. *Q.E.D.*

Proof of Lemma 8. Let $\mathscr{A} = \{(r, c) | r^0(c) \le r \le x, c \in [c_l, c_h]\}$ denote the region over which $\pi(\ell; c) = \pi(r; c)$ is defined. As showed in the text already, $\partial_c \pi(r; c) > 0$, so that $r_1 > r_2$ follows if $\pi(r_1; c_1) = \pi(r_2; c_2)$ as $c_1 < c_2$.

Moreover, if $\partial_c \Delta(r; c) = \bar{\mu}(1 - \theta^g) - (1 - \bar{\mu})(1 - \theta^b) \le 0$, then $\partial_c q^e(r; c) \le 0$. Since $r_1 > r_2$ and $c_1 < c_2$, it follows that $q_1 = q^e(r_1; c_1) > q_2 = q^e(r_2; c_2)$. Thus the only case needs a proof is $\partial_c \Delta(r; c) > 0$.

Consider any level curve of $\pi(r; c)$ within $\mathscr{A}$. Along this curve, $\pi(r; c)$ is constant, so that the total differentiation $\mathrm{d}\pi(r; c) = \partial_r \pi(r; c)\mathrm{d}r + \partial_c \pi(r; c)\mathrm{d}c = 0$ and the derivative at any point along this curve is given by

$$\frac{\mathrm{d}r}{\mathrm{d}c} = -\frac{\partial_c \pi}{\partial_r \pi} = -\frac{\bar{\mu} q^e (1 - \theta^g) + (1 - \bar{\mu})(1 - q^e)(1 - \theta^b)}{\bar{\mu} q^e \theta^g + (1 - \bar{\mu})(1 - q^e)\theta^b} < 0,$$

where $q^e = q^e(r; c)$. Fix a point (r_0, c_0) on this curve. Define a function $\Gamma_0(r; c) = \Delta(r; c) - \Delta(r_0; c_0)$ over $\mathscr{A}$. Then equation $\Gamma_0(r, c) = 0$ defines a line segment within $\mathscr{A}$ according $r = \kappa(c - c_0) + r_0$, where

$$\kappa = -\frac{\partial_c \Delta}{\partial_r \Delta} = -\frac{\bar{\mu}(1 - \theta^g) - (1 - \bar{\mu})(1 - \theta^b)}{\bar{\mu}\theta^g - (1 - \bar{\mu})\theta^b} < 0$$

as $\bar{\mu}(1 - \theta^g) - (1 - \bar{\mu})(1 - \theta^b) > 0$.

I claim that $\partial_c \pi / \partial_r \pi > \partial_c \Delta / \partial_r \Delta$. For this, observe that

$$\frac{\partial_c \pi}{\partial_r \pi} - \frac{\partial_c \Delta}{\partial_r \Delta} = \frac{\partial_c \pi \partial_r \Delta - \partial_r \pi \partial_c \Delta}{\partial_r \pi \partial_r \Delta},$$

of which the numerator is positive, therefore I only need to show that the denominator is positive. Some tedious algebra confirms that the denominator equals to $\bar{\mu}(1 - \bar{\mu})(\theta^g - \theta^b)$ which is indeed positive as $\theta^g > \theta^b$.

It follows from the claim that $\mathrm{d}r/\mathrm{d}c < \kappa < 0$. So the level curve going through (r_0, c_0) decreases faster than the line segment defined by $\Gamma_0(r, c)$ as $c \ge c_0$ increases. In particular, as (r_1, c_1) and (r_2, c_2) are on the same level curve, it follows that $r_2 < r_3$ where r_3 is such that $\Gamma_1(r, c) = \Delta(r; c) - \Delta(r_1; c_1) = 0$, and hence $\Gamma_1(r_2, c_2) < 0$ as $\Gamma_1(r, c_2)$ is increasing in r. As a result, $\Delta(r_2, c_2) < \Delta(r_1, c_1)$. Since $q^e(r; c) = (C')^{-1}(\Delta(r; c))$, I conclude that $q_1 = q^e(r_1; c_1) > q_2 = q^e(r_2; c_2)$. *Q.E.D.*

Proof of Proposition 4. When $\bar{\mu} \le \frac{1-\theta^b}{2-\theta^g-\theta^b}$, $\partial_c \Delta(r; c) \le 0$, and by $\partial_r \Delta(r; c) > 0$, it follows that $\Delta(r_h^*; c_h) < \Delta(r_l^*; c_l)$ as $r_h^* < r_l^* = x$. Since $q^e(r; c) = (C')^{-1}(\Delta(r; c))$ is increasing in Δ, $q_h^* = q^e(r_h^*; c_h) < q^e(r_l^*; c_l) = q_l^*$.

When $\bar{\mu} > \frac{1-\theta^b}{2-\theta^g-\theta^b}$, $\partial_c \Delta(r;c) > 0$. Thus the line segment defined by $0 = \Gamma_l(r,c) = \Delta(r;c) - \Delta(r_l^*;c_l)$ within $\mathscr{A}$ (see the proof of Lemma 8 for the notation) is downward sloping, goes through $(r_l^*;c_l) = (x;c_l)$, and intersects with line $c = c_h$ at a unique point $(\hat{r}, c_h)$ with $\hat{r} = x - \kappa(c_h - c_l) < x$. From the proof of Lemma 8, it is clear that the unique r_h^l such that $\pi(r_h^l;c_h) = \pi(x;c_l)$ is smaller than $\hat{r}$.

Following the same procedure as in the proof of Proposition 1, it can be established that

$$\pi(\ell_h^*;c_h) = \pi(r_h^*;c_h) = \frac{\gamma_l}{\chi(N,\delta) - \gamma_h}\pi_l^m.$$

As $\chi(N,\delta)$ varies over $[\bar{\pi}^m/\pi_h^m, 1]$, $\pi(r_h^*;c_h)$ ranges over $[\pi_l^m, \pi_h^m]$, and consequently r_h^* has a range of $[r_h^l, x]$ as $\pi(r;c_h)$ is strictly increasing in r. Given that $r_h^l < \hat{r} < x$, $\Delta(r_h^*;c_h) > \Delta(\hat{r};c_h) = \Delta(x;c_l)$ if $r_h^* > \hat{r}$, $\Delta(r_h^*;c_h) = \Delta(\hat{r};c_h) = \Delta(x;c_l)$ if $r_h^* = \hat{r}$, and $\Delta(r_h^*;c_h) < \Delta(\hat{r};c_h) = \Delta(x;c_l)$ if $r_h^* < \hat{r}$, which results in the desired characterization of q_h^* and q_l^*.

Lastly, as $\chi(N,\delta)$ is sufficiently close to 1, $\pi(r_h^*;c_h)$ is close to π_l^m, and thereby r_h^* is close to r_h^l and smaller than $\hat{r}$. As a result, q_h^* is always smaller than q_l^*. *Q.E.D.*

Proof of Proposition 5. Denote $\mathscr{V}_f = [0, \pi_h^m] \times [0, \pi_l^m]$ the feasible set of (v_h, v_l). Following Lemma 7, the associated linear program $\mathscr{P}_w$ is

$$\mathcal{B}(w) = \max_{(v_h,v_l)\in\mathscr{V}_f} \gamma_h v_h + \gamma_l v_l$$
$$\text{s.t.} \quad \chi(N,\delta)v_h \le w \quad \text{and} \quad \chi(N,\delta)v_l \le w,$$

$\forall w \in [0, \pi_h^m]$. The characterization of $\pi(\ell_s^*)$ and r_s^* $\forall s = h,l$ follows the same procedure as in the proof of Proposition 1. In particular, $r_h^* < r_l^* = x$ when $\bar{\pi}^m/\pi_h^m < \chi(N,\delta) \le 1$ follows from $\partial_r \pi(r;\bar{\mu}_h) > 0$ and $\pi(r_h^*;\bar{\mu}_h) < \pi(x;\bar{\mu}_h) = \pi_h^m$.

To characterize q_s^*, let $\Delta(r;\bar{\mu}) = \bar{\mu}\eta^g(r) - (1-\bar{\mu})\eta^b(r)$ and observe that $\partial_r \Delta(r;\bar{\mu}) = \bar{\mu}\theta^g - (1-\bar{\mu})\theta^b > 0$ by assumption and $\partial_{\bar{\mu}} \Delta(r;\bar{\mu}) = \eta^g(r) + \eta^b(r) \le 0$ by the extra condition $\eta^g(x) + \eta^g(x) \le 0$. For $\chi(N,\delta)$ in any region, $r_h^* \le r_l^*$ always holds, therefore $q_h^* = q^e(r_h^*;\bar{\mu}_h) \le q^e(r_l^*;\bar{\mu}_l) = q_l^*$ holds as $q^e(r;\bar{\mu}) = (C')^{-1}(\Delta(r;\bar{\mu}))$ and $\Delta(r_h^*;\bar{\mu}_h) \le \Delta(r_l^*;\bar{\mu}_l)$. In addition, strictly inequality $q_h^* < q_l^*$ holds except for the case where $r_s^* = x$ $\forall s = h,l$ and $\eta^g(x) + \eta^b(x) = 0$. *Q.E.D.*

Proof of Proposition 6. Let $\mathscr{V}_f = [0, \bar{v}_h] \times [0, \bar{v}_x] \times [0, \bar{v}_l]$ denote the feasible set of (v_h, v_x, v_l). Following Lemma 7, the associated linear program $\mathscr{P}_w$ is

$$\mathcal{B}(w) = \max_{(v_h,v_x,v_l)\in\mathscr{V}_f} \gamma_h v_h + \gamma_x v_x + \gamma_l v_l$$
$$\text{s.t.} \quad \chi(N,\delta)v_h \le w, \quad \chi(N,\delta)v_x \le w, \quad \text{and} \quad \chi(N,\delta)v_l \le w,$$

$\forall w \in [0, \bar{v}_h]$. Let V^* denote the maximum fixed point of $\mathcal{B}(w)$ and (v_h^*, v_x^*, v_l^*) denote the solution of $\mathcal{P}_{V^*}$.

(*a*) $\chi(N,\delta) \leq \bar{v}/\bar{v}_h$. It suffices to consider $w \in [\bar{v}, \bar{v}_x]$. As $\bar{v}_h > \bar{v}_x > \bar{v}_l$, it follows that $\chi(N,\delta)v_s \leq \chi(N,\delta)\bar{v}_s \leq \bar{v} \leq w$, so that $V^* = \bar{v}$ is the maximum fixed point of $\mathcal{B}(w)$ and $v_s^* = \ell_s^m \ \forall s$.

(*b*) $\bar{v}/\bar{v}_h < \chi(N,\delta) \leq [(\xi_h + \xi_x)\bar{v}_x + \xi_l\bar{v}_l]/\bar{v}_x$. It suffices to consider $w \in [(\xi_h + \xi_x)\bar{v}_x + \xi_l\bar{v}_l, \bar{v}]$. As $\bar{v}/\bar{v}_h < \chi(N,\delta)$, $\chi(N,\delta)v_h \leq w$ is a binding constraint, so that $v_h^* = w/\chi(N,\delta)$. Meanwhile, since $\bar{v}_l < \bar{v}_x$, $\chi(N,\delta) \leq [(\xi_h + \xi_x)\bar{v}_x + \xi_l\bar{v}_l]/\bar{v}_x$, and $w \geq [(\xi_h + \xi_x)\bar{v}_x + \xi_l\bar{v}_l]$, there is $v_x^* = \bar{v}_x$ and $v_l^* = \bar{v}_l$. As a result, $\mathcal{B}(w) = \xi_h w/\chi(N,\delta) + \xi_x\bar{v}_x + \xi_l\bar{v}_l$, and the V^* is determined by $V^* = \mathcal{B}(V^*)$. Solving for this equation, I obtain $V^* = \frac{\xi_x\bar{v}_x + \xi_l\bar{v}_l}{1-\xi_h/\chi(N,\delta)}$, and accordingly $v_h^* = \frac{\xi_x\bar{v}_x+\xi_l\bar{v}_l}{\chi(N,\delta)-\xi_h}$.

It can be easily verified that $v_h^* < \bar{v}_h$ as $\bar{v}/\bar{v}_h < \chi(N,\delta)$. The same reasoning as for Proposition 1 establishes that $r_h^* < r_x^* = x$ and $q_h^* < q_x^*$. Moreover, since $\bar{\mu} \leq \frac{1-\theta^b}{2-\theta^g-\theta^b}$, the same reasoning as for Proposition 4 establishes that $q_x^* < q_l^*$. This completes the proof for part (*b*).

(*c*) $[(\xi_h + \xi_x)\bar{v}_x + \xi_l\bar{v}_l]/\bar{v}_x < \chi(N,\delta) \leq 1$. It suffices to consider $w \in [\bar{v}_l, (\xi_h + \xi_x)\bar{v}_x + \xi_l\bar{v}_l]$. As $[(\xi_h + \xi_x)\bar{v}_x + \xi_l\bar{v}_l]/\bar{v}_x < \chi(N,\delta)$, $\chi(N,\delta)v_s \leq w$ is binding for $s = h, x$, thus $v_s^* = w/\chi(N,\delta)$. Meanwhile, as $\chi(N) \leq 1$ and $w \geq \bar{v}_l$, there is $v_l^* = \bar{v}_l$. It follows that $\mathcal{B}(w) = (\xi_h + \xi_x)w/\chi(N,\delta) + \xi_l\bar{v}_l$, and V^* satisfies $V^* = \mathcal{B}(V^*)$. As a result, $V^* = \frac{\xi_l\bar{v}_l}{1-(\xi_h+\xi_x)/\chi(N,\delta)}$, and accordingly $v_h^* = v_x^* = \frac{\xi_l\bar{v}_l}{\chi(N,\delta)-(\xi_h+\xi_x)}$.

Under the one-to-one mapping $v_s = z_s\pi(\ell_s; c_s)$, it follows that $\pi(\ell_h^*; c_h) < \pi(\ell_x^*; c_h)$ as $v_h^* = v_x^*$. Therefore the same reasoning as for Proposition 1 establishes that $r_h^* < r_x^*$ and $q_h^* < q_x^*$. Moreover, it can be easily verified that $v_x^* < \bar{v}_x < \bar{v}_h$ as $[(\xi_h + \xi_x)\bar{v}_x + \xi_l\bar{v}_l]/\bar{v}_x < \chi(N,\delta)$, so that $r_x^* < r_l^* = x$. Lastly, since $\bar{\mu} \leq \frac{1-\theta^b}{2-\theta^g-\theta^b}$, the same reasoning as for Proposition 4 establishes that $q_x^* < q_l^*$. This completes the proof for part (*c*).

(*d*) $1 < \chi(N,\delta)$. The proof is almost identical to the one for Proposition 1. *Q.E.D.*

Proof of Proposition 7. Straightforward calculation shows that

$$(1-\delta P)^{-1} = \begin{bmatrix} 1-\delta(1-\beta) & \delta\alpha \\ \delta\beta & 1-\delta(1-\alpha) \end{bmatrix},$$

so that

$$(1-\delta)P(1-\delta P)^{-1} \equiv \begin{bmatrix} \zeta_h & \zeta_l \\ \xi_h & \xi_l \end{bmatrix} = \frac{1}{1-\delta\rho}\begin{bmatrix} 1-\delta\rho-\alpha & \alpha \\ \beta & 1-\delta\rho-\beta \end{bmatrix}.$$

Since $\alpha, \beta < \frac{1}{2}$ and $\rho = 1-\alpha-\beta > 0$, $1-\delta\rho-\alpha = (1-\alpha)(1-\delta)+\delta\beta > 0$, so is $1-\delta\rho-\beta$. Note that $\zeta_h + \zeta_l = \xi_h + \xi_l = 1$, thus $(1-\delta)P(1-\delta P)^{-1}$ is a transition matrix as well. Letting

$\mathcal{V}_f = [0, z_h \pi^m] \times [0, z_l \pi^m]$, I write the linear program associated with the optimal SSPE explicitly as

$$\max_{(v_h, v_l) \in \mathcal{V}_f} \gamma_h v_h + \gamma_l v_l$$

$$\text{s.t.} \quad \mathcal{V}_h : [\chi(N, \delta) - \zeta_h] v_h \leq \zeta_l v_l \quad \text{and} \quad \mathcal{V}_l : [\chi(N, \delta) - \xi_l] v_l \leq \xi_h v_h,$$

where $\mathcal{V}_h, \mathcal{V}_l \subset \mathcal{V}_f$ denote the constraint sets of the IIC in state h, l. For expositional purpose, I first prove part (*c*), followed by (*a*) and (*b*). To simplify notation, denote $\chi(N, \delta)$ by χ; accordingly, let $\kappa_h = (\chi - \zeta_h)/\zeta_l$ and $\xi_h/(\chi - \xi_l)$ whenever $\chi - \xi_l \neq 0$.

Part (*c*) $\chi > 1$. In this case, both κ_h and κ_l are positive. It is easy to see geometrically that for $\mathcal{V}_h \cap \mathcal{V}_l$ to contain points other than $(0, 0)$ the inequality $\kappa_h \leq \kappa_l$ must hold. However, this is impossible since $\chi > 1$ implies that $\kappa_h > 1 > \kappa_l$. Thus in this case $\mathcal{V}_h \cap \mathcal{V}_l = (0, 0)$, i.e., the constraint set is a singleton at the origin.

Part (*a*) $\chi \leq \bar{\chi}$. First I show in this case that $(z_h \pi^m, z_l \pi^m) \in \mathcal{V}_h$. Observe that this is true iff $\kappa_h \geq z_l / z_h$, which in turn is equivalent to

$$\chi \geq \zeta_h + \zeta_l z_l / z_h = 1 - \frac{\alpha}{1 - \delta\rho} \frac{z_h - z_l}{z_h}.$$

To see that the RHS of the last equality equals to $\bar{\chi}$, note that $\mathbb{E} z_t = 1$ implies $\beta = \alpha(1 - z_l)/(z_h - 1)$, so that $\rho = 1 - \alpha - \beta = 1 - \alpha(z_h - z_l)/(z_h - 1)$, i.e., $\alpha = (1 - \rho)(z_h - 1)/(z_h - z_l)$. It follows that

$$1 - \frac{\alpha}{1 - \delta\rho} \frac{z_h - z_l}{z_h} = 1 - \frac{1 - \rho}{1 - \delta\rho} \frac{z_h - 1}{z_h} = \bar{\chi}.$$

Next I show that $(z_h \pi^m, z_l \pi^m) \in \mathcal{V}_l$. If $\xi_l < \chi \leq \bar{\chi}$, then $\kappa_l > 1 > \kappa_h$ as $\chi < 1$; and if $\chi \leq \xi_l$, then the IIC in state l ceases to be binding and $\mathcal{V}_l = \mathcal{V}_f$. Thus in both cases I have $(z_h \pi^m, z_l \pi^m) \in \mathcal{V}_l$, which is also the maximizing point.

Part (*b*) $\bar{\chi} < \chi \leq 1$. As in part (*a*), if $\chi > \xi_l$, then $\kappa_l \geq 1 \geq \kappa_h$ as $\chi \leq 1$; and if $\chi \leq \xi_l$, then $\mathcal{V}_l = \mathcal{V}_f$. Therefore the only binding constraint is the IIC in state h. It follows that the program is solved at $v_l^* = z_l \pi^m$ and $v_h^* = z_l \pi^m \zeta_l / (\chi - \zeta_h)$. Lastly, all comparative statics regarding r_s^* and q_s^* follows directly from the results in Proposition 1. *Q.E.D.*

Online Appendix (for online publication)

B Non-emptiness of the Parameter Space

In this subsection, I provide a simple sufficient condition for the existence of a screening cost function $C(\cdot)$ such that both Assumption 2 and 3 are satisfied.

Consider a cost function $C(\cdot)$ which is twice continuously differentiable with $C\left(\frac{1}{2}\right) = C'\left(\frac{1}{2}\right) = 0$ and $C'(\cdot) \geq 0$, $C''(\cdot) > 0$ over $\mathbb{Q}$. Suppose that $C_0 \leq C''(\cdot) \leq C_1$ over Q. I shall find a sufficient condition for the existence of $C_0 \leq C_1$ such that whenever $C''(\cdot)$ satisfies the lower and the upper bound C_0, C_1, it satisfies Assumption 2 and 3. In the process, I always assume that Assumption 1 holds.

First, simple calculus shows that

$$\frac{1}{2}C_0 \leq C'(1) = \int_{\frac{1}{2}}^{1} C''(x)\mathrm{d}x \leq \frac{1}{2}C_1,$$
$$\frac{1}{8}C_0 \leq C(1) = \int_{\frac{1}{2}}^{1}\int_{\frac{1}{2}}^{y} C''(x)\mathrm{d}x\mathrm{d}y \leq \frac{1}{8}C_1.$$

Thus, two sufficient conditions for Assumption 2 to hold are: (*i*) $C_1 \leq 8\min\{\bar{\mu}\eta^g(x), -(1-\bar{\mu})\eta^b(x)\}$, and (*ii*) $C_0 \geq 2[\bar{\mu}\eta^g(x) - (1-\bar{\mu})\eta^b(x)]$. Since $\eta^g(x) > 0 > \eta^b(x)$ by Assumption 1, it follows that $\bar{\mu}\eta^g(x) - (1-\bar{\mu})\eta^b(x) > [\bar{\mu} - (1-\bar{\mu})\theta^b/\theta^g]\eta^g(x)$. Consequently, Assumption 3 is satisfied whenever (*ii*) holds.

As a result, a sufficient condition for the required C_0, C_1 to exist is

$$\bar{\mu}\eta^g(x) - (1-\bar{\mu})\eta^b(x) \leq 4\min\{\bar{\mu}\eta^g(x), -(1-\bar{\mu})\eta^b(x)\},$$

or equivalently

$$\begin{cases} \bar{\mu}\eta^g(x) - (1-\bar{\mu})\eta^b(x) \leq 4\bar{\mu}\eta^g(x) \\ \bar{\mu}\eta^g(x) - (1-\bar{\mu})\eta^b(x) \leq 4(1-\bar{\mu})\eta^g(x). \end{cases}$$

This condition can be written more succinctly as

$$-3\frac{\bar{\mu}}{1-\bar{\mu}}\eta^g(x) \leq \eta^b(x) \leq -\frac{1}{3}\frac{\bar{\mu}}{1-\bar{\mu}}\eta^g(x),$$

with the interpretation that the net loss from a bad project should be within a comparable range of the net gain from a good project. I thereby conclude that whenever this condition is satisfied, the parameter space satisfying all assumptions is non-empty.

C Equilibrium Characterization under Risk Distribution Shock

For the case of the collateral value shock, Lemma 8 guarantees a relatively simple characterization of equilibrium screening intensity $q_s^* \; \forall s = h, l$. As made evident in the associated proof of that result, such a simple characterization is possible because there exists a simple relationship between the iso-profit curve and the level curve of the marginal benefit of screening. In contrast, I show here that such a simple relationship no longer exists for the risk distribution shock, unless the additional condition $\eta^g(x) + \eta^b(x) \leq 0$ is imposed.

Let $\mathcal{A} = \{(r, \bar{\mu}) | r^0(\bar{\mu}) \leq r \leq x, \bar{\mu} \in [\bar{\mu}_l, \bar{\mu}_h]\}$ denote the region over which $\pi(r; \bar{\mu})$ is defined. Fix a point $(r, \bar{\mu}) \in \mathcal{A}$ and consider the iso-profit curve through $(r, \bar{\mu})$ within in $\mathcal{A}$. Some algebra shows that the derivative of the iso-profit curve at $(r, \bar{\mu})$ equals to

$$\kappa_\pi = -\frac{\partial_{\bar{\mu}} \pi(r; \bar{\mu})}{\partial_r \pi(r; \bar{\mu})} = -\frac{q^e \eta^g - (1 - q^e)\eta^b}{\bar{\mu} q^e \theta^g + (1 - \bar{\mu})(1 - q^e)\theta^b} < 0,$$

where $q^e = q^e(r; \bar{\mu})$ and $\eta^\theta = \eta^\theta(r) \; \forall \theta = g, b$.

It can be showed that the derivative of the level curve of $\Delta(\cdot)$ at $(r, \bar{\mu})$ equals to

$$\kappa_\Delta = -\frac{\partial_{\bar{\mu}} \Delta(r; \bar{\mu})}{\partial_r \Delta(r; \bar{\mu})} = -\frac{\eta^g + \eta^b}{\bar{\mu}\theta^g - (1 - \bar{\mu})\theta^b}.$$

When $\eta^g(x) + \eta^b(x) \leq 0$, $\eta^g + \eta^b \leq 0$ for any r and $\kappa_\Delta \geq 0 > \kappa_\pi$. This implies that $\Delta(r'; \bar{\mu}') \leq \Delta(r; \bar{\mu})$ for any point $(r', \bar{\mu}')$ on the same iso-profit curve and to right of $(r, \bar{\mu})$, which leads to the characterization of q_s^* in Proposition 5. However, when $\eta^g(x) + \eta^b(x) > 0$, $\eta^g + \eta^b > 0$ for a range of r. In this case, there is no longer a simple relationship between κ_Δ and κ_π, as showed by the following equation

$$\kappa_\pi - \kappa_\Delta = \frac{(1 - \bar{\mu})\theta^b \eta^g + \bar{\mu}\theta^g \eta^b}{(\bar{\mu} q^e \theta^g + (1 - \bar{\mu})(1 - q^e)\theta^b)(\bar{\mu}\theta^g - (1 - \bar{\mu})\theta^b)}.$$

Assumption 1 requires that $(1 - \bar{\mu})\theta^b < \bar{\mu}\theta^g$, yet $\eta^g + \eta^b > 0$ implies $\eta^g > -\eta^b > 0$, so that the sign of $\kappa_\pi - \kappa_\Delta$ is undetermined. Accordingly, the change of $\Delta(r; \bar{\mu})$ along an iso-profit curve is undetermined as well. As a result, there is no simple characterization of q_s^* in this case.

D Technical Lemmas

Lemma D.1. *Suppose two random variables x and y satisfy following joint distribution*

Prob.	$\varphi\gamma_h$	$(1-\varphi)\gamma_h$	γ_l
x	x_h	x_h	x_l
y	y_h	y_l	y_l

where $\gamma_l, \gamma_h > 0,\ \gamma_h + \gamma_l = 1,\ \varphi \in [0,1],\ x_h > x_l$, and $y_h > y_l$. Then the correlation coefficient between x and y is

$$\rho = \rho(\varphi) \equiv \sqrt{\frac{\varphi\gamma_l}{1-\varphi\gamma_h}},$$

and $\rho(\varphi)$ is strictly increasing in $\varphi \in [0,1]$ with $\rho(0) = 0$ and $\rho(1) = 1$.

Proof. By definition, $\mathrm{cov}(x,y) = \mathbb{E}xy - \mathbb{E}x\mathbb{E}y$, so that

$$\begin{aligned}
\mathrm{cov}(x,y) &= \varphi\gamma_h x_h y_h + (1-\varphi)\gamma_h x_h y_l + \gamma_l x_l y_l - [\varphi\gamma_h y_h + (1-\varphi)\gamma_h y_l + \gamma_l y_l]\mathbb{E}x \\
&= [\varphi y_h + (1-\varphi)y_l]\gamma_h(x_h - \mathbb{E}x) + \gamma_l y_l(x_l - \mathbb{E}x) \\
&= \varphi(y_h - y_l)\gamma_h(x_h - \mathbb{E}x) + \gamma_h y_l(x_h - \mathbb{E}x) + \gamma_l y_l(x_l - \mathbb{E}x) \\
&= \varphi(y_h - y_l)\gamma_h(x_h - \mathbb{E}x) + y_l(\gamma_h x_h + \gamma_l x_l - \mathbb{E}x) \\
&= \varphi(y_h - y_l)\gamma_h(x_h - \mathbb{E}x).
\end{aligned}$$

By $\gamma_h x_h + \gamma_l x_l = \mathbb{E}x$, $x_l = (\mathbb{E}x - \gamma_h x_h)/\gamma_l$, and therefore $\mathrm{var}(x) = \mathbb{E}x^2 - (\mathbb{E}x)^2$ becomes

$$\begin{aligned}
\mathrm{var}(x) &= \gamma_h x_h^2 + \gamma_l x_l^2 - (\mathbb{E}x)^2 \\
&= \gamma_h x_h^2 + \tfrac{1}{\gamma_l}[\gamma_h^2 x_h^2 - 2\gamma_h x_h \mathbb{E}x + (\mathbb{E}x)^2] - (\mathbb{E}x)^2 \\
&= \tfrac{1}{\gamma_l}[\gamma_h\gamma_l x_h^2 + \gamma_h^2 x_h^2 - 2\gamma_h x_h \mathbb{E}x + (\mathbb{E}x)^2 - \gamma_l(\mathbb{E}x)^2] \\
&= \tfrac{\gamma_h}{\gamma_l}[x_h^2 - 2x_h\mathbb{E}x + (\mathbb{E}x)^2] = \tfrac{\gamma_h}{\gamma_l}(x_h - \mathbb{E}x)^2.
\end{aligned}$$

Moreover, since $\mathbb{E}y^2 = \varphi\gamma_h y_h^2 + (1-\varphi)\gamma_h y_l^2 + \gamma_l y_l^2 = \varphi\gamma_h(y_h^2 - y_l^2) + y_l^2$, $\mathrm{var}(y) = \mathbb{E}y^2 - (\mathbb{E}y)^2$ becomes

$$\begin{aligned}
\mathrm{var}(y) &= \varphi\gamma_h(y_h^2 - y_l^2) + y_l^2 - \varphi^2\gamma_h^2(y_h - y_l)^2 - 2\varphi\gamma_h y_l(y_h - y_l) - y_l^2 \\
&= \varphi\gamma_h(y_h - y_l)[y_h + y_l - \varphi\gamma_h(y_h - y_l) - 2y_l] \\
&= \varphi\gamma_h(y_h - y_l)^2(1 - \varphi\gamma_h).
\end{aligned}$$

As $\rho = \mathrm{cov}(x,y)/\sqrt{\mathrm{var}(x)\mathrm{var}(y)}$, it follows that

$$\rho(\varphi) = \frac{\varphi}{\sqrt{(1-\varphi\gamma_h)/\gamma_l}} = \sqrt{\frac{\varphi\gamma_l}{1-\varphi\gamma_h}}. \qquad \textit{Q.E.D.}$$

Lemma D.2. *Suppose $\{z_t\}$ is a two-state stationary Markov process with transition matrix*

$$\begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}.$$

Then the first order autocorrelation coefficient ρ equals to $1 - (\alpha + \beta)$

Proof. Let z_h and z_l denote the two states with stationary probability $\frac{\beta}{\alpha+\beta}$ and $\frac{\beta}{\alpha+\beta}$, and $\bar{z}$ denote $\mathbb{E}z_t$ under the stationary distribution. (I do not assume $\bar{z} = 1$.) It follows that

$$\begin{aligned} \operatorname{cov}(z_t, z_{t-1}) &= \frac{\alpha z_l^2 + \beta z_h^2 - \alpha\beta(z_h - z_l)^2 - (\alpha+\beta)\bar{z}^2}{\alpha+\beta}, \\ \operatorname{var}(z_t) &= \frac{\alpha z_l^2 + \beta z_h^2 - (\alpha+\beta)\bar{z}^2}{\alpha+\beta}. \end{aligned}$$

Since $\rho = \operatorname{cov}(z_t, z_{t-1})/\operatorname{var}(z_t)$, there is

$$\rho = 1 - \frac{\alpha\beta(z_h - z_l)^2}{\alpha z_h^2 + \beta z_l^2 - (\alpha+\beta)\bar{z}^2}.$$

Since $\alpha z_l = (\alpha+\beta)\bar{z} - \beta z_h$, the above expression becomes

$$\begin{aligned} \rho &= 1 - \frac{\beta(\alpha z_h - \alpha z_l)^2}{\alpha^2 z_l^2 + \alpha\beta z_h^2 - \alpha(\alpha+\beta)\bar{z}^2} \\ &= 1 - \frac{\beta(\alpha+\beta)^2(z_h - \bar{z})^2}{(\alpha+\beta)^2\bar{z}^2 - 2(\alpha+\beta)\beta\bar{z}z_h + \beta^2 z_h^2 + \alpha\beta z_h^2 - \alpha(\alpha+\beta)\bar{z}^2} \\ &= 1 - \frac{\beta(\alpha+\beta)(z_h - \bar{z})^2}{(\alpha+\beta)\bar{z}^2 - 2\beta\bar{z}z_h + \beta z_h^2 - \alpha\bar{z}^2} \\ &= 1 - \frac{\beta(\alpha+\beta)(z_h - \bar{z})^2}{\beta(z_h - \bar{z})^2} = 1 - (\alpha+\beta). \end{aligned}$$

Q.E.D.

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